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Dynamically Stable Single and Multirod Linear Resonators with Large-Volume Fundamental Mode

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Abstract. Multirod dynamically stable linear resonators with equal beam spot sizes within each rod, despite of the individual thermal optical power of the rods, are introduced. It is shown that reduced optical power of the thermal lenses allows for reliable high-power operation. Ways to control power density loading of the intracavity optical elements as well as resonator misalignment sensitivity are discussed.

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1 Introduction

A great number of scientific and industrial applications of lasers requires generation of high-power diffraction-limited laser beams. However achieving high-power beams of high quality is still an open problem primarily due to the thermooptical effects in solid-state laser media. Thermal focusing and birefringence of the pumped rod affect very strong operation of lamp-pump Nd:YAG lasers, especially in the case of single transverse mode operation.

Heat dissipation through the side-rod surface in a solid-state laser necessarily establishes radial thermal gradient that makes it a positive graded index lens of considerable optical power $D = 1/f$ m^{-1} (f is the lens focal length). The lens dioptric power is proportional to the input pump power P_{in}. For uniformly pumped rod the commonly used expression is [1,2]

$$\frac{1}{f} = \frac{\eta_d k}{\pi r^2} P_{in}, \qquad (1)$$

where η_d is the ratio between the heat dissipated in the rod and the pump power, k is a constant dependent on the optomechanical properties of the material, and

r is the rod radius. Rod thermal lensing converts the solid-state laser resonators into resonators containing lenses of variable focal length. The properties of stable linear resonators with a single variable lens are well investigated, including the so-called dynamically stable resonators in a very generalized manner [1-9]. In this work we use the generalized theory created by Magni *et al.* [4-8].

2 Basic Results for Resonators with Single Variable Lens

The main results are summarized in [5]. In general the resonator is considered to consist of two curved mirrors, thin variable lens and two arbitrary optical systems between them that include the distances to the mirrors and the lens. The curved mirrors are presented as a combination of a plane mirror and the corresponding lens of focal length $f_i = R_i$ $(i = 1, 2)$ (see Figure 1a). Then the generic resonator is modeled, as shown in Figure 1b, as plane-plane resonator that encloses a variable lens sandwiched between two generic optical systems. Generic optical systems are described with suitable 2×2 ray transfer matrices associated with 2×1 misalignment vectors.

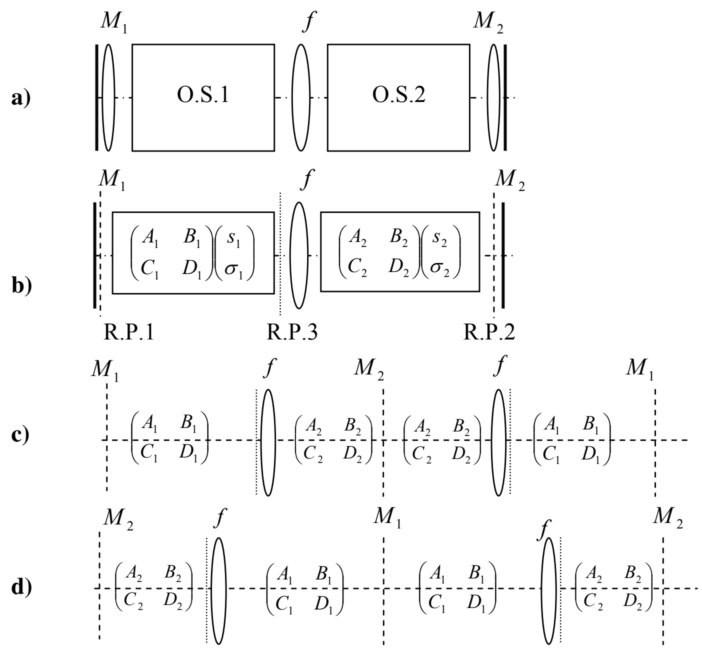


Figure 1. **a)** Resonator with a thin variable lens and two arbitrary optical systems. The curved mirrors are presented as combination of a plane mirror and a corresponding lens of focal length; **b)** Plane-plane resonator with a variable lens sandwiched between two generic optical systems. The dashed lines are reference planes; **c)** Resonator with two active elements and $u_1 > u_2$; **d)** Resonator with two active elements and $u_2 > u_1$.

A. Stability zones – these resonators have always two stability zones of equal width in terms of dioptric power. The resonator stability limits are listed in Table 1, where A_i, B_i, C_i, D_i $(i = 1, 2)$ are the elements of the corresponding matrix.

Table 1. Resonator stability limits

Stability limit	Rod dioptric power	Stability zone $\|B_1D_1\| > \|B_2D_2\|$	Stability zone $\|B_1D_1\| < \|B_2D_2\|$
$A = 0$	$\frac{1}{f} = \frac{C_1}{D_1} + \frac{A_2}{B_2}$	**I**	**II**
$D = 0$	$\frac{1}{f} = \frac{A_1}{B_1} + \frac{C_2}{D_2}$	**II**	**I**
$B = 0$	$\frac{1}{f} = \frac{A_1}{B_1} + \frac{A_2}{B_2}$	**I**	**I**
$C = 0$	$\frac{1}{f} = \frac{C_1}{D_1} + \frac{C_2}{D_2}$	**II**	**II**

B. Spot sizes – according to the behavior of the fundamental mode spot size at the variable lens, *i.e.* the rod, these zones are mirror symmetric and within each stable zone the rod spot size reaches a minimum of equal magnitude (see Figure 2). Around these minima the rod spot size is, at the first order, insensitive to fluctuations of the lens focal length caused by the fluctuations of the input pump power, and the laser operates in the so-called dynamically stable regime [2]. It is obvious that solid-state laser cannot work at the stable zones boundaries, where the rod spot size becomes large. In this region pump fluctuations involve uncontrollable variation of the mode size at the rod aperture connected with significant output fluctuations. The rod spot size ω_{30} at the point of dynamical stability (Figure 2) is inversely proportional to the width of the stability zone and this generalized relationship is independent from the number of the intracavity elements [5]

$$\omega_{30}^2 = \frac{2\lambda}{\pi} \frac{1}{\Delta \frac{1}{f}} , \tag{2}$$

where λ is the laser wavelength and $\Delta(1/f)$ is the dioptric width of the zones. The range of stability and the position of the two dynamically stable points are predefined by the resonator parameters excluding the variable lens. The spot size on the mirror M_1 at the dioptric points of the lens spot extreme has the same quantity in the two zones of stability. The stationary values of the spot size on the mirror M_2 are not the same in the two zones.

C. Misalignment sensitivity – it plays an important role in narrow stable range operation. It has been shown [2,5,6] that the two stable zones have different behavior in the presence of any (small) resonator misalignment. In one of the

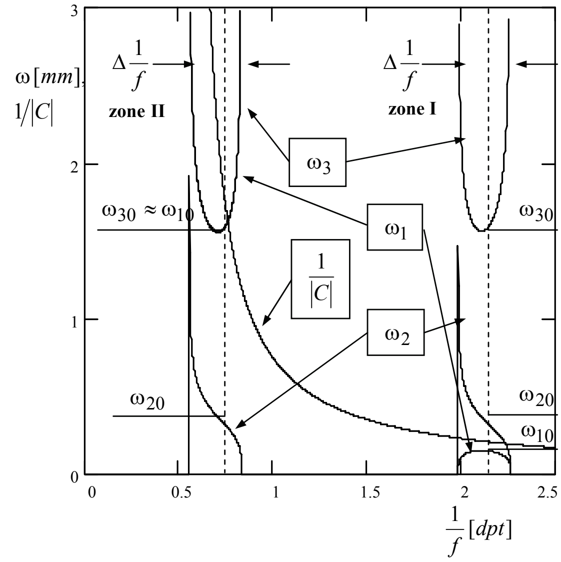


Figure 2. Spot sizes and misalignment sensitivity as a function of dioptric power of the variable lens.

zones (called zone II) the resonator reaches at one of the zone ends the equivalent concentric (or plane-plane) configuration and the resonator misalignment tends to infinity, (the term $|1/C|$ in Figure 2). At the other zone end the resonator misalignment sensitivity falls to moderate quantities. This fact means that when the zone power width is comparable to the range of the pump power fluctuations (in presence of any small resonator misalignment), the absolute displacement of the ray axis at the position of variable lens will vary in wide range. So there must take place significant fluctuations of the input power. In the other zone (zone I) misalignment sensitivity varies slowly remaining essentially lower than misalignment sensitivity for the zone II. Any small resonator misalignment cannot involve significant additional variations in the output power. Consequently zone I has another capability to be dynamically stable against resonator misalignment. The resonator reaches the maximum of the intracavity optical power to which corresponds the minimal misalignment sensitivity. Working in this zone is mandatory when the zone width is comparable to the range of input fluctuations. However in this zone the beam is permanently focused at the surface of one of the two resonator mirrors.

These problems, together with thermally induced birefringence, limit the beam quality improvement. Hence the average output power increasing of such type of lasers is consequently limited. Reducing the net value of the thermal lens optical power inside the resonator seems to be the general manner to overpass the above problems. An effective way to reduce the variable optical power inside the resonator is to distribute it within a multirod resonator of a proportionally extended length. The conception of multirod resonators for increasing the output power of solid-state lasers without reducing the beam quality has been introduced in [10], where it has been shown that only symmetric resonator configurations have properties similar to those of the equivalent single rod resonator. The properties of multirod resonators are well investigated in [11-13]. In this paper we show that multirod dynamically stable resonators have properties similar to the dynamically stable resonator with a single active element. The main expected advantage with this resonator conception is extending the output power proportionally to the number of the rods at the same beam quality as for an equivalent single rod resonator.

3 Multirod Resonators

In order to analyze a multirod resonator we initially consider a single rod resonator optimized for dynamically stable operation and with a predefined rod mode volume as a generic resonator. Two transfer matrices, two misalignment vectors, and a variable thin lens between them, are illustrated in Figure 1b. Then we spread them as an infinite periodic lens guide as it is shown in Figure 1c. The arbitrary cut between both mirror planes that contains the desired number of variable lenses (rods), forms a corresponding multirod resonator. In the real systems at the end mirror planes are placed mirrors of corresponding curvatures. At the internal mirrors positions must be placed thin lenses of focal length

$$f_i = R_i/2\,, \tag{3}$$

where R_i, $i = 1, 2$, are the radiuses of the curvature of the end mirrors of the generic resonator, respectively in linear arrangement of the resonator, or true mirrors of the corresponding curvature if the resonator is folded at the planes of the internal mirrors positions.

Considering the simple two mirrors generic resonator the transfer matrices according to Figure 1b become

$$\begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} = \begin{pmatrix} 1 & L_i \\ -\dfrac{1}{R_i} & 1 - \dfrac{L_i}{R_i} \end{pmatrix}\,, \qquad i = 1, 2. \tag{4}$$

Then it is convenient to parameterize the system with the early-involved useful parameters [2]

$$u_i \equiv B_i D_i = L_i\Big(1 - \frac{L_i}{R_i}\Big)\,, \qquad i = 1, 2 \tag{5}$$

with the following physical interpretation:

$$\Delta\frac{1}{f} = \min\Big(\frac{1}{|u_1|}, \frac{1}{|u_2|}\Big), \tag{6}$$

and the dioptric distance between the stable zones beginnings (^{b}) or ends (^{e}) of

$$\left|\left(\frac{1}{f}\right)_I^{b,e} - \left(\frac{1}{f}\right)_{II}^{b,e}\right| = \max\left(\frac{1}{|u_1|}, \frac{1}{|u_2|}\right). \quad (7)$$

Also the dioptric gap between the zones is

$$\left(\Delta\frac{1}{f}\right)_{I,II} = \left|\frac{1}{|u_1|} - \frac{1}{|u_2|}\right|. \quad (8)$$

The variable $1/f$ is also convenient to parameterize as it has been done in [2]

$$x = \frac{1}{f} - \frac{1}{L_1} - \frac{1}{L_2}\,. \quad (9)$$

With this parametrization the single pass matrix from mirror 1 to mirror 2 becomes

$$T = \begin{pmatrix} g_1^* & L_{eff} \\ \dfrac{g_1^* g_2^* - 1}{L_{eff}} & g_2^* \end{pmatrix} = (-)\begin{pmatrix} \dfrac{L_2}{L_1}(1+xu_1) & L_1 L_2 x \\ \dfrac{u_1+u_2+xu_1u_2}{L_1L_2} & \dfrac{L1}{L_2}(1+xu_2) \end{pmatrix}. \quad (10)$$

Resonators containing even number of rods must have end mirrors of equal curvature and can be obtained in two different configurations. The resonators with $u_1 > u_2$ are noted with (I) and the resonators with $u_2 > u_1$ are noted with (II). This is illustrated in Figure 1c,d for two-rod resonators. According to the figure the resonator single-pass transfer matrices have the presentation

$$T_n^{(I)} = (T'T)^{n/2}\,, \quad (11)$$

$$T_n^{(II)} = (TT')^{n/2}\,, \quad (12)$$

where $n = 2m$ is the number of the rods, $m = 1, 2, 3\ldots$ is an integer number, T is the single pass transfer matrix of the generic resonator, T' is the same matrix in the opposite passing direction. The odd number of rods resonator can be present by the following single-pass matrix:

$$T_n^{(I)} = T\,(T'T)^m\,, \qquad n = 2m+1\,. \quad (13)$$

Then with the aid of the Sylvester's theorem we can obtain in a very generalized manner

$$T_{n=2m} = \begin{pmatrix} \cos(m\theta) & 2g_{2,1}^* L_{eff}\dfrac{\sin(m\theta)}{\sin\theta} \\ \dfrac{2g_{1,2}^*(g_1^* g_2^* - 1)}{L_{eff}}\dfrac{\sin(m\theta)}{\sin\theta} & \cos(m\theta) \end{pmatrix}, \quad (14)$$

$$T_{n=2m+1} = \begin{pmatrix} g_1^*\Big[\cos(m\theta)+2(g_1^*g_2^*-1)\dfrac{\sin(m\theta)}{\sin\theta}\Big] & L_{eff}\Big[\cos(m\theta)+2g_1^*g_1^*\dfrac{\sin(m\theta)}{\sin\theta}\Big] \\ \dfrac{g_1^*g_1^*-1}{L_{eff}}\Big[\cos(m\theta)+2g_1^*g_1^*\dfrac{\sin(m\theta)}{\sin\theta}\Big] & g_2^*\Big[\cos(m\theta)+2(g_1^*g_2^*-1)\dfrac{\sin(m\theta)}{\sin\theta}\Big] \end{pmatrix}, \tag{15}$$

where

$$\cos\theta = 2g_1^*g_2^* - 1. \tag{16}$$

These expressions are for the area of the resonator stability. In the unstable region the functions **sin** and **cos** become corresponding hyperbolic functions. The resonators trajectories on the stability diagram are explicit functions derived by eliminating x in equations

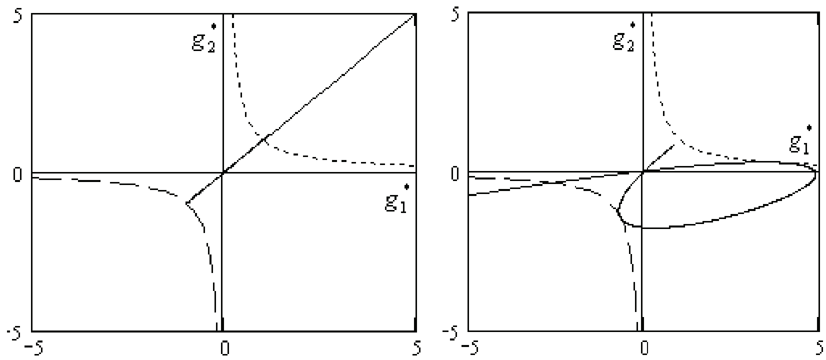
$$g_{1(n)}^* = A_n\,, \qquad g_{2(n)}^* = D_n\,, \tag{17}$$

where $g_{1(n)}^*$ and $g_{2(n)}^*$ sign the equivalent Kogelnic's g-parameters of the corresponding n-rods resonator. The trajectories are curves of the n-th order. The two types of even number rods resonators move equally on a straight line of the corresponding order as it is shown in Figure 3a for a four-rods resonator. Odd number resonators move on non-degenerated curves (see Figure 3b), which is for a three-rod resonator. The main relationships for the resonators can be outlined analyzing the resonator stability condition

$$A_nB_nC_nD_n < 0, \tag{18}$$

equating to zero each element

$$A_n = 0, \quad B_n = 0, \quad C_n = 0, \quad D_n = 0, \tag{19}$$



a) **b)**

Figure 3. **a)** Stability diagram for resonators with even number of rods; **b)** Stability diagram for resonators with odd number of rods.

and applying inductive approach. The roots common for the pair equations $A_n = 0$, $D_n = 0$ give the crossing confocal point from the resonator at the origin of the g-diagram. The roots common for the pair $B_n = 0$, $C_n = 0$ become double root of the equation

$$A_n B_n \equiv g^*_{1(n)} g^*_{2(n)} - 1 = 0, \tag{20}$$

and consequently they are the touching point of the resonator configuration on the hyperbola branches $g_1^* g_2^* = 1$. Thus, the solutions that are roots give the stability zone boundaries. So we have stable zone boundaries of

$$x = \begin{cases} 0 \\ -\dfrac{1}{u_1} \\ -\dfrac{1}{u_2} \\ -\Big(\dfrac{1}{u_1} + \dfrac{1}{u_2}\Big) \end{cases} . \tag{21}$$

The zones of stability are two, with the same width and separation as for the generic resonator, according to Eqs. (6), (7) and (8) for all considered cases. Also there is a turning point of the resonator trajectory $(g^*_{1(n)} g^*_{2(n)} = \max)$ at the point

$$x = -\frac{1}{2}\Big(\frac{1}{u_1} + \frac{1}{u_2}\Big), \tag{22}$$

exactly in the middle of the dioptric gap between stable zones in all considered cases. For resonators with an even number of rods the curve passes through the confocal point $n/2$ times and touches $m/2 - 1$ times hyperbola $g_1^* g_2^* = 1$ branches in each stable zone. Odd numbers rods resonators $(n = 2m + 1)$ do it $m/2$ and $m/2$ times, respectively.

Let the matrix $M_1(i)$ be the round-trip transfer matrix at an arbitrary reference plane marked by (i) inside the generic resonator. The elements of this matrix are related with a common procedure [14] to the spot size and the wave front radius of the eigenbeam at the reference plane. It is easy to verify that the corresponding round trip matrix for the considered multirod resonators is

$$M_n\,(i) = [M_1\,(i)]^n\,, \tag{23}$$

where i signs there all equivalent positions of the reference plane that take place in the multirod resonator (Figure 1c). Such multirod resonator has the same intermediate field distribution as its single-rod origin in the corresponding sections. Eq. (2) also takes place and parameters u_1 and u_2 are the same tools for the rods spot-sizes predefining, so such type of resonators can solve the task we have set.

4 Misalignment Sensitivity

In this section we consider the resonators as the first-order misaligned optical systems [5] by any small mirror tilts and lenses shifting in transverse direction. The position and the slope (x_i, θ_i) of the eigenmode axis at the lens of number i in the considered multirod resonators can be obtained from [5]

$$\begin{pmatrix} x_3 \\ \theta_3 \end{pmatrix} = -\frac{1}{C} \begin{bmatrix} D_2\sigma_1 + D_1\sigma_2 \\ -C_2\sigma_1 + (C_1 - D_1/f)\sigma_2 \end{bmatrix}, \tag{24}$$

where C is the element 2,1 of the matrix T (Eq. 10). The element $|1/C|$ is the absolute value of the focal length of the optics between the mirrors ('mirrors' power included) that obviously determines the overall resonator misalignment sensitivity.

If any resonator component is misaligned, the mode axis displacement on any arbitrary variable lens (rod) is always contained in the term $1/C$. Consequently this term that for the n-rod resonator is $1/C_n$ stands for the overall resonator misalignment sensitivity.

For two-rod resonators with Eqs. (10) and (14) we obtain the expressions

$$\left|\frac{1}{C_2}\right|^{(I)} = \frac{L_1^2}{2u_1^2 u_2 \left(x + \frac{1}{u_1}\right)\left(x + \frac{1}{u_1} + \frac{1}{u_2}\right)}, \tag{25}$$

$$\left|\frac{1}{C_2}\right|^{(II)} = \frac{L_2^2}{2u_1 u_2^2 \left(x + \frac{1}{u_2}\right)\left(x + \frac{1}{u_1} + \frac{1}{u_2}\right)}. \tag{26}$$

They show that although these resonators have equal zones widths and separation, and, consequently equal rod spots distributions in the dioptric scale, they have quite different behavior of the misalignment sensitivity. Let $u_1 > u_2$. In this case parameter u_1 determines the minimal rod spot size. When this parameter is connected with the end mirrors – resonator type I, the term $|1/C_2|$ goes to infinity at $x = -1/u_1$ in the end of each stable zone. Consequently, this resonator has no dynamical stability against transverse misalignment, although its configuration offers some design possibilities for reduction of end mirrors power-density-loading. The resonators of type II have always a dynamically stable against misalignment zone.

It is seen from Eqs. (14) and (15) that all resonators with number of rods $n > 2$ have points in which the absolute value of the intracavity optical power cancels out or the mode axis displacement goes to infinity in both stable zones. These are touching points to the hyperbola $g_1^* g_2^* - 1$ whose positions and number has already been indicated in previous section. So we outline the fact that symmetric multirod resonators containing more than two rods have no dynamical stability against misalignment and cannot work reliable when the zones width is of the range of the pump power fluctuations. Also the absolute deviation of the mode axis from the rods axes must increase more rapidly with increasing the number of adjustable elements and spaces between them according to Eq. (24). So multiplying the number of the rod has additional limits due to the considered problem. Intracavity optical inhomogeneousness increases with the number of the rods so we think that resonators containing more than two rods cannot be used for large-volume high-power single mode operation. But some of these resonators, especially four-rod configuration (see Figure 3a) can be optimized with widened stable zones for reliable high average power generation of multi-mode beams with improved spatial quality.

5 Sensitivity to Asymmetry in Resonator Parameters

Another problem consequential of the concept of symmetric resonators is the sensitivity of the resonator parameters to its asymmetry. Such can be distances

$$L_i' = L_i + \Delta(L_i), \tag{27}$$

mirror radii or their equivalents

$$R_i' = R_i + \Delta(R_i), \tag{28}$$

and thermal optical power distribution

$$1/f_i{}' = 1/f_i + \Delta(f_i), \tag{29}$$

where the index i stands for the position of the corresponding element. The deviations $\Delta(X_i)$ may be either positive or negative. The deviated element matrices we present as matrix sums

$$\begin{pmatrix} 1 & L_i' \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & L_i \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \Delta(L_i) \\ 0 & 0 \end{pmatrix} \tag{30}$$

$$\begin{pmatrix} 1 & 0 \\ -\dfrac{1}{R_i'} & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{R_i} & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \dfrac{\Delta(R_i)}{R_i^2} & 0 \end{pmatrix} \tag{31}$$

$$\begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f_i'} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f_i} & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\Delta(f_i) & 0 \end{pmatrix}. \tag{32}$$

Inserting whichever of matrices Eqs. (30), (31) and (32) into matrix products Eqs. (11), (12) and (13) at their assumed position one can establish that even single element non-symmetry adds a linear according to Δ component to each element of the resonator single-pass transfer matrix. So gaps of resonator instability appear around the points of singularity and the hyperbola touching points (Eq. 20). Each stable zone splits into n-subzones. Also there are some shifting and changes in the total width of the stable region. To estimate the correlation between the dioptric power magnitudes of the corresponding zones alterations and the absolute single element asymmetry, we made more detailed analysis for two-rod resonator. We suppose the parameter u_1 is always connected with the end mirrors. The results are as follows:

A) The single length difference $L_1' = L_1 + \Delta$.

Provided that all dimensions are in meters, for the resonator of type (I): $|u_1| > |u_2|$ (which is the case of high resonator misalignment sensitivity in both stable zones), with approximations $|u_1| \gg |u_2|$ and $R_1 \gg L_1$, we have obtained a gap of the resonator instability around the confocal point of dioptric power width

$$\Delta x_{L_1} = \frac{|\Delta|}{2L_1^2} \tag{33}$$

for both stable zones. For the resonator of type (II): $|u_2| > |u_1|$ with analogous approximations $|u_2| \gg |u_1|$ and $R_1 \gg L_1$ for mechanical sensitive zone, we have obtained

$$\Delta x_{L_1} = \frac{|\Delta|}{4u_2^2}, \tag{34}$$

(very tight gap), and for the dynamically stable against misalignment zone

$$\Delta x_{L_1} = \frac{|\Delta|}{L_1^2}, \tag{35}$$

(two times higher than this for the resonator of type (I). The dioptric zones shifting and changes in their widths are of the same order. The changes in the width of the zones are of the same order.

B) The end mirror curvature non-symmetry $R_1{}' = R_1 + \Delta$.

The case $|u_1| \gg |u_2|$ gave dioptric gap of

$$\Delta x_{R_1} = \frac{|\Delta|}{2(R_1 - L_1)^2} \tag{36}$$

for both zones, where $|R_1 - L_1| \gg 0$. The case $|u_2| \gg |u_1|$ gave

$$\Delta x_{R_1} = \frac{|\Delta|}{4u_2^2} \frac{L_1^2}{R_1^2} \tag{37}$$

for mechanical sensitive zone and

$$\Delta x_{R_1} = \frac{|\Delta|}{(R_1 - L_1)^2} \tag{38}$$

for insensitive zone also with $|R_1 - L_1| \gg 0$.

The obtained expressions give quantitative criteria for the two-rod resonator longitudinal alignment accuracy and lens-to-mirror matching requirements. The corresponding requirements for symmetric resonators containing more than two rods seem to be of the same or higher order.

C) Differences in variable lenses dioptric power $1/f_1{}' = 1/f_1 + \Delta$.

This is the most difficult to control the resonator parameter. The zones shifting around the confocal singularity is $\Delta x_f = -\Delta/2$. The dioptric gap between two zones of stability is

$$\Delta x_f = \Delta \frac{u_2}{\sqrt{u_1^2 + u_2^2}}. \tag{39}$$

From Eq. (39) it is seen that resonator of type (I) ($|u_1| > |u_2|$), which has no dynamical stability against misalignment, is less sensitive to differences in thermal power distribution

$$\Delta x_f \approx \Delta \frac{u_2}{u_1} \tag{40}$$

than resonator of type (II) $|u_2| > |u_1|$, which has one zone dynamically stable against misalignment

$$\Delta x_f \approx \Delta. \tag{41}$$

The symmetric resonators reduce the considered gap to

$$\Delta x_f = \Delta/\sqrt{2}. \tag{42}$$

The dioptric difference almost converts directly into dioptric gap between subzones.

It seems that with increasing the number of the resonator sections, *i.e.* the rod number, the dioptric gaps between subzones are almost of the same magnitude.

D) Thermal lens spot sizes deviation

At the presence of any asymmetry the distributions of the spot sizes at any individual thermal lens become rather different. This fact is very undesirable because due to different diffraction losses at the rod apertures the generated beam obtains a form that cannot utilize the all-available power from the rods.

The main deviations in rod spots are around the dioptric gaps between sub zones. It seems that affected area increases with the dioptric power width of the resonator instability gaps between the subzones. That is why we analyze the influence of the most difficult to control dioptric power differences. We obtained analytical expressions for nonsymmetrical spot-sizes distribution in the case of two-rod resonator.

We inserted a small deviation in the second of the two lenses according to Eq. (32). We obtained the rod-spot sizes from the common expression

$$\frac{\lambda}{\pi\omega_{1,2}^2} = \frac{\sqrt{1 - m_{1,2}^2}}{B_{1,2}}, \tag{43}$$

where subscripts 1 and 2 sign the first and the second lens respectively, $m_{1,2}$ are the half traces of the resonator round-trip matrices at the reference planes just before the corresponding lenses, $B_{1,2}$ are the corresponding round-trip equivalent lengths of the resonator. Applying corresponding calculations we have obtained that the two perturbed by Δ_f matrices have equal half-traces of

$$m_1 = m_2 = 2m^2 - 1 + 2Bm\Delta_f - u_1(2u_2x + 1)(B - 2u_2)\Delta_f^2, \tag{44}$$

where

$$m = [2x(xu_1u_2 + u_1 + u_2) + 1], \tag{45}$$

$$B = -2(2xu_1u_2 + u_1 + u_2), \tag{46}$$

m is the round-trip matrix half trace and B is the effective length at the lens reference plane of the generic single-rod resonator. We have also obtained that the two lenses "see" different resonator round-trip lengths

$$B_1 = -[2Bm + 2(B^2 - 2u_1^2)\Delta_f], \tag{47}$$

$$B_2 = -2Bm + (B^2 - 4u_2^2)\Delta_f. \tag{48}$$

It becomes clear that the difference of the effective resonator lengths for the two lenses in presence of such perturbation (due to the lens combination law) is the main reason for nonsymmetrical rod-spot-sizes distribution. Due to this reason any other compensating scheme except to fully symmetrize the resonator does not seem to be able to solve the problem. Another estimation can be done if the effective length deviation is small. Then the ratio of the spots areas becomes

$$\frac{\omega_2^2}{\omega_1^2} = \frac{2Bm + (B^2 - 4u_2^2)\Delta_f}{2Bm + 2(B^2 - 2u_1^2)\Delta_f} \approx 1 - \frac{(B^2 - 4u_1^2 + 4u_2^2)\Delta_f}{2Bm}. \tag{49}$$

Inside the stable zones and near to their boundaries, expression Eq. (49) becomes

$$\frac{\omega_2^2}{\omega_1^2} \approx 1 \pm 2u_2\Delta_f, \tag{50}$$

where the sign $\pm$ depends on the zone number and the position just before the corresponding zone boundary. The ratio in Eq. (50) tends close to one with decreasing the lens dioptric difference and decreasing the effective distance between the lenses as in our choice u_2 stands for the last. This result is in agreement with numerical results and conclusions obtained in [14]. Also it explains the selected resonator advantages in successful experimental work for high brightness high average power generation with two Nd:YAG rod with birefringence compensation [15].

Symmetric resonators containing more than two rods also can be analyzed from the same point of view. But the changes of rod-spot sizes distributions have much more variety difficult for generalization and cannot be presented with simple expressions. It seems that every case must be investigated separately. Numerical calculations for particular cases show approximately the same conclusions as for two-rod resonators. Also it seems that symmetric resonators containing more than two rods cannot offer schemes that minimize the lens-spot-size differences according to Eq. (50) except the short schemes that generate beams of low spatial quality.

6 Conclusions

We have analyzed multirod dynamically stable resonators and shown that they have two zones of stability of equal width and separation between them exactly as in the case of the generic single-rod resonators. These zones are not dynamically stable against transverse misalignment by the resonators of the type (I) ($u_1 > u_2$), but the resonators of the type (II) ($u_2 > u_1$) have always dynamically stable zone. The resonators with an even number of rods move on a straight line, such as the symmetric resonators. The resonators with odd number of rods move on the curve of n-th order. The estimation of the resonator instability around the confocal point at the presence of asymmetry in the resonator parameters has been made (in the case of two rod resonator). The ratio of the spots areas when difference in the dioptric power of the thermal lenses exists has been estimated also.

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References

[1] J. Steffen, J. P.Lortscher, and G. Herziger (1972) *IEEE J. Quantum Electron.* **QE-8** 239.

[2] P. Lortscher, J. Steffen, and G. Herziger (1975) *Opt. Quantum Electron.* **7** 505.

[3] H. Kogelnik (1965) *Bell. Syst. Tech. J.* **44** 455.

[4] V. Magni (1986) *Appl. Opt.* **25** 107.

[5] V. Magni (1987) *J. Opt. Soc. Amer.* **A4** 1962.

[6] V. Magni, G. Valentini, and S. De Silvestri (1991) *Opt. Quantum Electron.* **23** 1105.

[7] S. De Silvestri, P. Laporta, and V. Magni (1987) *IEEE J. Quantum Electron.* **QE-23** 1999.

[8] G. Cerullo, S. De Silvestri, V. Magni, and O. Svelto (1993) *Opt. Quantum Electron.* **25** 489.

[9] M.I. Demchuk, I. A. Manichev, V.P. Mikhailov, G.A. Pribytok, and A.K. Khoroshun (1972) *Ukrainian J. Quantum Electron.* **19** 175.

[10] V.R. Kushnir, A.N. Nemkov, and N.V. Shkunov (1975) *Sov. J. Quantum Electron.* **5** 713.

[11] H.P Kortz, R. Iffländer, and H. Weber (1981) *Appl. Optics* **20** 4124.

[12] Th. Graf, J.E. Balmer, R. Weber, and H.P. Weber (1997) *Optics Comm.* **135** 171.

[13] M.P. MacDonald, T. Graf, J.E. Balmer, and H.P. Weber (1998) *IEEE J. Quantum Electron.* **QE-34** 366.

[14] H.P Kortz, R. Iffländer, and H. Weber (1981) *Appl. Optics* **20** 4124.

[15] Koji Yasui (1996) *Appl. Opt.* **35** 2566.