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Digital Pulse Acoustic Lensless Fourier Method for Ultrasonic Imaging

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Abstract. The profile of a periodic surface is computer visualized only by registration of the amplitude of the transmitted through the object pulse acoustic field. A computer Fourier transformation of the digital data of the amplitude field is performed as a precondition for converging lens. The Fourier series of the object obtained as digital data after transformation is multiplied with a term, describing the angle distribution of the field in spatial frequencies. The image reconstruction is performed by reverse transformation, i.e. summing up in all spatial frequencies. The profile of the studied object is compared by shape with the theoretical one.

It is shown that the computer reconstruction of objects by the digital Fourier transformation is modern and effective method for ultrasonic imaging compared with the theory of wave scattering [1]. It is necessary the object lengths to be of the same order of magnitude as the wavelength. For visualization of objects with different parameters, this condition can be reached in the acoustics by different liquids or changing the frequency of the RF pulse.

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1 Introduction

The ultrasonic imaging has attained a special importance in all science areas during the recent years. The development of this area is stimulated by the creation of flexible systems using acoustic phase array (matrix) technology. This is of great importance for applications in the systems for imaging of human organs and nondestructive testing.

The processing of the registered scattered (or transmitted) field by the object is mostly in the ultrasonic imaging. At the processing of the acoustic field, a mathematical model is used, which is in accordance with the applicable theory. The mathematical model connects the scattered or transmitted fields with the object size for the following computer reconstruction and visualization on flat screen.

One of the applicable theories, connecting the scattered field with the surface object parameters is the theory of the wave scattering. If the function describing the object is expanded in Fourier series (series on spatial frequencies). the scattered and transmitted fields can be approximated as diffraction from separated sinusoidal gratings [1]. Taking into account arbitrary diffraction orders (higher than the first) is of great importance for a more correct reconstruction. In the case of random rough surface the statistical parameters can be determined more correctly. This number depends on the parameters of the studied object and wavelength. The theory of wave scattering is comparatively complicated in the most common case. One special case for solving of Helmholtz integral, describing the field on the object surface [2], is the case when the Kirchhoff condition for "locally flat" surface is realized. Then the field on the surface can be presented as a sum of incident and mirror reflected fields [2]. When the wavelength is of the same order of magnitude as the surface length, the mathematical model is easier. The number of diffraction orders that must be taken into account is smaller [3]. In this case the computer processing of the phase and the amplitude of each grating is faster.

The digital Fourier holography is another method for object reconstruction. The traditional Fourier holograms are obtained in optics, when the Fourier transformation and the reference beam interfere on the recording plane [4]. When the object is set on the front focal plane of the lens, the field at the back focal plane presents systems of plane waves. These waves form correct Fourier image of the object, i.e. a two-dimensional transformation of the object waves is realized by the lens. The recording plane is the back focal plane of the converging lens. If the converging lens is not used, the Fourier transformation of the field diffracted from the object can be performed as precondition for converging lens. In general, the basic aims in the digital Fourier holography are two:

(i) Recording of the Fourier transformation of the object, using the coherent source.

(ii) Phase and amplitude retrieval of the transformation components in the series and digital reconstruction of the image.

The reverse Fourier transformation can be performed digitally and the object is visualized on the computer screen.

It must be noted, that the Fourier method is practically effective when the wavelength is of the same order of magnitude as the object length. At simulation of lens the Fourier transformation will be performed with the step of registration of the field in digital form. When this step is smaller or the number of the values is higher, the object will be reconstructed more correctly. In optics, the digital Fourier holography founds the basic application in the Holographic Microscopy.

The problem for obtaining of object images immersed in opaque media gives possibility for the development of ultrasonic holography as a method for ultrasonic imaging. The holography is a powerful method for high spatial resolution real time 3D ultrasound imaging and diagnostics [5]. The authors in [5] used transmissive ultrasound with a scanning needle hydrophone for full 3D holographic imaging.

When the liquid, in which the object is immersed, responds to the ultrasonic pressure instantly, the acoustic signal can be pulsed at the television frame rates. The imaging of invisible objects with the help of acoustic holography finds application in medicine and nondestructive testing too.

Ultrasonic imaging is developed largely by the ultrasonic nondestructive testing industry too. The author in [6] has discussed the similarities between imaging and acoustic microscopy, the roles of the focusing lens, the pulse frequency, and the imaging of the studied materials with respect to the final resolution of an acoustic image. It is discussed on the difficulties to establish operational differences between imaging and acoustic microscopy because the commercial ultrasonic imaging systems use transducers producing focused beams for images with high resolution. The differences are available in the case of the interference measurement and the gigahertz frequencies used at the higher frequency scanning acoustic microscopes.

A lensless acoustic Fourier hologram can be made in a standard way – the reference source is placed coplanar with the object point. The hologram formed in this way presents two-dimensional Fourier transformation of the object [7]. The image can be reconstructed by a reverse Fourier transformation of the hologram. Failing of this method is that the image is perfect when the hologram is located in the far field only. The image points, which are not in the reference plane, will be with aberration. Other failing is that a reference beam is required.

Another alternative is to measure electronically only the amplitude and the phase of the diffracted by the object acoustic field without using a reference beam [7]. Then in the computer the reference beam is added by appropriate phase value to each measured value in the set of data. The image is produced by inverse Fourier transformation. After that the synthetic Fourier transform hologram is visualized on the computer screen. The procedure of mixing of the reference and the diffracted beams may be performed in the digital oscilloscope too.

Hildebrand [7] suggested a method for digital reconstruction of the object image and he named it "Imaging by backward wave reconstruction". The essence of this idea is that the acoustic wave front in the data plane can be approximated in such a way that to consist of the sum of angular distribution of plane waves. The angular distribution of plane waves in the data plane can be found by multiplying the angle spectrum of plane waves on the object plane by exponential factor accounting for the phase change due to the distance propagated. The reconstructed object in [7] is a row of flat-bottomed holes through 75 mm of steel.

Transmission or reflection ultrasound image can be recorded by using acoustic lens to focus the object onto a detector array (camera) sensitive to the ultrasound [8].

The computer reconstruction of object by digital lensless Fourier method is the purpose of this paper. FFT (Fast Fourier Transformation) is on the recorded in digital form amplitude of the transmitted field applied. This replaces the use of the lens to obtain a Fourier image of the object. The processing and elaboration of the acoustic lens is more complicated than the optical lens. The lens material must be selected too in accordance with the liquid in which the object is immersed (the acoustic impedances in both materials have to be close). The amplitude of the registered acoustic field presents the array of the amplitudes of the echo-signals, registered at separate points of scanning. It is not necessary to take into account factor, giving the phase change due to the distance propagated. In acoustics this factor defines the difference of the delays of the echo-signals reflected (transmitted) from different places of the object. Every amplitude can be registered in accordance with delay time. If each complex value obtained after Fourier transformation on the amplitude field is multiplied with term, describing the angle distribution of the field in spatial frequencies, the result presents Fourier series of the object. The reconstruction of the image is performed by reverse transformation – summing up in all spatial frequencies. The requirement the wavelength to be of the same order of magnitude as the object lengths can be achieved by different liquids or by changing of the wavelength.

Digital Fourier holography in optics is a fast method for reconstruction of images. Since in optics the wave intensity is registered, the phase information is converted into the amplitude modulation. The phase problem in physics is how the phase can be recovered from the Fourier data. In contrast to the optics, in acoustics the amplitude and the phase of the received echo-signal or the amplitude at defined phase displacement can be measured.

By development of the digital techniques, computer technologies, and the presence of software for Fast Fourier transformation, the digital lensless Fourier method will find much more applications for imaging of invisible objects.

2 Theory

The complex amplitude of a monochromatic wave in the $z = 0$ plane is a function $g(x, y)$ composed of harmonic components of different spatial frequencies as it is shown in Figure 1. Each harmonic component corresponds to a plane wave. The plane wave travelling at angles $\theta_x = \arcsin(\lambda\nu_x)$ and $\theta_y = \arcsin(\lambda\nu_y)$ corresponds to the components with spatial frequencies ν_x and ν_y, and has an amplitude $G(\nu_x, \nu_y)$, the Fourier transform of $g(x, y)$.

At a great distance, a single plane wave contributes to the total amplitude at each point in the output plane.

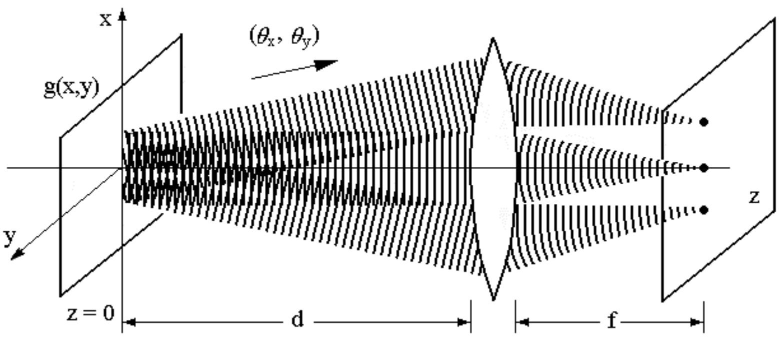

Figure 1. The Fourier transformation of the input wave in the far field.

- Fourier components are separated naturally.
- This suggests that diffraction pattern in the far field can be thought of as the Fourier transform of the two-dimensional function $g(x, y)$.

The Fourier transform properties of the far-field can also be accomplished by using of lens to focus each of the plane waves into a single point. When d is large, the only plane wave that contributes to the complex amplitude at (x, y) in the output plane is the wave with direction making angles $\theta_x \cong x/d$ and $\theta_y \cong y/d$ with the principal axis (in the optics -optical axis).

This is the wave with wave vector components $k_x \cong (x/d)k$ and $k_y \cong (y/d)k$ and amplitude $F(\nu_x, \nu_y)$ with $\nu_x \cong x/\lambda d$ and $\nu_y \cong y/\lambda d$. The complex amplitudes $U(x, y)$ and $g(x, y)$ of the wave at the $z = 0$ and $z = d$ planes are related by

$$U(x, y) \cong h_0 F\left(\frac{x}{\lambda d}, \frac{y}{\lambda d}\right), \tag{A.1}$$

where $F(\nu_x, \nu_y)$ is the Fourier transform of $g(x, y)$ and $h_0 = (j/\lambda d)\exp(-jkd)$.

The plane-wave components may also be separated by using of lens. A thin lens transforms the plane wave into a wave focused to a point in the lens focal plane.

- If the plane wave arrives at small angles θ_x and θ_y, the wave is centered about the point $(\theta_x f, \theta_y f)$, where f is the focal length.
- The lens maps each direction (θ_x, θ_y) into a single point $(\theta_x f, \theta_y f)$ in the focal plane and thus separates the contributions of the different plane waves so that

$$U\,(x, y) = h_l F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right). \tag{A.2}$$

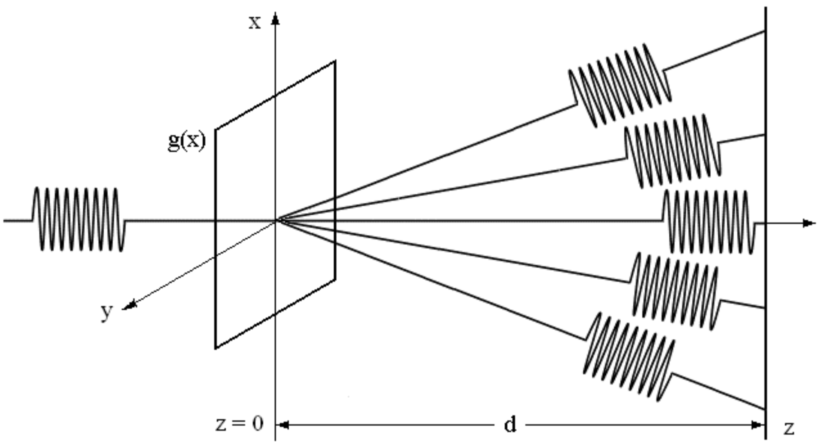

Figure 2. Separation of the harmonic components by the delays.

In the case where $d = f$, $h_l = (j/\lambda f)$, and $g = g(x)$, the amplitude $U(x)$ in the back focal plane is [4]

$$U\left(x\right) = \frac{j}{2\pi}\int\limits_{-\infty}^{\infty} g\left(x\right)\exp\left(jK_x x\right)dK_x, \tag{A.3}$$

where $K_x = 2\pi\nu_x$.

Substituting the integral with a sum, we obtain

$$U(x) = \sum_{n=-\infty}^{\infty} C_n \exp(jnK_n x), \tag{A.4}$$

$$U(x) = \sum_{n=-\infty}^{\infty} |z|_n \exp[j((K_n x + \varphi_n)], \tag{A.5}$$

where C_n is the complex amplitude of the n-th harmonic in the series, $K_n = nK$ is the n-th spatial wave number and K is the spatial wave number of the base harmonic. The complex amplitudes C_n in (4) can be presented in the form $|z|n\exp(j\varphi_n)$, where $|z|_n$ is its absolute value, and φ_n is its phase.

Equation (4) states that the field at the back focal plane is composed of harmonic components of different spatial frequencies. The lens separates plane-wave components and arranges the Fourier components in spatial frequencies – from low to high ones.

If the pulse acoustic signal is incident on the studied object and a lens is not used, then the complex amplitude of the n-th harmonic component in the data

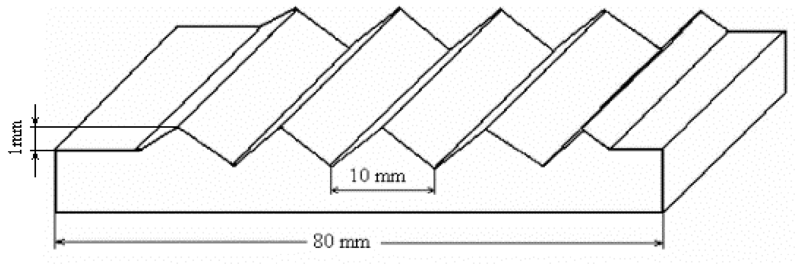

Figure 3. The surface profile of the studied object.

plane ($z = d$) will be $A_n \exp(j\omega t_n)$, where $\omega = 2\pi f_{\mathrm{RF}}$, f_{RF} is the frequency of the incident acoustic signal, t_n is the delay of the echo-signal with amplitude A_n in comparison with the incident one. The separated harmonic components will be registered with different delays t_n as it is shown in Figure 2.

If the amplitudes A_n are registered in digital form (as array of n discrete values), the applied Fourier transformation on this array will play the role of the lens, i.e. it arranges the Fourier components on spatial frequencies. The result will present an array of complex values C_n. It is obviously, that to obtain Fourier series of the studied object in spatial frequencies, each of the complex amplitudes C_n must be multiplied by a term, giving angle distribution to the corresponding spatial frequency. This term is $\exp(j_n x)$. Then the expanded in Fourier series function, which describes the object, is (5). The object image can be obtained after summing up in all spatial frequencies.

If the object has periodic surface and we want to display this surface, the n-th spatial wave number K_n in this case will be $2\pi n/D$, where D is the period of basic harmonic in the series. The calculations in the case of random rough surface in the frame of the biggest period and the corresponding periods of the harmonic can be in the range of 2π and the corresponding periods of the rest harmonics, divisible by this period. The studied object has periodic surface as it is shown in Figure 3, with base period $D = 10$ mm and the lowest spatial frequency $f = 1/D$.

3 Experimental Setup

The experimental setup for registration of the amplitude of transmitted field is shown in Figure 4. The electric signal is passed from the amplifier by a coaxial cable to the ultrasonic transducer (the diametrical size is 3 cm), which radiates RF pulse. This transducer is located in a glass ripple tank, which is full of water and the transducer radiates in the water acoustic wave with compression polarization. The studied object is set to be parallel to the receiving and radiating transducers. The distance between the object and the data plane is 85 mm.

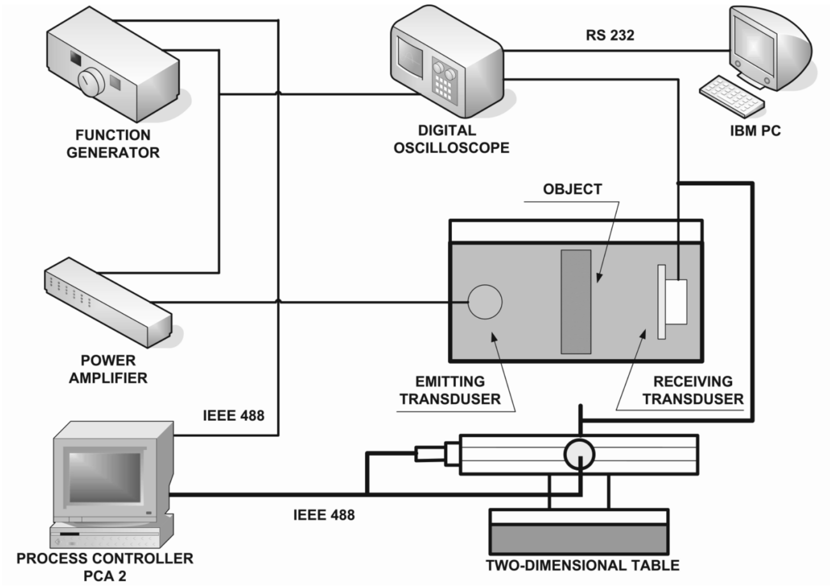

Figure 4. Experimental setup.

The radiated ultrasonic pulse incidents perpendicularly to the object surface. A part of this signal is scattered, the other is transmitted through the sample into the water again. The receiving transducer is moved by the computer controlled two-dimensional coordinate table. This transducer registers the amplitude of the transmitted echo-signal, which has compression polarization too. The diffracted field is registered in both cases of RF frequency. This corresponds to different wavelengths.

In the first case, the step of a movement of the transducer is 0.6 mm, the RF frequency is 2.2 MHz (the wavelength is 0.6818 mm) and the transmitted field is registered with 45 discrete values. In the second case, the RF frequency is 1.1 MHz (the wavelength is 1.3636 mm) and the same field is registered at a step of 1 mm, and with 80 values at a step of 0.5 mm. A blend is on the receiving transducer placed in order to register the amplitude of the diffracted field at each step of moving. The width of the blend is 1 mm. The obtained echo-signal (i.e. the registered echo-signal) is passed to the digital oscilloscope Tektronix 11201. The signal is digitized and it is averaged with a digital filter to improve the ratio signal/noise. For each step of the movement of the transducer, the echo-signal amplitude is registered in defined time-window. The window may be in the digital oscilloscope program controlled for the measured purposes. The digital oscilloscope is controlled by the MatLab program. Each registered amplitude is transferred by the interface RS 232 to PC in digital form, where it is written in a file for a further processing. The transfer to PC is performed by the MatLab program. This program performs the following data processing.

The Function Generator type ROHDE & SCHWARZ AFG generates RF signal (sine burst) with the following parameters: frequency $f = 1.1$ MHz ($\lambda = 1.3636$ mm), rise time $t = 159.6$ ns, fall time $t = 168.2$ ns, output level $U = 1$ V, phase offset 0°, interval between the pulses $T = 10$ ms, number of the sinusoids $N = 2$. The continuance of the pulse is 1.81 μs.

The amplifier type ENI has the following characteristics: frequency range 150 kHz – 300 MHz, $U_i^{\max} = 1$ V, $P_{\max} = 10$ W.

4 Discussion

The Fourier transformation of the registered in a digital form transmitted field is performed by the MatLab program and its function FFT (Fast Fourier Transformation). The array of values after FFT operation presents complex values. The absolute value and the phase of each complex value are the amplitude and the phase of each harmonic in the series. Fourier transformation of the studied object in spatial frequencies is obtained, as the complex amplitude C_n is multiplied by a term, giving angle distribution of the corresponding spatial frequency – $\exp(jK_nx)$. The real part of the n-th harmonic in (5) will consist of $\cos(K_nx + \varphi_n)$, and the imaginary one: $\sin(K_nx + \varphi_n)$. The reconstruction of the image is performed by reverse transformation – summing up in all spatial frequencies Eq.(5), i.e. for each value of x performs summing up in all harmonics in the series of the interval $(0 - D)$.

The obtaining data after reverse transformation presents array of a complex value. The image can be plot with the imaginary part of the values. It is known that one of the methods to expand in Fourier series is

$$f\left(x\right) = \sum_{n=-\infty}^{\infty} \delta_n \sin\left(K_nx + \varphi_n\right), \tag{A.6}$$

where $f(x)$ is the function, describing the surface profile, δ_n, φ_n, and $K_n = n/D$ are the amplitude, the phase, and the spatial wave number of the n-th harmonic correspondingly, $1/D$ is the base spatial frequency ($D = 10$ mm).

The reconstruction profile of the object surface at RF frequency 2.2 MHz and 45 discrete values (the step of registration is 0.6 mm) of the registered field is presented in Figure 5. The profile is compared with the theoretical one (calculated with 45 harmonics in the Fourier series) only by shape. The Fourier series of the studied object is

$$f\left(x\right) = \frac{8h}{\pi^2} \sum_{n=0}^{\infty} \frac{\left(-1\right)^n}{\left(2n+1\right)^2} \sin\left\{2\pi\left(x/D\right)\left(2n+1\right)\right\}, \tag{A.7}$$

where h is the amplitude, in the experiments $h = 0.5$ mm and $D = 10$ mm.

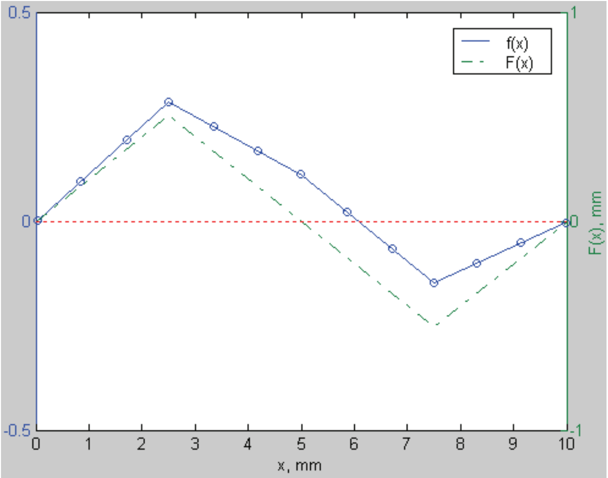

Figure 5. The reconstructed surface profile of the studied object with 45 registered values of the amplitude field and $\lambda = 0.6818$ mm – $f(x)$; the same profile theoretically calculated with 45 harmonics in the Fourier series – $F(x)$.

The reconstruction profile of the object surface at RF frequency 1.1 MHz, 40 and 80 values of the registered field is presented in Figures 6 and 7, correspondingly. The reconstruction profiles are compared with the theoretical one (by shape), calculated from Eq. (7). The theoretical profile in Figure 6 is calculated with

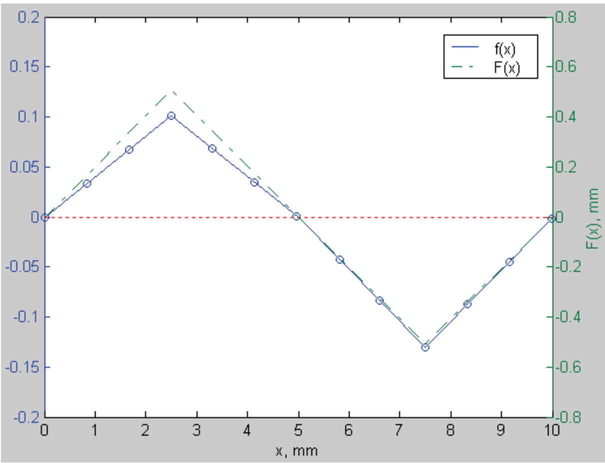

Figure 6. The reconstructed surface profile of the studied object with 40 registered values of the amplitude field and $\lambda = 1.3636$ mm – $f(x)$; the same profile theoretically calculated with 40 harmonics in the Fourier series – $F(x)$.

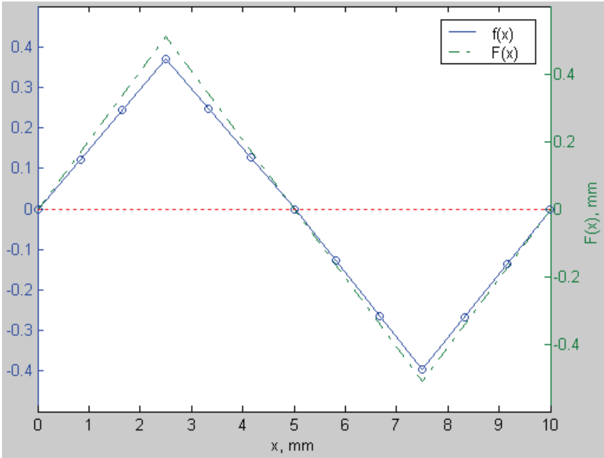

Figure 7. The reconstructed surface profile of the studied object with 80 registered values of the amplitude field and $\lambda = 1.3636$ mm – $f(x)$; the same profile theoretically calculated with 80 harmonics in the Fourier series – $F(x)$.

40 harmonics ($n = 40$) in the Fourier series (the step of registration is 1 mm) and in Figure 7 with 80 harmonics ($n = 80$) (the step of registration is 0.5 mm). Better reconstruction (Figure 7) can be obtained by more points of registration or smaller step of registration. In accordance with Nyquist's sampling criterion, for distinct images it must sample the wave front at least twice in the shortest spatial wavelength in the wave front. The studied profile (see Figure 3) has the base spatial wavelength $D = 10$ mm and the restriction $\sin(\theta_m) = m\lambda/D = 1$, $(m = \pm 1, \pm 2, \pm 3, \ldots)$ to be realized, the shortest possible spatial wavelength is λ [6]. Then $\lambda/2$ is suitable to sample the acoustic field. The step 20 μm is the minimal step of moving of the coordinate table. The shortest sampling length and sound wavelength depend on the liquid or on changing the frequency of the RF pulse. The accuracy of accounting of the delay with a digital oscilloscope Tektronix is one percent. This means that at sound velocity in water 1500 m/s, the shortest difference in the paths (between two harmonic components of different spatial frequencies) will be 15 μm. The distance between the object and the data plane is chosen in accordance with the shortest difference in the paths.

The studied profile is visualized and reconstructed, using another mathematical model for retrieval of the amplitude and the phase of the harmonics [1]. In this paper the function, describing the profile of the surface is presented in a Fourier series and the diffraction field is approximated as field diffracted from composition of sinusoidal gratings. The profile is reconstructed in the cases of the diffracted and the transmitted acoustic fields. In this case except the amplitude of the transmitted (diffracted) field, the delay time is registered for the purposes of the reconstruction. The mathematical model in our opinion is more complicated than in the case of Fourier holography. Especially in the scattering case, when at arbitrary angles of incident and wavelength in the liquids, the backscattered peaks can appear. Then the applicability of the mathematical model is not valid. The Fourier holography as method is simpler in the sense that, the lens for obtaining of Fourier transformation may not be used. The presence of fast FFT and IFFT accelerates the process of visualizing.

5 Conclussion

The pulse methodology used for a measurement of the transmitted (diffracted) field and the followed Fourier transformation (lensless technology) is a simple and fast method for purposes of ultrasonic imaging. The pulse methodology allows to measure directly the amplitudes of the separated harmonics not affected from interference. The geometry and experimental setup of transmission case is more convenient than the scattered one. The using of moving transducer is rather slow process. The acoustic array or matrix allows rapid electronic scanning of the fields, record and 3D imaging. The rapid development of technologies makes the real-time imaging without problems. Transmissive or reflective ultrasonic 3D imaging in “real time” is possible now, using an ultrasonic stereo camera and a real time viewing system [8].

The visualizing of the studied object on a flat screen gives possibility to be sent directly by Internet. By development of the digital techniques, computer technologies, and the presence of software for Fast Fourier Transformation, this method will find much more applications for imaging of invisible objects.

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