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# Semiclassical Strings in Lunin-Maldacena Background

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**Abstract.** The aim of this paper is to investigate semiclassical rotating string configurations in the recently found Lunin-Maldacena background. This background is conjectured to be dual to the Leigh-Strassler $\beta$-deformation of $\mathcal{N} = 4$ SYM and therefore a good laboratory for tests of the AdS/CFT correspondence beyond the well explored $AdS_5 \times S^5$ case. We consider different multispin configurations of rotating strings by allowing the strings to move in both the $AdS_5$ and the deformed $S^5$ part of the Lunin-Maldacena background. For all of these configurations we compute the string energy in terms of the angular momenta and the string winding numbers and thus provide the possibility of reproducing our results from a computation of the anomalous dimension of the corresponding dilatation operator. This can be achieved by means of the Bethe ansatz techniques for the relevant sectors of the corresponding Yang-Mills theory. We also compare our results with those for multispin rotating strings on $AdS_5 \times S^5$.

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## 1 Introduction

The idea for the correspondence between the large N limit of gauge theories and string theory was proposed over thirty years ago [1] but its realization was given when Maldacena conjectured the AdS/CFT correspondence [2–4]. Since then this became a major research area and many fascinating discoveries were made in the last years. These include the discovery of Gubser, Klebanov and Polyakov [6] that the energy of certain string configurations in the limit of large quantum numbers reproduces the behavior of the anomalous dimension of the corresponding SYM operator. Soon after that Minahan and Zarembo [24] provided a way to compute the anomalous dimension of a certain dilatation operator by presenting it as the Hamiltonian of an integrable spin chain. These two discoveries opened the door towards many qualitative and quantitative checks of the AdS/CFT beyond the supergravity approximation. On the string side of the correspondence many semiclassical string configurations on $AdS_5 \times S^5$ were studied and the classical energies as well as their quantum corrections were obtained * [11]- [23]. On the gauge theory side the anomalous dimensions of the dilatation operator in certain sectors of the $\mathcal{N} = 4$ SYM were found (see [27] and references therein). Moreover these dimensions coincide with the energies of the corresponding semiclassical strings, thus providing remarkable quantative proofs of the string theory/gauge theory correspondence [24]- [34]. Nevertheless the $AdS_5 \times S^5/\mathcal{N} = 4$ SYM is the main and best explored example. Since the $\mathcal{N} = 4$ superconformal gauge theory is not appropriate phenomenologically we would like to extend the above ideas to less supersymmetric Yang-Mills theories. Such attempts were made by studying semiclassical strings in less supersymmetric backgrounds (Maldacena-Nunez, Pilch-Warner and other confining and warped geometries [35]- [54]) but it was not completely clear how to reproduce the string theory results from the SYM side. Fortunately such a possibility emerged when Lunin and Maldacena [7] found the gravity background dual to the Leigh-Strassler $\beta$-deformation of $\mathcal{N} = 4$ SYM. A quantitative check of this correspondence for the $su(2)$ sector was made in [10], thus giving the hope that we can extend the remarkable previous results to this case. Moreover in [9] the integrability of the string Hamiltonian on the Lunin-Maldacena background was proven by finding a Lax pair. This suggests that the interplay between integrable structures in $AdS_5 \times S^5$ [26] and $\mathcal{N} = 4$ SYM is also present in this less supersymmetric case.

These recent developments give us the motivation to investigate semiclassical strings on the Lunin-Maldacena background. In [10] a simple two spin string ansatz was considered. In this paper we would like to investigate multispin rotating string solutions, having angular momenta both in the $AdS_5$ and the $S^5$ part of the background. Since the anti-de Sitter piece of the metric stays undeformed under the TsT transformation generating the Lunin-Maldacena background we expect that the string motion there will not differ from the $AdS_5 \times S^5$ case. However the case of rotating string with three spins in the deformed $S^5$ part will lead to some non-trivial results for the energy. We will consider the following cases: i)three spins on the deformed $S^5$ $(J_1, J_2, J_3)$; ii)two spins on $AdS_5$ and two spins on the deformed $S^5$ $(S_1, S_2, J_1, J_2)$ and iii)the most general configuration of five spins $(S_1, S_2, J_1, J_2, J_3)$. In all of these cases we compute the energy of the rotating string in terms of the angular momenta and the string winding numbers. If we take the limit $\tilde{\gamma} \to 0$ our results reproduce those found for the undeformed $AdS_5 \times S^5$ case [16]. This should be expected since this is

*see [14] and [15] for a review

exactly the limit in which the Lunin-Maldacena background reduces to the usual $AdS_5 \times S^5$. We would also like to note that it should be possible to reproduce our results from the SYM side. For example the three $S^5$ spin solution should be dual to operators from the $su(3)$ sector, their anomalous dimensions can be found by a spin chain computation in the same spirit as this was done for the $su(2)$ case in [10]*.

In the next Section we will briefly review the form of the Lunin-Maldacena background and find the energy for the rotating three spin string ansatz. In Section 3 the case of two spins on $AdS_5$ and two spins on $S^5$ will be presented. In Section 4 we will find the general rotating string solution with five spins and compute its energy. Finally in the last Section we will present our conclusions and some open problems.

## 2 Three Spin String Solution in Lunin-Maldacena Background

We will begin by presenting the form of the supergravity background found by Lunin and Maldacena

$$\begin{aligned} ds^2_{str} = R^2\sqrt{H}\Big\{ & ds^2_{AdS_5} \\ & + \sum_{i=1}^{3}(d\rho_i^2 + G\rho_i^2 d\phi_i^2) + (\tilde{\gamma}^2+\tilde{\sigma}^2)G\rho_1^2\rho_2^2\rho_3^2(\sum_{i=1}^{3} d\phi_i)^2\Big\}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} &\frac{1}{G} = 1 + (\tilde{\gamma}^2+\tilde{\sigma}^2)(\rho_1^2\rho_2^2+\rho_1^2\rho_3^2+\rho_2^2\rho_3^2), \\ &H = 1+\tilde{\sigma}^2(\rho_1^2\rho_2^2+\rho_1^2\rho_3^2+\rho_2^2\rho_3^2), \\ &B_2 = R^2(\gamma G w_2 - \sigma w_1 d\psi), \\ &\psi = \frac{\phi_1+\phi_2+\phi_3}{3}, \\ &dw_1 = \cos\alpha\sin^3\alpha\sin\theta\cos\theta d\alpha d\theta, \\ &w_2 = \rho_1^2\rho_2^2 d\phi_1 d\phi_2 + \rho_2^2\rho_3^2 d\phi_2 d\phi_3 + \rho_3^2\rho_1^2 d\phi_3 d\phi_1, \\ &\rho_1 = \sin\alpha\cos\theta, \\ &\rho_2 = \sin\alpha\sin\theta, \\ &\rho_3 = \cos\alpha. \end{aligned} \tag{2}$$

Now we are going to write the Polyakov action for strings staying at the center of $AdS_5$ and moving on the deformed five sphere (*i.e.* strings on $R_t \times S^5_\beta$).We

*After this paper was completed an interesting paper treating the three spin sector in the case of completely broken supersymmetry appeared [58].

will also suppose that the deformation parameter is real ($\tilde{\gamma} \neq 0$, $\tilde{\sigma} = 0$).

$$\begin{aligned}
S = -\frac{R^2}{2} \int \frac{d\tau d\sigma}{2\pi} \Big[\gamma^{\alpha\beta}\sqrt{H}\big( &- \partial_\alpha t \partial_\beta t + \partial_\alpha \alpha \partial_\beta \alpha + \sin^2\alpha \partial_\alpha \theta \partial_\beta \theta \\
&+ G \sin^2\alpha \cos^2\theta \partial_\alpha \phi_1 \partial_\beta \phi_1 + G \sin^2\alpha \sin^2\theta \partial_\alpha \phi_2 \partial_\beta \phi_2 \\
&+ G \cos^2\alpha \partial_\alpha \phi_3 \partial_\beta \phi_3 + \tilde{\gamma}^2 G \sin^4\alpha \cos^2\alpha \sin^2\theta \cos^2\theta \\
&\times (\partial_\alpha \phi_1 + \partial_\alpha \phi_2 + \partial_\alpha \phi_3)(\partial_\beta \phi_1 + \partial_\beta \phi_2 + \partial_\beta \phi_3)\big) \\
&- 2\tilde{\gamma} G \epsilon^{\alpha\beta}\big( \sin^4\alpha \sin^2\theta \cos^2\theta \partial_\alpha \phi_1 \partial_\beta \phi_2 \\
&+ \sin^2\alpha \cos^2\alpha \sin^2\theta \partial_\alpha \phi_2 \partial_\beta \phi_3 \\
&+ \sin^2\alpha \cos^2\alpha \cos^2\theta \partial_\alpha \phi_3 \partial_\beta \phi_1 \big)\Big]. \qquad (3)
\end{aligned}$$

Let us consider the following rotating string ansatz:

$$\begin{aligned}
\phi_1 &= \omega_1 \tau + m_1 \sigma, \\
\phi_2 &= \omega_2 \tau + m_2 \sigma, \qquad (4) \\
\phi_3 &= \omega_3 \tau + m_3 \sigma.
\end{aligned}$$

We impose also a constant radii condition, namely $\alpha = \theta = \pi/4$ and the global time is expressed through the world sheet time as $t = \kappa\tau$. Then the equations of motion for $\alpha$ and $\theta$ are simply the following relations between the frequencies $\omega_i$, the winding numbers $m_i$ and the real deformation parameter $\tilde{\gamma}$:

$$\omega_1^2 - m_1^2 - \omega_2^2 + m_2^2 + \frac{\tilde{\gamma}}{2}(\omega_3 m_2 - \omega_2 m_3 - \omega_3 m_1 + \omega_1 m_3) = 0 \qquad (5)$$

and

$$\begin{aligned}
m_1^2 - \omega_1^2 - \omega_2^2 + m_2^2 + 2\omega_3^2 - 2m_3^2 - \frac{\tilde{\gamma}^2}{8}(\omega_1 + \omega_2 + \omega_3)^2 \\
+ \frac{\tilde{\gamma}^2}{8}(m_1 + m_2 + m_3)^2 + \tilde{\gamma}(\omega_2 m_1 - \omega_1 m_2) = 0. \qquad (6)
\end{aligned}$$

We should also explore the Virasoro constraints which in our case have the following form:

$$\begin{aligned}
&\omega_1 m_1 + \omega_2 m_2 + 2\omega_3 m_3 \\
&\quad + \frac{\tilde{\gamma}^2}{8}(\omega_1 m_1 + \omega_2 m_2 + \omega_3 m_3 + \omega_1 m_2 + \omega_1 m_3 + \omega_2 m_3) = 0 \qquad (7)
\end{aligned}$$

and

$$\begin{aligned}
\frac{4\kappa^2}{G} &= \omega_1^2 + \omega_2^2 + 2\omega_3^2 + m_1^2 + m_2^2 + 2m_3^2 \\
&+ \frac{\tilde{\gamma}^2}{8}\big(\omega_1^2 + \omega_2^2 + \omega_3^2 + 2\omega_1\omega_2 + 2\omega_1\omega_3 + 2\omega_2\omega_3 \\
&+ m_1^2 + m_2^2 + m_3^2 + 2m_1 m_2 + 2m_1 m_3 + 2m_2 m_3\big). \qquad (8)
\end{aligned}$$

It is easily seen that if

$$\begin{aligned} m_1 &= m_2 = -m_3 = m, \\ \omega_1 &= \omega_2 = \omega_3 = \omega \end{aligned} \tag{9}$$

(5) and (7) are satisfied, from (6) follows that $3\omega = m$ and the second Virasoro constraint (8) gives and expression for $\kappa$ which is related to the energy in the following way (we just note that $R^2 = \sqrt{\lambda}$):

$$E = \sqrt{\lambda}\mathcal{E} = \frac{R^2}{2\pi}\int_0^{2\pi} d\sigma\kappa = R^2\kappa. \tag{10}$$

Before we calculate the energy let us first compute the three conserved charges corresponding to the three angle variables in our problem.

$$\begin{aligned} \mathcal{J}_1 &= \frac{J_1}{\sqrt{\lambda}} = \frac{G}{2}\Big(\omega + \frac{3\tilde{\gamma}^2}{8}\omega + \frac{3\tilde{\gamma}}{4}m\Big) \\ \mathcal{J}_2 &= \frac{J_2}{\sqrt{\lambda}} = \frac{G}{2}\Big(\omega + \frac{3\tilde{\gamma}^2}{8}\omega - \frac{3\tilde{\gamma}}{4}m\Big) \\ \mathcal{J}_3 &= \frac{J_3}{\sqrt{\lambda}} = G\Big(\omega + \frac{3\tilde{\gamma}^2}{16}\omega\Big). \end{aligned} \tag{11}$$

And thus for the full spin of our system we find:

$$\mathcal{J} = \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 = 2G\omega + \frac{9\tilde{\gamma}^2}{16}G\omega = 2G\omega + \frac{3\tilde{\gamma}^2}{16}Gm. \tag{12}$$

The equation for $\kappa$ is simply

$$\kappa = \sqrt{G(\omega^2 + m^2 + \frac{9\tilde{\gamma}^2}{32}\omega^2 + \frac{\tilde{\gamma}^2}{32}m^2)}\,. \tag{13}$$

Thus for the energy we end up with the following, messy at first sight,expression:

$$\begin{aligned} E = \sqrt{\lambda}[&\left(1 + \frac{5\tilde{\gamma}^2}{16}\right)\frac{\mathcal{J}^2}{4} - \frac{3\tilde{\gamma}^2}{32}\left(1 + \frac{9\tilde{\gamma}^2}{16}\right)m\mathcal{J} \\ &+ \left(1 + \frac{\tilde{\gamma}^2}{32} + \frac{9\tilde{\gamma}^4}{64(16+5\tilde{\gamma}^2)} + \frac{81\tilde{\gamma}^6}{2056(16+5\tilde{\gamma}^2)}\right)m^2]^{1/2}. \end{aligned} \tag{14}$$

Although it looks complicated this expression reproduces the result for the energy of a three spin string in pure $AdS_5 \times S^5$ if we take the limit $\tilde{\gamma} \to 0$ (as should be expected), namely:

$$E = \sqrt{\lambda}\sqrt{\frac{\mathcal{J}^2}{4} + m^2}\,. \tag{15}$$

## 3 Two Spins in Both $AdS_5$ and $S^5_{\tilde{\gamma}}$

Here we will investigate semiclassical strings with two spins on the $AdS_5$ part and two spins on the deformed $S^5$ part of the Lunin-Maldacena background. The relevant Polyakov action is

$$\begin{aligned} S = -\frac{R^2}{2}\int \frac{d\tau d\sigma}{2\pi}\Big[\gamma^{\alpha\beta}\sqrt{H}\big(-\cosh^2\rho\partial_\alpha t\partial_\beta t+\partial_\alpha\rho\partial_\beta\rho+\sinh^2\rho\partial_\alpha\psi\partial_\beta\psi \\ +\sinh^2\rho\cos^2\psi\partial_\alpha\psi_1\partial_\beta\psi_1+\sinh^2\rho\sin^2\psi\partial_\alpha\psi_2\partial_\beta\psi_2 \\ +\partial_\alpha\theta\partial_\beta\theta+G\cos^2\theta\partial_\alpha\phi_1\partial_\beta\phi_1+G\sin^2\theta\partial_\alpha\phi_2\partial_\beta\phi_2\big) \\ -2\tilde{\gamma}G\epsilon^{\alpha\beta}\sin^2\theta\cos^2\theta\partial_\alpha\phi_1\partial_\beta\phi_2\Big]. \end{aligned} \tag{16}$$

We have imposed $\alpha=\pi/2$ and thus

$$\begin{aligned} G^{-1} &= 1+\frac{1}{4}(\tilde{\gamma}^2+\tilde{\sigma}^2)\sin^2 2\theta, \\ H &= 1+\frac{\tilde{\sigma}^2}{4}\sin^2 2\theta. \end{aligned} \tag{17}$$

Now we will assume that $\tilde{\sigma}=0$, *i.e.* we will work in the real deformed $AdS_5\times S^5$ background. It can be easily checked that the following ansatz is compatible with the string equations of motion:

$$\begin{gathered} t=\kappa\tau, \quad \rho=\text{const}, \quad \alpha=\frac{\pi}{2}, \quad \theta=\frac{\pi}{4}, \\ \psi=\frac{\pi}{4}, \quad \psi_1=\nu_1\tau+n_1\sigma, \quad \psi_2=\nu_2\tau+n_2\sigma, \\ \phi_1=\omega_1\tau+m_1\sigma, \quad \phi_2=\omega_2\tau+m_2\sigma, \quad \phi_3=0. \end{gathered} \tag{18}$$

From the equations of motion for $\rho$, $\psi$ and $\theta$ follow some relations between the winding numbers and the frequencies:

$$\begin{aligned} &\omega_1^2-m_1^2=\omega_2^2-m_2^2, \\ &\nu_1^2-n_1^2=\nu_2^2-n_2^2, \\ &\kappa^2=\nu_1^2-n_1^2. \end{aligned} \tag{19}$$

The Virasoro constraints of our system adopt the following form:

$$\begin{aligned} &\frac{\sinh^2\rho}{2}(\nu_1n_1+\nu_2n_2)+\frac{G}{2}(\omega_1m_1+\omega_2m_2)=0 \\ &\kappa^2\cosh^2\rho=\frac{\sinh^2\rho}{2}(\nu_1^2+\nu_2^2+n_1^2+n_2^2)+\frac{G}{2}(\omega_1^2+\omega_2^2+m_1^2+m_2^2). \end{aligned} \tag{20}$$

The equations of motion and the first Virasoro constraint are satisfied if we choose

$$\begin{aligned}
\nu &= \nu_1 = \nu_2, \\
\omega &= \omega_1 = \omega_2, \\
n &= n_1 = -n_2, \\
m &= m_1 = -m_2.
\end{aligned} \tag{21}$$

For the two $AdS_5$ angular momenta ($\mathcal{S}_1$, $\mathcal{S}_2$) and the two $S^5$ angular momenta ($\mathcal{J}_1$, $\mathcal{J}_2$) we obtain

$$\begin{aligned}
\mathcal{S}_1 &= \mathcal{S}_2 = \frac{\sinh^2 \rho}{2}\nu, \\
\mathcal{J}_1 &= \mathcal{J}_2 = \frac{G}{2}\omega - \frac{\tilde{\gamma}G}{4}m.
\end{aligned} \tag{22}$$

The full $AdS_5$ and $S^5$ angular momenta are simply

$$\begin{aligned}
\mathcal{S} &= \mathcal{S}_1 + \mathcal{S}_2 = \sinh^2 \rho\nu, \\
\mathcal{J} &= \mathcal{J}_1 + \mathcal{J}_2 = G\omega - \frac{\tilde{\gamma}G}{2}m.
\end{aligned} \tag{23}$$

This leads to the following relation:

$$\omega = \mathcal{J} + \frac{\tilde{\gamma}}{2}\Big(m + \frac{\tilde{\gamma}}{2}\mathcal{J}\Big). \tag{24}$$

And thus the second Virasoro constraint can be expressed as

$$\kappa^2 \cosh^2 \rho = \sinh^2 \rho(\nu^2 + n^2) + \mathcal{J}^2 + \Big(m + \frac{\tilde{\gamma}}{2}\mathcal{J}\Big)^2 \tag{25}$$

The full energy of our system is $\mathcal{E} = \kappa \cosh^2 \rho$ and we find the following relation:

$$\frac{\mathcal{E}}{\kappa} - \frac{\mathcal{S}}{\nu} = 1. \tag{26}$$

Using this relation, the second Virasoro constraint and the relation coming from the equation of motion for $\rho$ we end up with

$$2\mathcal{E}\kappa - \kappa^2 = 2\sqrt{n^2 + \kappa^2}\mathcal{S} + \mathcal{J}^2 + \Big(m + \frac{\tilde{\gamma}}{2}\mathcal{J}\Big)^2. \tag{27}$$

This expression reduces to the analogous expression found by Arutyunov, Russo and Tseytlin [16] if we take the limit $\tilde{\gamma} \to 0$. This is exactly what one should expect because this limit reproduces the well known $AdS_5 \times S^5$ background, which is the case considered in [16].

## 4 Generalized Rotating String

Considering the results from the preceding sections it seems natural to combine them in order to investigate the most general rotating string ansatz. This is the case of two spins in the $AdS_5$ part and three spins in the $S^5$ part of the geometry $(S_1, S_2, J_1, J_2, J_3)$. Following the same procedure we start with the relevant string action:

$$
\begin{aligned}
S = -\frac{R^2}{2}\int\frac{d\tau d\sigma}{2\pi}\Big[&\gamma^{\alpha\beta}\sqrt{H}\Big(-\cosh^2\rho\partial_\alpha t\partial_\beta t+\partial_\alpha\rho\partial_\beta\rho+\sinh^2\rho\partial_\alpha\psi\partial_\beta\psi\\
&+\sinh^2\rho\cos^2\psi\partial_\alpha\psi_1\partial_\beta\psi_1+\sinh^2\rho\sin^2\psi\partial_\alpha\psi_2\partial_\beta\psi_2+\partial_\alpha\alpha\partial_\beta\alpha\\
&+\sin^2\alpha\partial_\alpha\theta\partial_\beta\theta+G\sin^2\alpha\cos^2\theta\partial_\alpha\phi_1\partial_\beta\phi_1\\
&+G\sin^2\alpha\sin^2\theta\partial_\alpha\phi_2\partial_\beta\phi_2+G\cos^2\alpha\partial_\alpha\phi_3\partial_\beta\phi_3\\
&+\tilde{\gamma}^2G\sin^4\alpha\cos^2\alpha\sin^2\theta\cos^2\theta(\partial_\alpha\phi_1+\partial_\alpha\phi_2+\partial_\alpha\phi_3)(\partial_\beta\phi_1+\partial_\beta\phi_2+\partial_\beta\phi_3)\Big)\\
&-2\tilde{\gamma}G\epsilon^{\alpha\beta}(\sin^4\alpha\sin^2\theta\cos^2\theta\partial_\alpha\phi_1\partial_\beta\phi_2+\sin^2\alpha\cos^2\alpha\sin^2\theta\partial_\alpha\phi_2\partial_\beta\phi_3\\
&+\sin^2\alpha\cos^2\alpha\cos^2\theta\partial_\alpha\phi_3\partial_\beta\phi_1)\Big]. \qquad (28)
\end{aligned}
$$

We will again examine the case of real deformation parameter and use the following ansatz:

$$
\begin{aligned}
&t=\kappa\tau,\quad \rho=\text{const},\quad \alpha=\frac{\pi}{4},\quad \theta=\frac{\pi}{4},\\
&\psi=\frac{\pi}{4},\quad \psi_1=\nu_1\tau+n_1\sigma,\quad \psi_2=\nu_2\tau+n_2\sigma, \qquad (29)\\
&\phi_1=\omega_1\tau+m_1\sigma,\quad \phi_2=\omega_2\tau+m_2\sigma,\quad \phi_3=\omega_3\tau+m_3\sigma.
\end{aligned}
$$

From the equations of motion for $\rho$, $\psi$, $\alpha$ and $\theta$ we can extract the following relations between the frequencies and the winding numbers:

$$
\begin{aligned}
&\nu_1^2-n_1^2=\nu_2^2-n_2^2,\\
&\kappa^2=\nu_1^2-n_1^2,\\
&\omega_1^2-m_1^2-\omega_2^2+m_2^2+\frac{\tilde{\gamma}}{2}(\omega_3m_2-\omega_2m_3-\omega_3m_1+\omega_1m_3)=0 \qquad (30)\\
&m_1^2-\omega_1^2-\omega_2^2+m_2^2+2\omega_3^2-2m_3^2-\frac{\tilde{\gamma}^2}{8}(\omega_1+\omega_2+\omega_3)^2\\
&\quad+\frac{\tilde{\gamma}^2}{8}(m_1+m_2+m_3)^2+\tilde{\gamma}(\omega_2m_1-\omega_1m_2)=0
\end{aligned}
$$

We should also impose the Virasoro constraints, they will provide one more relation between the parameters and an equation for $\kappa$. We will retain from presenting their explicit form here but the expressions are analogous to the previous two cases. In order to satisfy the constraints and the equations of motion we should choose

$$
\begin{aligned}
&\nu=\nu_1=\nu_2,\\
&\omega=\omega_1=\omega_2=\omega_3,\\
&n=n_1=-n_2,\\
&m=m_1=m_2=-m_3.
\end{aligned} \qquad (31)
$$

It is again straightforward to compute the angular momenta in both parts of the background

$$\begin{aligned}
&\mathcal{J} = \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 = 2G\omega + \frac{9\tilde{\gamma}^2}{16}G\omega = 2G\omega + \frac{3\tilde{\gamma}^2}{16}Gm, \\
&\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2 = \sinh^2\rho\nu.
\end{aligned} \tag{32}$$

We just remind that in this case $G^{-1} = 1 + \frac{5}{16}\tilde{\gamma}^2$. The equation for $\kappa$ following from the second Virasoro constraint is

$$\kappa^2\cosh^2\rho = \sinh^2\rho(\nu^2 + n^2) + G(\omega^2 + m^2 + \frac{9\tilde{\gamma}^2}{32}\omega^2 + \frac{\tilde{\gamma}^2}{32}m^2). \tag{33}$$

The energy of the rotating string is $\mathcal{E} = \kappa\cosh^2\rho$ and it is related to the $AdS$ angular momentum as

$$\frac{\mathcal{E}}{\kappa} - \frac{\mathcal{S}}{\nu} = 1 \tag{34}$$

and we can extract an analogous to (27) expression

$$\begin{aligned}
2\mathcal{E}\kappa - \kappa^2 = 2\sqrt{n^2+\kappa^2}\mathcal{S} + \left(1 + \frac{5\tilde{\gamma}^2}{16}\right)\frac{\mathcal{J}^2}{4} - \frac{3\tilde{\gamma}^2}{32}\left(1 + \frac{9\tilde{\gamma}^2}{16}\right)m\mathcal{J} \\
+ \left(1 + \frac{\tilde{\gamma}^2}{32} + \frac{9\tilde{\gamma}^4}{64(16+5\tilde{\gamma}^2)} + \frac{81\tilde{\gamma}^6}{2056(16+5\tilde{\gamma}^2)}\right)m^2.
\end{aligned} \tag{35}$$

This expression again reduces to the one known from the undeformed $AdS_5 \times S^5$ case if we take the limit $\tilde{\gamma} \to 0$.

## 5 Conclusions and Outlook

In this paper we have studied rotating strings configurations in the recently found Lunin-Maldacena background. These semiclassical strings are an important tool for proving the AdS/CFT beyond the supergravity approximation. We have found the energy of different rotating strings in terms of the angular momenta and the string winding numbers. In the limit of zero deformation parameter we reproduce the well known results from the $AdS_5 \times S^5$ case. This should be expected because this is the limit in which the Lunin-Maldacena background reduces to the usual $AdS_5 \times S^5$.

Our work can be extended in several ways *. First of all we can work with the most general ansatz with different frequencies and winding numbers and find the energy behavior in terms of the angular momenta. It is also important to consider fluctuations around this classical solutions. They will provide the corrections to the above found energies and also clarify the stability of these

*For other related work on the subject see [55]- [57]

solutions. Maybe the most important open problem is to reproduce our results from the gauge theory side. We believe that this could be done on the level of effective actions [29]. This was the approach in [10], where an exact agreement was found for the $su(2)$ sector and we expected that this agreement could be extended for the $su(3)$ sector considered in the second Section.

There is one more interesting class of semiclassical strings - pulsating strings. It is worth investigating such pulsating solutions in the Lunin-Maldacena background and see how the deformation affects the form of the solution. We plan to address this issue in a future paper.

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