

## Bianchi Type-III Anisotropic Cosmological Models with Varying $\Lambda$

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**Abstract.** Bianchi type-III cosmological models in the presence of a bulk viscous fluid are studied. The coefficient of bulk viscosity is assumed to be a power function of mass density. The cosmological constant  $\Lambda$  is found to be positive and is a decreasing function of time which is supported by results from recent supernovae observations. Some physical and geometrical features of the models are also discussed.

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### 1 Introduction

The study of Bianchi type cosmological models creates more interest because some of these models contain isotropic special cases and permit arbitrarily small anisotropy levels at some instant of cosmic times. Hence such property makes them to be known as a suitable model of our universe. While studying most of the cosmological models, we have assumed that the matter in the universe can be described by a “dust” (a pressureless distribution) or at the best by a “perfect fluid”. Israel and Vardalas [1], Klimek [2], Weinberg [3] believed that viscous effects did play a role at least at the early stage of the universe. Landau and Lifshitz [4] proved that the existence of the bulk viscosity is equivalent to restoring equilibrium states. Bulk viscosity is associated with the GUT phase transition and string creation. Grøn [5] reviewed cosmological models with bulk viscosity and proved that we should consider the presence of a material distribution other than a perfect fluid to have more realistic cosmological model of universe. In general theory of relativity, the effect of bulk viscosity on the cosmological evolution has been studied by many researchers [6-17].

Other researchers like Zeldovich [18], Bertolami [19,20], Ozer and Taha [21], Weinberg ([22], Carroll *et al.* [23], Carlberg *et al.* [24], Friemann and Waga [25] and Pradhan *et al.* [15] investigated more significant cosmological models with cosmological constant  $\Lambda$ . Ratra and Peebles [26] discussed in detail

the cosmological constant problem and cosmology with a time-varying cosmological constant. A.D. Dolgov [27-29], Sahni and Starobinsky [30] proved that in the absence of any interaction with the matter or radiation, the cosmological constant remains a “constant”, however, in the presence of any interaction with the matter and radiation, a solution of Einstein’s equation and the assumed equation of covariant conservation of stress-energy with a time varying cosmological constant could be found.

Perlmutter *et al.* [31] and Riess *et al.* [32] strongly favoured a significant and positive  $\Lambda$  in their observations made from the study of more than 50 supernovae of type Ia with red shifts in the range of  $0.10 \leq z \leq 0.83$  and suggested Friedmann models with negative pressure matter, such as a cosmological constant, domain walls or cosmic strings. Recently Carmeli and Kuzmenko [33] and Behar and Carmeli [34] have shown that the relativity theory predicts the value of cosmological constant  $\Lambda = 1.934 \times 10^{-32} \text{ s}^{-2}$  which is in excellent agreement with the measurements recently obtained by the high Z supernova team associated with supernova cosmological project [35,36]. Finally all these observations conclude that the expansion of the universe is accelerating.

Some of the researchers [37-45] have proposed several cosmological studies in which the  $\Lambda$ -term decays with time. The most significant study done by Chen & Wu [39] has been further modified by several authors [46-50].

Motivated by the situations discussed above, in this paper we have studied Bianchi type-III cosmological models in the presence of the bulk viscous fluid with varying  $\Lambda$ . Some physical and geometrical features of the models are discussed.

## 2 Field Equations

We consider Bianchi-III metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 dz^2, \quad (1)$$

where  $A, B, C$  are the functions of  $t$  and  $\alpha$  is a constant.

The energy momentum tensor in the presence of bulk stress has the form

$$T_i^j = (\rho + \bar{p})v_i v^j + \bar{p}g_i^j, \quad (2)$$

where

$$\bar{p} = p - \xi v_i^j. \quad (3)$$

Here  $\rho, p, \bar{p}$  and  $\xi$  are the energy density, isotropic pressure, effective pressure, bulk viscous coefficient, respectively and  $v^i$  is the flow vector satisfying the relation

$$g_{ij}v^i v^j = -1. \quad (4)$$

Here the comoving coordinates are taken to be  $v^1 = v^2 = v^3 = 0$  and  $v^4 = 1$ .

The Einstein's field equations with cosmological constants are

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -8\pi T_i^j \quad (5)$$

( $C = 1, G = 1$  in gravitational unit).

For the line element (1), the field equations (5) have been set up as

$$\frac{\alpha^2}{A^2} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{\alpha}{2} \left[ \frac{B_4}{B} - \frac{A_4}{A} \right] + \Lambda = -8\pi\bar{p} \quad (6)$$

$$\frac{A_{44}}{A} - \frac{\alpha^2}{A^2} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{\alpha}{2} \left[ \frac{B_4}{B} - \frac{A_4}{A} \right] + \Lambda = -8\pi\bar{p} \quad (7)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{\alpha}{2} \left[ \frac{B_4}{B} - \frac{A_4}{A} \right] + \Lambda = -8\pi\bar{p} \quad (8)$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} + \frac{\alpha}{2} \left[ \frac{B_4}{B} - \frac{A_4}{A} \right] + \Lambda = 8\pi\rho \quad (9)$$

$$\alpha \frac{A_4}{A} - \alpha \frac{B_4}{B} = 0 \quad (10)$$

The suffix 4 by the symbols  $A, B$  and  $C$  denote the differentiation with respect to  $t$ .

### 3 Solutions of the Field Equations

Equation (10) gives

$$B = \mu A. \quad (11)$$

Without loss of generality we have to take  $\mu = 1$ , so that

$$B = A. \quad (12)$$

The field equations (6) to (9) with the equation (12) reduce as

$$\frac{\alpha^2}{A^2} + \frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \Lambda = -8\pi\bar{p} \quad (13)$$

$$-\frac{\alpha^2}{A^2} + \frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \Lambda = -8\pi\bar{p} \quad (14)$$

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 + \Lambda = -8\pi\bar{p} \quad (15)$$

$$\left(\frac{A_4}{A}\right)^2 + 2\left(\frac{A_4 C_4}{AC}\right) + \Lambda = 8\pi\rho. \quad (16)$$

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Equations (13) to (16) are four equations with five unknowns  $A, C, \bar{p}, \rho$  and  $\Lambda$ .

To get a determinate solution we need extra conditions.

First we assume that

$$p + \rho = 0, \tag{17}$$

*i.e.* the fluid is anti-stiff fluid.

Therefore, equations (13) to (16) together with equation (17) give

$$\frac{A_{44}}{A} - \frac{A_4}{A} \frac{C_4}{C} = 0. \tag{18}$$

Secondly, we assume that

$$A = C^n, \tag{19}$$

where  $n$  is any real number.

The above equation reduces as

$$\frac{C_{44}}{C} + (n - 2) \frac{C_4}{C} = 0, \tag{20}$$

which on integration gives

$$C = (n - 1)^{\frac{1}{n-1}} (at + b)^{\frac{1}{n-1}}, \tag{21}$$

where  $a$  and  $b$  are constants of integration.

Hence we obtain

$$A^2 = (n - 1)^{\frac{2n}{n-1}} (at + b)^{\frac{2n}{n-1}} \tag{22}$$

$$B^2 = (n - 1)^{\frac{2n}{n-1}} (at + b)^{\frac{2n}{n-1}} \tag{23}$$

$$C^2 = (n - 1)^{\frac{2}{n-1}} (at + b)^{\frac{2}{n-1}}. \tag{24}$$

Therefore, the space-time (1) reduces to the form

$$ds^2 = -dt^2 + \left[ (n - 1)^{\frac{2n}{n-1}} (at + b)^{\frac{2n}{n-1}} \right] dx^2 \\ + \left[ (n - 1)^{\frac{2n}{n-1}} (at + b)^{\frac{2n}{n-1}} e^{-2\alpha x} \right] dy^2 + \left[ (n - 1)^{\frac{2}{n-1}} (at + b)^{\frac{2}{n-1}} \right] dz^2. \tag{25}$$

After a suitable transformation of coordinates, the metric (25) takes the form

$$ds^2 = -\frac{dT^2}{a^2} + (n - 1)^{\frac{2n}{n-1}} T^{\frac{2n}{n-1}} dX^2 \\ + (n - 1)^{\frac{2n}{n-1}} T^{\frac{2n}{n-1}} e^{-2\alpha x} dY^2 + (n - 1)^{\frac{2}{n-1}} T^{\frac{2}{n-1}} dZ^2, \tag{26}$$

where  $T = at + b$ .

The effective pressure  $\bar{p}$  and density  $\rho$  for the model (26) are given by

$$8\pi\bar{p} = 8\pi(p - \xi\theta) = \frac{-\alpha^2}{(n-1)\frac{2n}{n-1}T^{\frac{2n}{n-1}}} - \frac{a^2(n+2)}{(n-1)^2T^2} - \Lambda \quad (27)$$

$$8\pi\rho = \frac{a^2(n+2)}{(n-1)^2T^2} + \Lambda, \quad (28)$$

and  $\theta$  is the scalar expansion calculated for the flow vector  $v^i$  and is given by

$$\theta = \frac{(2n+1)a^2}{(n-1)T}. \quad (29)$$

For the specification of  $\xi$ , now we assume that the fluid obeys an equation of the state of the form

$$p = \gamma\rho, \quad (30)$$

where  $\gamma$  ( $0 \leq \gamma \leq 1$ ) is a constant.

In most of the investigations the bulk viscosity is assumed to be a simple power function of the energy density.

$$\xi(t) = \xi_0\rho^k, \quad (31)$$

where  $\xi_0$  and  $k$  are constants.

For  $k = 1$ , equation (31) may correspond to a radiative fluid.

However, more realistic models are based in the region of  $0 \leq \gamma \leq 1/2$ .

On using equations (27) and (31) we obtain

$$8\pi(p - \xi_0\rho^k\theta) = \frac{-\alpha^2}{(n-1)\frac{2n}{n-1}T^{\frac{2n}{n-1}}} - \frac{a^2(n+2)}{(n-1)^2T^2} - \Lambda, \quad (32)$$

where  $T = (at + b)$ .

#### Case I: ( $\xi = \xi_0$ )

When  $k = 0$  equation (31) reduces to  $\xi = \xi_0$ .

With the use of equations (28), (29), (30), the equation (32) reduces to

$$8\pi\gamma\rho = \frac{8\pi\xi_0(2n+1)a^2}{(n-1)T} - \frac{\alpha^2}{(n-1)\frac{2n}{n-1}T^{\frac{2n}{n-1}}} - \frac{a^2(n+2)}{(n-1)^2T^2} - \Lambda. \quad (33)$$

Eliminating  $\rho(t)$  between equations (28) and (33), we get

$$(1 + \gamma)\Lambda = \frac{8\pi\xi_0(2n+1)a^2}{(n-1)T} - \frac{a^2(n+2)(n\gamma+1)}{(n-1)^2T^2} - \frac{\alpha^2}{(n-1)\frac{2n}{n-1}T^{\frac{2n}{n-1}}} \quad (34)$$

Case II: ( $\xi = \xi_0\rho$ )

When  $k = 1$  equation (30) reduces to  $\xi = \xi_0\rho$ .

Using equations (28), (29), (30), the equation (32) reduces to

$$8\pi\rho \left[ \gamma - \frac{(2n+1)a^2\xi_0}{(n+1)T} \right] = \frac{\alpha^2}{(n-1)\frac{2n}{n-1}T^{\frac{2n}{n-1}}} - \frac{a^2(n+2)}{(n-1)^2T^2} - \Lambda \quad (35)$$

Eliminating  $\rho(t)$  between equations (28) and (35), we get

$$\Lambda = \frac{(n+1)\alpha^2}{(2n+1)a^2\xi_0(n-1)\frac{2n}{n-1}T^{\frac{n+1}{n-1}} - (1+\gamma)(n+1)(n-1)\frac{2n}{n-1}T^{\frac{n+1}{n-1}}} - \frac{a^2(n+2)}{(n-1)^2T^2}$$

$$\Lambda = \frac{(n+1)\alpha^2}{(n-1)\frac{2n}{n-1}T^{\frac{n+1}{n-1}} [(2n+1)a^2\xi_0 - (1+\gamma)(n+1)]} - \frac{a^2(n+2)}{(n-1)^2T^2} \quad (36)$$

From equations (27), (28), (32) and (33), it is observed that  $\rho$  and  $p$  vary as  $1/T$ . The models are singular at  $T = 0$  and as they evolve, the pressure and density decreases.

From equations (34) and (36), we observe that the cosmological constant is a decreasing function of time and it approaches a small value as time progresses (*i.e.* present epoch), which explains the small value of  $\Lambda$  at present. These positive values of  $\Lambda$  in both cases (I) and (II) are supported by results from recent supernovae observations [31-33, 35, 36].

**4 Physical and Geometrical Features of the Model**

The components of the shear tensor  $\sigma_i^j$  are given by

$$\sigma^2 = \frac{1}{9} \frac{(2n+1)^2 a^4}{(n-1)^2 T^2}, \quad \frac{\sigma}{\theta} = \frac{1}{3} \frac{\frac{(2n+1)a^2}{(n-1)T}}{\frac{(2n+1)a^2}{(n-1)T}} = \frac{1}{3} \quad (37)$$

$$\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0. \quad (38)$$

From equations (29) and (37), as  $T \rightarrow \infty$ , the expansion scalar  $\theta$  and shear scalar  $\sigma$  tends to zero. The volume is given by

$$V = \sqrt{-g} = \frac{e^{-\alpha x}}{a} [(n-1)T]^{\frac{2n+1}{n-1}}. \quad (39)$$

The rate of expansion  $H_i$  in the direction of  $X, Y, Z$  axes are

$$H_1 = H_2 = \frac{an}{(n-1)T} \quad (40)$$

$$H_3 = \frac{a}{(n-1)T}. \quad (41)$$

Since  $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$ , the model does not approach isotropy for large values of  $T$ .

The model is anisotropic. From equation (39), we get that when  $T \rightarrow \infty$ , the volume tends to infinity. The model, in general, represents shearing and rotating universe. The model starts expanding with a Big bang at  $T = 0$  and the expansion in the model increases as time decreases.

### Particular models

**Model I.** If we set  $n = 2$ , then the geometry of the space-time (26) will be

$$ds^2 = -\frac{dt^2}{a^2} + T^4 dx^2 + T^4 e^{-2\alpha x} dy^2 + T^2 dz^2. \quad (42)$$

The cosmological constant, effective pressure and density of model (42) are

$$\Lambda = \frac{3\alpha^2}{T^3 [5a^2\xi_o - 3 - 3\gamma]} - \frac{4a^2}{T^2} \quad (43)$$

$$\bar{p} = -\frac{\alpha^2}{8\pi T^3} \left[ \frac{1}{T} + \frac{3}{5a\xi_o - 3 - 3\gamma} \right] \quad (44)$$

$$\rho = \frac{1}{8\pi T^2} \left[ 4a^2 + \frac{3\alpha^2}{T(5a\xi_o - 3 - 3\gamma)} \right] \quad (45)$$

The scalar expansion  $\theta$  is obtained as

$$\theta = \frac{5a^2}{T}. \quad (46)$$

From equation (43) it is observed that the cosmological constant  $\Lambda$  is found to be positive and is a decreasing function of time.

From equations (44) and (45)  $\bar{p}$  and  $\rho$  vary as  $1/T$ . The model is singular at  $T = 0$ , and they evolve as the pressure and density decreases.

The rate of expansion in the direction of  $X, Y, Z$  axes is given by

$$H_1 = H_2 = \frac{2a}{T} \quad (47)$$

$$H_3 = \frac{a}{T} \quad (48)$$

The non vanishing component of shear tensor is obtained as

$$\sigma = \frac{5a^2}{3T}.$$

Now

$$\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} = \frac{1}{3} \neq 0. \quad (49)$$

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Since  $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$ , the model does not approach isotropy for large values of  $T$ .

The model in general represents shearing and rotating universe.

The model starts expanding with a Big bang at  $T = 0$  and the expansion in the model decreases as time increases and expansion in the model becomes zero at  $T = \infty$ .

Both density and pressure in the model become zero at  $T = \infty$ .

**Model II.** If we set  $n = -2$ , then the geometry of space-time (26) will be

$$ds^2 = -\frac{dT^2}{a^2} + 3^{4/3}T^{4/3}dx^2 + 3^{4/3}T^{4/3}e^{-2\alpha x}dy^2 + 3^{-2/3}T^{-2/3}dz^2. \quad (50)$$

The cosmological constant, effective pressure and density of model (50) are

$$\Lambda = \frac{\alpha^2}{3^{4/3}T^{1/3} [3a^2\xi_o - 1 - \gamma]} \quad (51)$$

$$\bar{p} = -\frac{\alpha^2}{8\pi T^3} \left[ \frac{1}{T} + \frac{3}{5a^2\xi_o - 3 - 3\gamma} \right] \quad (52)$$

$$\rho = \frac{1}{8\pi T^2} \left[ 4a^2 + \frac{3\alpha^2}{T(5a^2\xi_o - 3 - 3\gamma)} \right]. \quad (53)$$

The scalar expansion  $\theta$  is obtained as

$$\theta = \frac{a^2}{T}. \quad (54)$$

From equation (51), the cosmological constant  $\Lambda$  is found to be positive and it is decreasing function of time.

From equations (52) and (53)  $\bar{p}$  and  $\rho$  vary as  $1/T$ .

The rate of expansion  $H_i$  in the direction of  $X, Y, Z$  axes is

$$H_1 = H_2 = \frac{2a}{3T} \quad (55)$$

$$H_3 = -\frac{a}{3T}. \quad (56)$$

The non vanishing component of shear tensor is obtained as

$$\sigma = \frac{a^2}{3T}.$$

Now

$$\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} = \frac{1}{3}. \quad (57)$$

Since  $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$ , the model does not approach isotropy for large values of  $T$ . The model in general represents shearing and rotating universe. The model starts expanding with a Big bang at  $T=0$  and the expansion in the model decreases as time increases and expansion in the model becomes zero at  $T = \infty$ .

Both density and pressure in the model become zero at  $T = \infty$ .

**Model III.** If we set  $n = 3$ , then the geometry of space time (26) will be

$$ds^2 = -\frac{dT^2}{a^2} + 8T^3 dx^2 + 8^3 e^{-2\alpha x} dy^2 + 2T dz^2 \quad (58)$$

The cosmological constant, effective pressure and density of model (58) are

$$\Lambda = \frac{1}{2T^2} \left[ \frac{\alpha^2}{7a\xi_0 - 4 - 4\gamma} - \frac{5a^2}{2} \right] \quad (59)$$

$$\bar{p} = \frac{\alpha^2}{64\pi T^2} \left[ \frac{4}{4 + 4\gamma - 7a^2\xi_0} - \frac{1}{T} \right] \quad (60)$$

$$\rho = \frac{1}{16\pi T^2} \left[ \frac{\alpha^2}{7a^2\xi_0 - 4 - 4\gamma} \right] \quad (61)$$

The scalar expansion  $\theta$  is obtained as

$$\theta = \frac{7a^2}{2T}. \quad (62)$$

From equation (59) it is observed that the cosmological constant  $\Lambda$  is found to be positive and it is a decreasing function of time.

From equations (60) and (61)  $\bar{p}$  and  $\rho$  vary as  $1/T$ .

The model is singular at  $T = 0$  and they evolve as the pressure and density decrease.

The rate of expansion  $H_i$  in the direction of  $X, Y, Z$  axes is given by

$$H_1 = H_2 = \frac{3a}{2T} \quad (63)$$

$$H_3 = \frac{a}{2T}. \quad (64)$$

The non vanishing component of shear tensor is obtained as

$$\sigma = \frac{7a^2}{6T}.$$

Now

$$\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} = \frac{1}{3}. \quad (65)$$

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Since  $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$  the model does not approach isotropy for large values of  $T$ .

The model in general represents shearing and rotating universe.

The model starts expanding with a Big bang at  $T = 0$  and the expansion in the model decreases as time increases and expansion in the model becomes zero at  $T = \infty$ .

Both density and pressure in the model become zero at  $T = \infty$ .

**Model IV.** If we put  $n = -3$ , then the geometry of space time (26) will be

$$ds^2 = -\frac{dT^2}{a^2} + 8T^{3/2}dx^2 + 8^{3/2}e^{-2\alpha x}dy^2 - \frac{1}{2}T^{-1}dz^2. \quad (66)$$

The cosmological constant, effective pressure and density of model (66) are

$$\Lambda = \frac{\alpha^2}{4T^{1/2}[2 + 2\gamma - 5a^2\xi_0]} + \frac{a^2}{16T^2} \quad (67)$$

$$\bar{p} = -\frac{\alpha^2}{32T^{1/2}} \left[ \frac{1}{5a^2\xi_0 - 2 - 2\gamma} + \frac{1}{2T} \right] \quad (68)$$

$$\rho = \frac{1}{32\pi} \left[ \frac{5a^2}{4T^2} - \frac{2\alpha^2}{T^{1/2}(5a^2\xi_0 - 2 - \gamma)} \right] \quad (69)$$

The scalar expansion  $\theta$  is obtained as

$$\theta = \frac{7a^2}{2T}. \quad (70)$$

From equation (67) it is observed that the cosmological constant  $\Lambda$  is found to be positive and it is a decreasing function of time.

From equations (68) and (69),  $\bar{p}$  and  $\rho$  vary as  $1/T$ .

The model is singular at  $T = 0$  and it evolves as the pressure and density decrease.

The rate of expansion  $H_i$  in the direction of  $X, Y, Z$  axes are given by

$$H_1 = H_2 = \frac{3a}{2T} \quad (71)$$

$$H_3 = \frac{a}{2T}. \quad (72)$$

The non vanishing component of shear tensor is obtained as

$$\sigma = \frac{7a^2}{6T}.$$

Now

$$\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} = \frac{1}{3}. \quad (73)$$

Since  $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$ , the model does not approach isotropy for large values of  $T$ .

The model in general represents shearing and rotating universe.

The model starts expanding with a Bigbang at  $T = 0$  and the expansion in the model decreases as time increases and expansion in the model becomes zero at  $T = \infty$ .

Both density and pressure in the model become zero at  $T = \infty$ .

## 5 Conclusion

We have studied a new class of Bianchi type-III anisotropic cosmological models with a bulk viscous fluid as the source of matter. The models are expanding and shearing. In these models we have observed that they do not approach isotropy for large value of time  $t$ .

The cosmological constant in all cases is a decreasing function of time and approaches a small value as time increases (*i.e.* the present epoch). The values of cosmological constant for these cases are found to be small and positive which are supported by the results from supernovae observations recently obtained by the High-Z supernova team and supernova cosmological project. We have verified a physically relevant decay law for the cosmological constant  $\Lambda$ .

The model exists for every integer  $n$  except for  $n = 1$  and  $n = -1$ .

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