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Attosecond Quantum Entanglement of Protons and Electrons in Condensed Matter – Neutron and Electron Compton Scattering

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Abstract. The "anomalous" scattering of neutrons and electrons from protons in the electron-volt energy range is described, and related experimental results are presented. Due to the very short characteristic scattering time, there is no well defined separation of time scales of electronic and protonic motions. An outline of a proposed theoretical interpretation is presented, which is based on the fact that scattering protons represent *open* quantum systems.

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1 Introductory Remarks

The counter-intuitive phenomenon of entanglement [1] between two or more quantum systems has emerged as the most emblematic feature of quantum mechanics. Experiments investigating entanglement, however, are mainly focused on collections of few simple (two- or three-level) quantum systems thoroughly isolated from their environment (e.g., atoms in high-Q cavities and optical lattices). These experimental conditions are necessary due to the decoherence of entangled states [2, 3]. In short, decoherence refers to the suppression of quantum superpositions caused by the environment.

By contrast, quantum entanglement (QE) in condensed and/or molecular matter at ambient conditions is usually assumed to be inaccessible experimentally. However, applying two new scattering techniques (NCS and ECS, see below) which operate in the sub-femtosecond time scale, we provided results indicating that short-lived entangled states may affect measurements in condensed matter even at room temperature [4–6].

The first direct experimental evidence of short-lived (that is, attosecond) QE involving protons in condensed matter was provided by means of the novel neutron Compton scattering (NCS) method. Starting in 1995, our experiments on

liquid water and H_2O-D_2O mixtures [4] have revealed the following striking effect: The intensity of neutrons scattered from protons exhibits a considerable "anomalous" shortfall which can be as much as 30%. This finding unequivocally contradicts conventional theory [4]. Further NCS experiments confirmed the existence of this effect in quite different condensed matter systems, e.g., urea dissolved in D_2O [7], metallic hydrides [8], polymers [5, 9], "soft condensed matter" [10], liquid benzene [9], and most recently in liquid H_2-D_2 mixtures and HD [11].

Additionally, the effect under consideration was independently confirmed using electron-proton Compton scattering (ECS) [5]. ECS-investigations from a solid polymer showed the same shortfall in scattered electrons from hydrogen nuclei, comparable to the shortfall of scattered neutrons in accompanying NCS experiments on the same polymer. Thus this effect was shown to be independent of the two fundamental interactions involved (i.e., the electromagnetic and strong interactions) [5, 6].

Here we present some experimental results which illustrate the aforementioned striking effect. In our theoretical considerations, short-lived QE and decoherence [2, 3] (especially, between a struck proton and its adjacent electrons) play a central role and represent our "working hypothesis", which a proposed theoretical interpretation is based on. This provides an extension of conventional neutron scattering theory (in which decoherence plays no role). The presented derivations also reveal the crucial importance of the very short (that is, attosecond) "scattering time" of the experiments.

2 Neutron and Electron Compton Scattering from Protons

The experimental method utilized here is neutron Compton scattering (NCS), provided by the electron-volt spectrometer Vesuvio (formerly eVS) at the ISIS neutron spallation source, Rutherford Appleton Laboratory, UK. This technique is particularly suitable for the investigation of short-lived QE since its "time window" (i.e., the scattering time τ_{sc}) lies in the attosecond range, see below. Taking full advantage of the high flux of high energy neutrons, this "inverse geometry" time of flight (TOF) instrument (see [12] for details) was originally constructed to measure directly the momentum distribution $n(p)$ of nuclei and is particularly useful for the investigation of H, D and He [13].

An example of a measured TOF spectrum of a solid polymer (formvar, monomer: $C_8O_2H_{14}$) is given in Figure 1. The high energy and momentum transfers applied,

$$\Delta E \approx 1 - 100 \ \text{eV} , \qquad |\mathbf{q}| \approx 30 - 200 \ \text{Å}^{-1} , \tag{1}$$

have an important consequence: The H and the joint C/O recoil peaks are well separated, permitting the direct determination of the ratio of peak areas of the

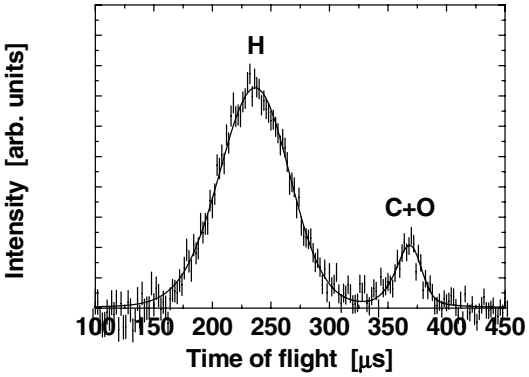

Figure 1. NCS TOF-spectrum of formvar (self-supporting foil, 0.1 mm thick) at scattering angle $\theta = 51.27^{\circ}$, corresponding to a mean momentum transfer (for the neutron-proton collision) with $q = 60.7$ Å^{-1}; taken from [5].

atoms in the sample. For an account in detail of data analysis and experimental procedures, see [12].

We now shortly discuss the striking decrease of NCS-intensity from H, which is represented by the violation of the basic equation $R_{exp} = R_{conv}$, see below. As an example, let us consider the scattering results [5] from formvar. From a measured TOF spectrum, the data analysis procedure [12] determines the double differential cross section $d^2\sigma/dEd\Omega$. The relevant peak areas A_X (with X=H, C, O) are then extracted from $d^2\sigma/dEd\Omega$. Thus one determines the ratio

$$R_{exp} \equiv A_H/(A_C + A_O). \tag{2}$$

According to standard NCS theory [13, 14] the conventionally expected value R_{conv} of this ratio is calculated with (cf. [4, 12])

$$R_{conv} = N_H\sigma_H/(N_C\sigma_C + N_O\sigma_O). \tag{3}$$

(σ_i: *bound*-atom total cross-section [15]). N_X is the number density of atom X (=H, C, O), which is for formvar N_H:N_C:N_O = 14:8:2. Instead of the conventionally expected equality $R_{exp} = R_{conv}$, however, the experimental results presented below show that this equation is strongly violated: $R_{exp} < R_{conv}$.

Using electrons with energies $15 - 30$ keV and a scattering angle of $\theta = 44.3^{\circ}$, electron-proton energy transfers in the range of about $2 - 12$ eV have been recently achieved [5, 16]. The energy loss spectra obtained show that the recoil peak of protons is well resolved from the combined peak of the heavier C and O. In this physical context, this electron-proton Compton scattering (ECS) method is the electron analog to NCS. We emphasize that ECS always refers to *electron-nucleus* scattering only, and not to electron-electron scattering.

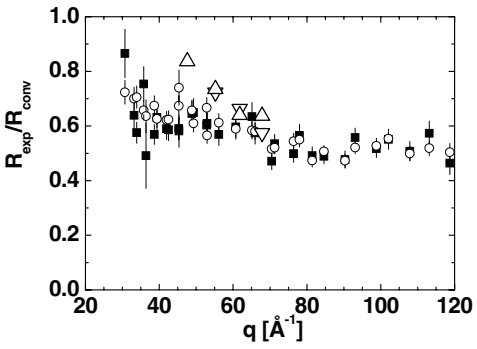

Figure 2. Anomalous reduction of NCS and ECS intensity from H of formvar, as a function of applied momentum transfer $\hbar q$. The q-range shown corresponds to scattering times $\tau_{sc} \approx 200 - 1000 \times 10^{-18}$ s. Small squares and circles: Values of rations R_{exp} of NCS peak-areas measured in the detector angular range $32^{\circ} - 68^{\circ}$, relative to R_{conv}, see Eqs. (2)-(3). Full squares (open circles) represent results for foils with 0.1 mm (0.2 mm) thickness. Large open triangles: Rations R_{exp}/R_{conv} as measured by ECS from films of $50 - 100$ Å thickness, using electrons with kinetic energies $15 - 30$ keV. Note the strong (ca. $20 - 50\%$) shortfall of the ratio R_{exp}; taken from Ref. [5].

According to conventional theory, the aforementioned equation $R_{exp} = R_{conv}$ should be also valid for ECS. Here, however, the cross section for electron scattering from hydrogen, carbon and oxygen is simply the Rutherford cross section: $\sigma_X \propto Z_X^2$ (Z_X: atomic number of atom X). Calculations of the cross section based on the electronic structure show that screening effects are not important under these conditions [16, 17]. The obtained ECS results reveal that, as in the case of NCS, the measured ratio R_{exp} of the hydrogen peak and the joint oxygen/carbon peak is considerably decreased: $R_{exp} < R_{conv}$.

Comparative results from ECS and NCS measurements from formvar were presented [5]. In Figure 2 the ratios R_{exp}/R_{conv} are given as functions of $q = |\mathbf{q}|$ for both ECS and NCS. The effect revealed by NCS is between 25% and 50%, and increases with increasing momentum transfer, corresponding to decreasing scattering time τ_{sc}; see Eq. (5). The ECS data reveal a corresponding "anomalous" decrease of R_{exp} of 15-45%. The results derived from the ECS spectra were found to be in very good agreement with associated NCS data [5], see Figure 2. (For full experimental details, see [17].) Thus this effect appears to be independent of the two fundamental interactions involved (i.e., electromagnetic and strong), which also implies that it reveals a genuine property of matter [5,6].

Considerable efforts to identify possible sources of NCS-experimental errors have been made during the last five years; see [12]. Moreover, Senesi et al. [18] investigated in the most rigorous way all the criticisms raised in Refs. [19] concerning the effects of instrumental resolution and filter absorption profile, by

applying directly, and for the first time, the so-called "exact method" of data analysis proposed in Refs. [19]. These results demonstrated unequivocally the presence of the same anomaly in the scattering from protons, in agreement with the results of the standard data analysis procedure of ISIS [18].

3 On Scattering Time

In the context of NCS, as provided by the Vesuvio setup, the Impulse Approximation (IA) is valid [13, 14] and the characteristic time scale — often termed "scattering time", τ_{sc} — of the neutron-proton scattering process is very short [5, 8–12],

$$\tau_{sc} \sim 100 - 1000 \text{ as} \tag{4}$$

(as: attosecond). These sub-femtosecond scattering times are a consequence of the large energy and momentum transfers attained with the Vesuvio instrument, Eqs. (1), and they follow from the theoretical result valid in the IA [13, 14]

$$\tau_{sc} \, |\mathbf{q}| \, v_0 \approx 1 \, , \tag{5}$$

where v_0 is the root-mean-square (rms) velocity of the nucleus and $\hbar\mathbf{q}$ is the momentum transfer from the neutron to the proton. The time τ_{sc} is given by the t-width of the intermediate correlation function $F(q,t)$, which is related to the dynamic structure factor $S(q,\omega)$ by Fourier transform [14]

$$S(q,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(-i\omega t\right) F(q,t) \, dt \, . \tag{6}$$

It is interesting to note that the "actual duration" of a neutron-proton interaction may be even shorter, as a classical estimate indicates. E.g. a neutron with kinetic energy $E_0 \approx 10$ eV will pass a distance of 10^{-5} Å (i.e. the range of the strong interaction) in a much shorter time, i.e. $10^{-19} - 10^{-20}$ s.

However, this is not in conflict with the above estimate, for the following reason. As standard theory shows [13], $S(q,\omega)$ is peaked around the nuclear recoil energy $E_q = \hbar^2 q^2 / 2m$. The scattering time τ_{sc} is also given by the inverse of the width ΔE of $S(q,\omega)$, and $S(q,\omega)$ plays the role of the probability density distribution for transferring energy $\hbar\omega$ from the neutron to the proton, when the momentum transfer is $\hbar q$. That is, $\tau_{sc} \approx \hbar / \Delta E$. (In addition, it can also be shown that τ_{sc} is about the inverse of the energy spread of the proton wave function after collision [20].) For a typical value $\Delta E \approx 10$ eV, one gets $\tau_{sc} \approx 10^{-16} - 10^{-17}$ s. In other words, the scattering time τ_{sc} gives a statistical measure of the length of the time interval during which an elementary neutron-proton collision may occur, in the same way that the spatial extent of a particle wave function (or wave packet) gives a statistical measure of the extent of the region in which the particle may be found.

To shed more light upon the issue of "relevant scattering time", one may also refer to the celebrated Margolus-Levitin theorem [21]. Let us consider the neutron-proton system during the collision process. Obviously, the initial and final states of it are very different and so they can safely be assumed to be orthogonal. This theorem asserts that in takes at least a time $T_{\perp} \geq (\pi\hbar)/(2E_s)$ for the system to evolve from its initial to any orthogonal final state. E_s is the system's average energy minus its ground state energy. $T_{\perp}$ provides a strict bound for the considered dynamical process [21]. Note that in NCS one has $E_s \approx E_0$ and thus E_s is larger than, but of similar order as, the aforementioned energy spread ΔE. Thus it is revealing that also this time $T_{\perp}$ is very similar to the aforementioned scattering time, i.e. $T_{\perp} \lesssim \tau_{sc}$.

In addition, the Brunetti-Fredenhagen construction of an observable (as a positive operator valued measure) characterizing the "time of occurrence of an effect" [22] should be mentioned, which seems to provide a similar magnitude of "scattering time", too.

4 Theoretical Considerations

The thus far existing theoretical models attribute this scattering effect to:

(A) Shortfall of scattering intensity caused by quantum exchange correlations between pairs of identical particles in the scattering system [23, 24].

(B) Contribution of electronic degrees of freedom to the dynamics of a struck proton (deuteron). I.e., breakdown of the Born-Oppenheimer (BO) approximation, in connection with $(B.1)$ additional excitations of the electronic system [20, 25], and/or $(B.2)$ decoherence accompanying short-lived spatial entanglement of a struck proton with adjacent electrons and perhaps also nuclei [10, 26, 27]. In the models of category (B), quantum exchange correlations play no role.

In order to test the relevance of these competing theoretical models, we recently carried out NCS experiments [11] from *(a)* the equimolar mixture of H_2 and D_2, and *(b)* the mono-molecular liquid HD, using the same experimental setup. Both systems have the same overall atomic composition, that is, H:D=1:1. System *(a)* contains homonuclear molecules and thus exhibits exchange correlations between indistinguishable particles; e.g., these lead to the well known para- and ortho-states of H_2. In contrast, there is no reason to assume that the heteronuclear system HD comprises *the same amount* of such correlations between pairs of protons belonging to different HD molecules.

Interestingly, the considered "anomalous" NCS from protons was found to exist in both samples, and even with the same magnitude [11]. Thus we may conclude that quantum exchange correlations between pairs of protons play no significant role in NCS. This conclusion agrees with related theoretical results of Colognesi [28] and Sugimoto et al. [29]. In contrast, the experimental findings [11] do not contradict the theoretical models of category (B).

4.1 Effects of short-lived QE and decoherence on scattering

In the following we present an outline of the theoretical interpretation of the considered NCS (and ECS) results, which belongs to the aforementioned category $(B.2)$; for more details see [27].

Let us consider neutron scattering from a system consisting of N particles with the same scattering length b, and the N-body Hamiltonian $H_{total} = H_0 + V$ with the interaction

$$V(\mathbf{r}) = \lambda\, n(\mathbf{r}), \quad \lambda = \frac{2\pi\hbar^2}{m}\, b\,. \tag{7}$$

m is the neutron mass, $n(\mathbf{r})$ is the particle density operator

$$n(\mathbf{r}) = \frac{1}{V}\sum_{j=1}^{N} \delta(\mathbf{r} - \mathbf{R}_j)\,, \tag{8}$$

where V is the volume, and $\mathbf{R}_j$ is the spatial position of the j-th particle; cf. the textbook [15].

In the *interaction* picture, the Schrödinger equation is now (setting for simplicity $\hbar = 1$) $i\partial_t \Psi = \lambda\, n(\mathbf{r},t)\Psi$, with the perturbative solution

$$\Psi(t) = \Psi(0) - i\lambda \int_0^t n(\mathbf{r}, t')\, dt' \Psi(0). \tag{9}$$

We write the transition probability $W(t)$ between initial states ψ_i (with probability P_i) and final states ψ_f of the scattering system to be given by

$$W(t) = \sum_{i,f} \mid \langle \psi_f \mid \lambda \int_0^t n(\mathbf{r}, t')\, dt' \mid \psi_i \rangle \mid^2 P_i. \tag{10}$$

It should be noted that ψ_i and ψ_f are eigenstates of the N-body Hamiltonian H_0 omitting the probe system [15, 30]. The transition probability is then given in the form

$$W(t) = \lambda^2 \int_0^t dt' \int_0^t dt'' \sum_f \langle \psi_f \mid n(\mathbf{r},t')\, \rho\, n(\mathbf{r},t'') \mid \psi_f \rangle, \tag{11}$$

with $\rho = \sum_i \mid \psi_i \rangle P_i \langle \psi_i \mid$, where we have noted that $n^\dagger(\mathbf{r}, t) = n(\mathbf{r}, t)$.

In an actual scattering experiment from condensed matter, we do not measure the cross-section for a process in which the scattering system goes from a specific initial state ψ_i to another state ψ_f, both being unobserved states of the many-body system. Therefore, one takes an appropriate average over all these states [15, 30], as done in Eq. (10).

Furthermore, the initial ($\mathbf{k}_0$) and final ($\mathbf{k}_1$) momenta of an impinging neutron may be assumed to be well defined [15, 30]. Introducing the momentum transfer $\mathbf{q} = \mathbf{k}_0 - \mathbf{k}_1$ from the probe particle to the scattering system, the Fourier transform of the particle density reads

$$n(\mathbf{r}, t) = \frac{1}{(2\pi)^3} \int d\mathbf{q}\, n(\mathbf{q}, t) \exp\left(i\, \mathbf{q} \cdot \mathbf{r}\right) \ , \tag{12}$$

where, in the case of neutron scattering, cf. Eq. (8),

$$n(\mathbf{q}, t) = \sum_j \exp\left[-i\mathbf{q} \cdot \mathbf{R}_j(t)\right] \ . \tag{13}$$

Since $n(\mathbf{r}, t)$ is Hermitian, we have $n^\dagger(\mathbf{q}, t) = n(-\mathbf{q}, t)$ and one obtains from Eq. (10)

$$W(t) = \lambda^2 \int_0^t dt' \int_0^t dt'' \sum_f \langle \psi_f | n(\mathbf{q}, t')\, \rho\, n(-\mathbf{q}, t'') | \psi_f \rangle. \tag{14}$$

At this stage one traditionally assumes that the sum over ψ_f runs over all possible eigenstates of H_0 which constitute a complete set, i.e. $\Sigma_f |\psi_f\rangle\langle\psi_f| = \mathbf{1}$; see [15, 30]. Hence

$$\sum_f \langle \psi_f | n(\mathbf{q}, t')\, \rho\, n(-\mathbf{q}, t'') | \psi_f \rangle = Tr\left[n(\mathbf{q}, t')\, \rho\, n(-\mathbf{q}, t'')\right] \ , \tag{15}$$

where $Tr[...]$ denotes the trace operation. As done in standard theory [15, 30], in Eq. (15) one first sums over all final states, keeping the initial state ψ_i fixed, and then averages over all ψ_i (see e.g. [15], p. 19). The right-hand-side of Eq. (15) contains the density operator ρ of the system before collision. Also, if the integration in Eq. (14) is extended over all times (i.e., $t \to \infty$), this ensues over-all energy conservation. This reproduces the well known result of standard neutron scattering theory, cf. [15, 30].

Here, however, it is important to retain the finite duration of the scattering time. This introduces an additional freedom into the theory, because we may be able to observe the influence of the decoherence on the scattering yield. The result will be expressed in terms of the correlation function

$$C(\mathbf{q}, \tau) = Tr[n(\mathbf{q}, t)\, \rho\, n(-\mathbf{q}, t + \tau)] = Tr[n(\mathbf{q}, 0)\, \rho\, n(-\mathbf{q}, \tau)] \ , \tag{16}$$

where we have utilized the fact that the scattering system is stationary.

By introducing the so-called scattering time τ_{sc}, representing the time interval in which the scattering process may happen, we find

$$W(\tau_{sc}) = \lambda^2 \int_0^{\tau_{sc}} dt' \int_0^{\tau_{sc}} dt'' C(q, t'' - t') = \lambda^2 \int_0^{\tau_{sc}} dt' \int_0^{t'} d\eta \left[C(q, \eta) + C(q, -\eta)\right] . \tag{17}$$

Here we use the stationarity of the correlation function [15]. If we assume this function to be real, and that $C(q,\eta) \approx 0$ for $\eta \gtrsim \tau_{sc}$, we obtain the result

$$W(\tau_{sc}) \approx 2\lambda^2 \tau_{sc} \int_0^{\tau_{sc}} d\eta\, C(q,\eta). \tag{18}$$

Now we can introduce the transition rate, $\dot{W}$ say, which is defined as

$$\dot{W} \equiv \frac{W(\tau_{sc})}{\tau_{sc}} = 2\lambda^2 \int_0^{\tau_{sc}} d\eta\, C(q,\eta). \tag{19}$$

Here the correlation function is analogous to the so-called intermediate function of neutron scattering theory [15]. This result for the scattering yield is analogous to that of standard theory.

4.2 Dynamics of open systems and scattering

We now introduce a set of preferred coordinates $\{\,|\xi\rangle\}$, cf. [2, 31, 32]. These represent the relevant degrees of freedom coupled to the neutron probe. The density matrix needed in (16) is then the *reduced* one in the space spanned by these states, and it is obtained by tracing out the (huge number of the) remaining degrees of freedom belonging to the "environment". To simplify notations, we denote this reduced density matrix by ρ too.

In the *subspace* spanned by the preferred coordinates (also termed "pointer basis"), we may assume the relevant density matrix to obey a Lindblad-type equation [3](a) of the form

$$\partial_t \rho = -i\,[H,\rho] + \mathcal{R}\rho \equiv \mathcal{L}\rho \tag{20}$$

with the formal solution: $\rho(t) = e^{\mathcal{L}t}\rho(0)$. Let us look at a time-dependent expectation value

$$\langle A(t)\rangle \equiv Tr\,(\rho(t)A) = Tr\left(e^{\mathcal{L}t}\rho(0)A\right) = Tr\left(\rho(0)e^{\mathcal{L}^\dagger t}A\right)\,, \tag{21}$$

where we define $\mathcal{L}^\dagger$ by setting $Tr\,((\mathcal{L}X)\,Y) = Tr\left(X\,\left(\mathcal{L}^\dagger Y\right)\right)$. Thus we obtain a Lindblad time evolution for the operators too by writing

$$\partial_t A(t) = \mathcal{L}^\dagger A(t)\,. \tag{22}$$

cf. [3](a). Note that this assumes that $\mathcal{L}$ does not depend on time.

We may use this formalism to calculate correlation functions like the one in Eq. (16). It holds

$$\langle A(t)B\rangle = Tr\left[\rho(0)\left(e^{\mathcal{L}^\dagger t}A\right)B\right] = Tr\left[Ae^{\mathcal{L}t}\,(B\rho(0))\right] \equiv Tr\,(A\rho_B(t))\,, \tag{23}$$

where $\rho_B(t)$, as defined in Eqs. (23), obeys the equation

$$\partial_t \rho_B(t) = \mathcal{L}\rho_B(t) \tag{24}$$

and the initial condition $\rho_B(0) = B\rho(0)$. Thus, except for the initial condition, we have to solve the same equation of motion as for the density matrix, Eq. (20).

Let us assume here a simple Lindblad-type Ansatz for the master equation in the relevant subspace of the preferred coordinates. In order to show the effect of decoherence, we simply assume one Lindblad variable X; in the real system we would have a multitude of such variables. We set

$$\partial_t \rho = -i\,[H, \rho] - K\,[X, [X, \rho]] = \mathcal{L}\rho\,, \tag{25}$$

where $K > 0$, H is the reduced (or relevant) Hamiltonian of a microscopic or mesoscopic scattering system, and the double commutator term describes decoherence (and/or dephasing). For simplicity of the further calculations, we here assume that we can take the preferred coordinates to commute with the total Hamiltonian

$$H \mid \xi\rangle = \mathcal{E}_\xi \mid \xi\rangle\,, \qquad X \mid \xi\rangle = \xi \mid \xi\rangle\,. \tag{26}$$

This time evolution is now introduced into the correlation function $C(\mathbf{q}, \tau)$, Eq. (16). A short straightforward calculation (see Ref. [27] for full details) yields for the transition rate the result

$$\dot{W} = 2\lambda^2 \int_0^{\tau_{sc}} \sum_{\xi,\xi'} \exp\left[-i\,(\mathcal{E}_{\xi'} - \mathcal{E}_\xi)\,\tau\right]\, \exp\left[-K\,(\xi' - \xi)^2\,\tau\right] \times \langle \xi \mid n(-\mathbf{q}, 0) \mid \xi'\rangle\langle \xi' \mid n(\mathbf{q}, 0)\,\rho(0) \mid \xi\rangle\, d\tau\,. \tag{27}$$

Obviously, the decoherence-free limit of this result, i.e. with $K = 0$, corresponds to the conventional result of scattering theory. The oscillating factors $\exp\left[-i\,(\mathcal{E}_{\xi'} - \mathcal{E}_\xi)\,\tau\right]$ are characteristic for the "unitary-type" dynamics caused by the commutator part $-i\,[H, \rho]$ of the master equation (25) for the reduced (or relevant) density matrix ρ. These factors have the absolute value 1 and are well known from standard theory [15]. On the other hand, decoherence is present in the case $K^{-1} \sim \tau_{sc}$, leading to the additional restrictive factors $\exp(-K\,(\xi' - \xi)^2\,\tau) \leq 1$, which can be seen to cause a decrease of the transition rate and thus of the associated cross-section. This can be illustrated in physical terms as follows.

Let us first assume that the reduced density operator $\rho(0)$ can be chosen to be *diagonal* in the preferred $\xi-$representation (which corresponds to the usual random phase approximation at $t = 0$). Then each term of Eq. (27) contains the factor

$$\langle \xi | n(-\mathbf{q}, 0) | \xi'\rangle\langle \xi' | n(\mathbf{q}, 0)\,\rho(0) | \xi\rangle = |\langle \xi \mid n(-\mathbf{q}, 0) \mid \xi'\rangle|^2 \langle \xi | \rho(0) | \xi\rangle \geq 0. \tag{28}$$

If the assumption of diagonal form for $\rho(0)$ would be considered as being "too strong", one may note the following: The decoherence factors $\exp(-K\,(\xi' - \xi)^2\,\tau)$ imply that only terms with $\xi \approx \xi'$ contribute significantly to the transition rate. Thus we may conclude that, by continuity, all associated terms with $\xi \approx \xi'$ in Eq. (27) should be positive, too. The further terms with ξ being much different from ξ' can be positive or negative. But they may be approximately neglected, since they decay very fast and thus contribute less significantly to $\dot{W}$; cf. [2](b).

The main conclusion from the preceding considerations is that the time average over τ_{sc} in Eq. (27) always decreases the value of $\dot{W} \equiv W(\tau_{sc})/\tau_{sc}$, due to the presence of the exponential factors $\exp(-K\,(\xi' - \xi)^2\,\tau) \leq 1$. In other words, the effect of decoherence (and/or dephasing) during τ_{sc} plays a crucial role and may lead to an "anomalous" decrease of the transition rate and the associated scattering intensity. This result is in line with that of Ref. [26], which investigated the standard expression of the double differential cross-section of neutron scattering theory [15] by assuming decoherence of final and initial states of the scattering system.

Obviously, in the limit of "vanishing" decoherence, $K \rightarrow 0$, the anomalous scattering effect disappears, i.e. the preceding scattering results are predicted to agree with the conventional theoretical results [13, 15].

In the opposite limiting case of "very fast" decoherence, $K \rightarrow \infty$, one immediately sees that only the "diagonal" terms with $\xi = \xi'$ survive in Eq. (27) — which is due to the action of the factors $\exp(-K\,(\xi' - \xi)^2\,\tau)$. For $\xi = \xi'$ it also holds: $\exp\left[-i\,(\mathcal{E}_{\xi'} - \mathcal{E}_{\xi})\,\tau\right] = 1$. Consequently, Eq. (27) goes over to the standard expression Eq. (19) in the limit of short scattering times. Also this result is in line with conventional expectations.

5 Discussion and Conclusions

It was mentioned that the ultra-short time window of our NCS and ECS experiments, given by the scattering time τ_{sc}, lies in the attosecond regime. Obviously, any technique with "long" (e.g. neutron interferometry [33]) or not well-defined (e.g. neutron transmission) characteristic time window is irrelevant for testing the appearance of our effect [34].

As mentioned above, one may expect NCS and ECS to be governed by the dynamics of the "relevant" *open* quantum system consisting of the struck proton and its adjacent particles (electrons and perhaps other nuclei), up to a distance of a few Å apart. Note also that, during the scattering process, there is no well defined separation of time scales of electronic and protonic motions. However, QE and decoherence constitute uncommon, if not unknown, terms in the scientific literature of neutron scattering (cf. [13, 15]) and their real existence is often questioned (cf. [28, 35]). A fortiori, the dynamics of open quantum sys-

tems might then be regarded to be irrelevant in the considered physical context. In contrast, our theoretical considerations (Section 4) are based on the physical insight that micro- and/or mesoscopic scattering systems ought to be treated as open quantum systems.

The preceding striking NCS and ECS results obtained from a variety of condensed matter systems indicate that attosecond entanglement (and its decoherence) involving protons are quantum phenomena of broader significance and relevance than realized so far.

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References

[1] A. Einstein, B. Podolsky, N. Rosen (1935) *Phys. Rev.* **47** 777.
[2] (a) H.-P. Breuer, F. Petruccione (2002) *The Theory of Open Quantum Systems* (Oxford University Press, Oxford);
(b) M.B. Mensky (2003) *Quantum Measurements and Decoherence* (Dordrecht, Kluwer).
[3] (a) G. Lindblad (1976) *Comm. Math. Phys.* **48** 119;
(b) E. Joos, H.D. Zeh (1985) *Z. Phys.* **B59** 223.
[4] C.A. Chatzidimitriou-Dreismann, T. Abdul-Redah, R.M.F. Streffer, J. Mayers (1997) *Phys. Rev. Lett.* **79** 2839.
[5] C. A. Chatzidimitriou-Dreismann, M. Vos, C. Kleiner, T. Abdul-Redah (2003) *Phys. Rev. Lett.* **91** 057403.
[6] (a) *Physics Today* (2003) Physics Update **56**(9) 9;
(b) (2003) *Scientific American* **289**(4) 20;
(c) The American Institute of Physics Bulletin of Physics News, Physics News Update, 2003 (31 July), No. **648**.
[7] C.A. Chatzidimitriou-Dreismann, T. Abdul Redah, R.M.F. Streffer (1998) *Ber. Bunsenges. Phys. Chem.* **102** 544.
[8] (a) E.B. Karlsson *et al.* (1999) *Europhys. Lett.* **46** 617;
(b) T. Abdul-Redah *et al.* (2000) *Physica* **B 276-278** 824;
(c) E.B. Karlsson *et al.* (2003) *Phys. Rev.* **B67** 184108;
(d) T. Abdul-Redah, C.A. Chatzidimitriou-Dreismann (2004) *Physica* **B350** E1035.
[9] C.A. Chatzidimitriou-Dreismann, T. Abdul-Redah, R.M.F. Streffer, J. Mayers (2002) *J. Chem. Phys.* **116** 1511.
[10] C.A. Chatzidimitriou-Dreismann, T. Abdul-Redah, B. Kolarić (2001) *J. Am. Chem. Soc.* **123** 11945.
[11] C.A. Chatzidimitriou-Dreismann, T. Abdul-Redah, M. Krzystyniak (2005) *Phys. Rev.* **B72** 054123.
[12] J. Mayers, T. Abdul-Redah (2004) *J. Phys.: Condens. Matter* **16** 4811.
[13] G.I. Watson (1996) *J. Phys.: Condens. Matter* **8** 5955.

[14] V.F. Sears (1984) *Phys. Rev.* **B30** 44.
[15] G.L. Squires (1996) *Introduction to the Theory of Thermal Neutron Scattering* (Mineola, Dover).
[16] M. Vos (2002) *Phys. Rev.* **A65** 012703.
[17] M. Vos, C.A. Chatzidimitriou-Dreismann, T. Abdul-Redah, J. Mayers (2005) *Nucl. Instr. Meth.* **B227** 233.
[18] R. Senesi, D. Colognesi, A. Pietropaolo, T. Abdul-Redah (2005) *Phys. Rev.* **B72** 054119.
[19] (a) J.J. Blostein, J. Dawidowski, J.R. Granada, *Physica* (2001) **B304** 357; (2003) **B334** 257;
(b) J.J. Blostein, J. Dawidowski, J.R. Granada (2005) *Phys. Rev.* **B71** 054105.
[20] N.I. Gidopoulos (2005) *Phys. Rev.* **B71** 054106.
[21] N. Margolus, L.B. Levitin (1998) *Physica* **D120** 188.
[22] R. Brunetti, K. Fredenhagen (2002) *Phys. Rev.* **A66** 044101; (2002) *Rev. Math. Phys.* **14** 897.
[23] E.B. Karlsson, S.W. Lovesey (2000) *Phys. Rev.* **A61** 062714; (2002) *Phys. Scr.* **65** 112.
[24] E.B. Karlsson (2003) *Phys. Rev. Lett.* **90** 095301.
[25] G.F. Reiter, P.M. Platzman (2005) *Phys. Rev.* **B71** 054107.
[26] C.A. Chatzidimitriou-Dreismann (2005) *Laser Physics* **15** 780.
[27] C.A. Chatzidimitriou-Dreismann, S. Stenholm (2005) in: *Decoherence, Entanglement and Information Protection in Complex Quantum Systems*, NATO Science Series II – Vol. 189, eds. V.M. Akulin, A. Sarfati, G. Kurizki, S. Pellegrin (Springer, Dordrecht) pp. 555-562.
[28] D. Colognesi (2004) *Physica* **B344** 73.
[29] H. Sugimoto, H. Yuuki, A. Okumura (2005) *Phys. Rev. Lett.* **94** 165506.
[30] L. van Hove (1954) *Phys. Rev.* **95** 249.
[31] (a) H.D. Zeh (1973) *Found. Phys.* **3** 109;
(b) O. Kübler, H.D. Zeh (1973) *Ann. Phys. (NY)* **76** 405.
[32] W.H. Zurek (1981) *Phys. Rev.* **D24** 1516; (1982) *Phys. Rev.* **D26** 1862; (2003) *Rev. Mod. Phys.* **75** 715.
[33] A. Ioffe *et al.* (1999) *Phys. Rev. Lett.* **82** 2322; (2000) *Phys. Rev. Lett.* **84** 2037 (Reply).
[34] C.A. Chatzidimitriou-Dreismann, T. Abdul Redah, R.M.F. Streffer, B. Hessmo (2000) *Phys. Rev. Lett.* **84** 2036 (Comment).
[35] R. A. Cowley (2003) *J. Phys.: Condens. Matter* **15** 4143.