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A Taub-Like Plane-Symmetric Solutions in Bimetric Relativity

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Abstract. A Taub-like plane-symmetric solutions are obtained in bimetric theory of relativity when the sources of matter are domain walls, cosmic strings and Maxwell's field, respectively. Here we have shown that plane-symmetric Taub-like metric does not accommodate cosmic strings and domain walls in Rosen's bimetric theory of relativity. As we know that at the early stage of evolution of the universe domain walls as well as cosmic strings do appear which lead to formation of galaxies. Thus, it may be said that the bimetric relativity does not help to describe the early era of the universe. And further we cannot obtain the exact solutions with plane-symmetry.

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1 Introduction

Many authors have investigated the plane-symmetric solutions of Einstein–Maxwell equations in General Relativity. Some of them have paid attention to the solutions in which both metric g_{ij} and the electromagnetic field tensor F_{ij} are plane symmetric, *i.e.* they remain invariant under the 3-parameter group of motions which characterizes plane-symmetry [1-4]. However, it is also possible that F_{ij} not having full plane symmetry can also yield a plane-symmetric g_{ij} [2]. Here, we study the line element of a plane-symmetric metric in Taub-like coordinates in alternative theory of gravitation, *i.e.* Bimetric Theory of Relativity proposed by N. Rosen in 1973[5]. In this theory, there are two metric tensors at each point of space-time – g_{ij} which describes gravitation and the background metric γ_{ij}, which enters into the field equations and interacts with g_{ij} but does not interact directly with matter. Accordingly, at each space-time point, one has two line elements $ds^2 = g_{ij}dx^i dx^j$ and $d\sigma^2 = \gamma_{ij}dx^i dx^j$, where ds is the interval between two neighboring events as measured by a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that exists if no matter is present.

Here we have studied the plane symmetric solutions with cosmic strings, domain walls, and Maxwell field in bimetric relativity, respectively. And obtained nil contribution of them in this theory.

2 Field Equations and Plane-Symmetric Space-Time

The field equations of bimetric relativity derived from variational principles are

$$K_i{}^j - N_i{}^j - {}^1\!/_2 N g_i{}^j = -8\pi k T_i{}^j, \tag{1}$$

where

$$N_i{}^j = {}^1\!/_2 \gamma^{\alpha\beta}(g^{hj} g_{hi}|_\alpha)|_\beta, \tag{2}$$

$$N = N_\alpha{}^\alpha, \quad k = (g/\gamma)^{1/2}, \tag{3}$$

$$g = \det g_{ij}, \quad \gamma = \det \gamma_{ij}\,. \tag{4}$$

Here a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}. $T_i{}^j$ is the energy momentum tensor for the matter.

Let us consider that the line element of a plane-symmetric metric in Taub-like coordinates is

$$ds^2 = E(-dt^2 + dz^2) + G(dx^2 + dy^2), \tag{5}$$

where E and G are functions of t and z. The plane-symmetric property of the metric is ensured by the existence of three Killing fields $(\partial/\partial x)^a$, $(\partial/\partial y)^a$ and $(\partial/\partial\varphi)^a = x(\partial/\partial y)^a - y(\partial/\partial x)^a$, the latter representing rotations in x–y plane. Here $x^{1,2,3,4} = x, y, z, t$.

Corresponding to (5), let us consider the background metric

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2. \tag{6}$$

3 Non-existence of Cosmic Strings

In this Section we show that the non-existence of cosmic cloud strings in plane symmetric metric in Taub-like coordinates in bimetric relativity. Here we consider the energy momentum tensor for cosmic strings as

$$T_i{}^j = T_i{}^j{}_{\text{strings}} = \rho v_i v^j - \lambda x_i x^j. \tag{7}$$

Here ρ is the rest energy density for a cloud with particle attached along the extension, thus $\rho = \rho_p + \lambda$, where ρ_p is the particle energy density, λ is the tension density of the strings. As pointed out by Letelier [6], λ may be positive or negative. And v_i is the flow vector of matter. The flow of the matter is taken orthogonal to the hyper-surface of homogeneity so that $v_4 v^4 = -1$ and

x^i representing the direction vector of anisotropy, *i.e.* x-axis $\Rightarrow x_1x^1 = 1$ and $v_iv^i = 0$. Using the equations (1)–(7), we get

$$\Big[\Big(\frac{E''}{E} - \frac{E'^2}{E^2}\Big) - \Big(\frac{E^{\cdot\cdot}}{E} - \frac{E^{\cdot 2}}{E^2}\Big)\Big] = -16\pi k\lambda, \tag{8}$$

$$\Big[\Big(\frac{E''}{E} - \frac{E'^2}{E^2}\Big) - \Big(\frac{E^{\cdot\cdot}}{E} - \frac{E^{\cdot 2}}{E^2}\Big)\Big] = 0, \tag{9}$$

$$\Big[\Big(\frac{G''}{G} - \frac{G'^2}{G^2}\Big) - \Big(\frac{G^{\cdot\cdot}}{G} - \frac{G^{\cdot 2}}{G^2}\Big)\Big] = 0, \tag{10}$$

$$\Big[\Big(\frac{G''}{G} - \frac{G'^2}{G^2}\Big) - \Big(\frac{G^{\cdot\cdot}}{G} - \frac{G^{\cdot 2}}{G^2}\Big)\Big] = 16\pi k\rho. \tag{11}$$

From the equations (8)–(11) we have

$$\rho = 0 = \lambda. \tag{12}$$

Thus, one can state that in the plane-symmetric Taub-like metric cosmic strings do not exist.

4 Non-existence of Domain Walls

In this Section one can obtain the non-existence of Taub-like plane-symmetric thick domain walls in bimetric relativity. Thick domain walls are characterized by the energy momentum tensor

$$T_{ij} = \rho(g_{ij} + \omega_i\omega_j) + p\omega_i\omega_j \qquad \text{with} \quad \omega_1\omega^1 = -1. \tag{13}$$

Here ρ is the energy density of the wall, p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction. Here the energy momentum tensor components in the commoving coordinates for thick domain walls are given by

$$T_1{}^1 = -p, \quad T_2{}^2 = T_3{}^3 = T_4{}^4 = \rho, \quad T_1{}^0 = 0, etc. \tag{14}$$

The field equations for the metric (5) with the help of equations (1)–(4) and (13)–(14) reduce to

$$\Big[\Big(\frac{E''}{E} - \frac{E'^2}{E^2}\Big) - \Big(\frac{E^{\cdot\cdot}}{E} - \frac{E^{\cdot 2}}{E^2}\Big)\Big] = -16\pi kp, \tag{15}$$

$$\Big[\Big(\frac{E''}{E} - \frac{E'^2}{E^2}\Big) - \Big(\frac{E^{\cdot\cdot}}{E} - \frac{E^{\cdot 2}}{E^2}\Big)\Big] = 16\pi k\rho, \tag{16}$$

$$\Big[\Big(\frac{G''}{G} - \frac{G'^2}{G^2}\Big) - \Big(\frac{G^{\cdot\cdot}}{G} - \frac{G^{\cdot 2}}{G^2}\Big)\Big] = 16\pi k\rho. \tag{17}$$

From equations (15)–(16) we have

$$\rho + p = 0. \tag{18}$$

But in view of reality, conditions $\rho \geq 0$, $p \geq 0$ give us

$$\rho = 0 = p. \tag{19}$$

Hence, thick domain wall does not exist in plane-symmetric Taub-like metric in bimetric relativity.

5 Plane-Symmetric Maxwell Solutions

Here we will study the Taub-like plane-symmetric metric when the source of matter is electromagnetic field. Energy momentum tensor for Maxwell field is defined as

$$T_{ij} = F_i{}^{\alpha} F_{j\alpha} - {}^1/_4 g_{ij} F_{\alpha\beta} F^{\alpha\beta}. \tag{20}$$

And non-vanishing components of electromagnetic tensor are

$$\begin{aligned} &F^{12} = -\frac{E_1}{(EG)^{1/2}}\,, \quad F^{13} = \frac{E_2}{(EG)^{1/2}}\,, \quad F^{14} = -\frac{E_3}{E} \\ &F^{23}[= -\frac{B_3}{G}\,, \quad F^{24} = \frac{B^2}{(EG)^{1/2}}\,, \quad F^{34} = -\frac{B_1}{(EG)^{1/2}}\,, \end{aligned} \tag{21}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields measured in a local inertial system.

Using the equations (20), (21) and (5) the non-vanishing T_{ij} are

$$\begin{aligned} T_{11} &= (-G/2)(E_1^2 - E_2^2 - E_3^2 + B_1^2 - B_2^2 - B_3^2) \\ T_{22} &= -(G/2)(-E_1^2 + E_2^2 - E_3^2 - B_1^2 + B_2^2 - B_3^2) \\ T_{33} &= (E/2)(E_1^2 + E_2^2 - E_3^2 + B_1^2 + B_2^2 - B_3^2) \\ T_{44} &= (E/2)(E_1^2 + E_2^2 + E_3^2 + B_1^2 + B_2^2 + B_3^2) \\ T_{01} &= (EG)^{1/2}(B_2E_3 - E_2B_3) \\ T_{02} &= (EG)^{1/2}(E_1B_3 - B_1E_3) \\ T_{03} &= E(E_2B_1 - E_1B_2) \\ T_{12} &= -G(E_1E_2 + B_1B_2) \\ T_{13} &= -(EG)^{1/2}(E_1E_3 + B_1B_3) \\ T_{23} &= -(EG)^{1/2}(E_2E_3 + B_2B_3) \end{aligned} \tag{22}$$

Since g_{ij} are independent of x^1 and x^2 so T_{ij} and E_1, E_2, E_3, B_1, B_2, B_3, as can be seen from equation (22).

Using the equations (20)–(21) with (1)–(6) the field equations are

$$\left[\left(\frac{E''}{E}-\frac{E'^2}{E^2}\right)-\left(\frac{E^{\cdot\cdot}}{E}-\frac{E^{\cdot 2}}{E^2}\right)\right]=16\pi k{T_1}^1, \tag{23}$$

$$\left[\left(\frac{E''}{E}-\frac{E'^2}{E^2}\right)-\left(\frac{E^{\cdot\cdot}}{E}-\frac{E^{\cdot 2}}{E^2}\right)\right]=16\pi k{T_2}^2, \tag{24}$$

$$\left[\left(\frac{G''}{G}-\frac{G'^2}{G^2}\right)-\left(\frac{G^{\cdot\cdot}}{G}-\frac{G^{\cdot 2}}{G^2}\right)\right]=16\pi k{T_3}^3, \tag{25}$$

$$\left[\left(\frac{G''}{G}-\frac{G'^2}{G^2}\right)-\left(\frac{G^{\cdot\cdot}}{G}-\frac{G^{\cdot 2}}{G^2}\right)\right]=16\pi k{T_4}^4 \tag{26}$$

and all other $T_{ij}=0$.

Combining the equations (22)–(26) one obtains

$$\begin{aligned} &{E_1}^2+{B_1}^2={E_2}^2+{B_2}^2\\ &B_2E_3=E_2B_3\\ &E_1B_3=B_1E_3\\ &E_1E_3=-B_1B_3\\ &E_2E_3=-B_2B_3\\ &E_1E_2=-B_1B_2 \end{aligned} \tag{27}$$

The set of equations (27) is the necessary condition imposed on F_{ij} for getting a plane-symmetric metric. There can be only two cases which satisfy equations (27):

Case 1:

$$E_1=E_2=B_1=B_2=0. \tag{28}$$

Here F_{ij} are invariant under rotation about z-axis, so F_{ij} have the full symmetric property. Many authors have studied this condition [4].

Case 2:

$$-E_3=B_3=0,\quad {E_1}^2+{B_1}^2={E_2}^2+{B_2}^2 \text{ and } E_1E_2+B_1B_2=0. \tag{29}$$

From the last two equations of (29), we get ${E_1}^2={B_2}^2$, thus equation (29) is equivalent to

$$E_3=B_3=0,\quad {E_1}^2+{E_2}^2={B_1}^2+{B_2}^2 \text{ and } E_1B_1+E_2B_2=0. \tag{30}$$

It can be physically interpreted as a requirement that the electric and magnetic fields measured in a local inertial system are equal in magnitude but orthogonal to each other in direction. And it is to be noted that in case 2 F_{ij} are not invariant under rotation about z-axis. By solving Maxwell equations [2], we can obtain

$$\begin{aligned}
E_1 &= [\varepsilon_1/(EG)^{1/2}]\, sin(\omega t)\sin(\omega z),\\
E_2 &= [\varepsilon_2/(EG)^{1/2}]\sin(\omega t) sin(\omega z),\\
B_1 &= -[\varepsilon_2/(EG)^{1/2}]\cos(\omega t)\cos(\omega z),\\
B_2 &= [\varepsilon_1/(EG)^{1/2}]\cos(\omega t)\cos(\omega z),
\end{aligned} \tag{31}$$

where ε_1 and ε_2 are constants.

However, these components do not satisfy the necessary condition, and it is impossible to obtain an exact solution with plane symmetry. Hence, one can find an approximate solution by taking the space- and time-average of T_{ij}. It happens that the average satisfies the necessary condition, thus ensuring the approximate solution g_{ij} is plane-symmetric (for details one may refer [7]).

Hence, we can say that from case 1, F_{ij} are invariant under rotation about z-axis, so F_{ij} have the full symmetric property. And from case 2, we get F_{ij} are not invariant under rotation about z-axis. We cannot obtain the exact solutions with plane symmetry.

6 Conclusion

Here we have shown that plane-symmetric Taub-like metric does not accommodate cosmic strings and domain walls in Rosen's bimetric theory of relativity.

As we know that, at the early stage of evolution of the universe domain walls as well as cosmic strings do appear which leads to formation of galaxies. Thus, it may be said that the bimetric relativity does not help to describe the early era of the universe.

And further we cannot obtain the exact solutions with plane symmetry.

References

[1] S. Patnaik (1970) *Proc. Camb. Phil. Soc.* **67** 127.

[2] P.A. Amundsen, Ø. Grøn (1983) *Gen. Rel. Grav.* **15** 315.

[3] A.H. Taub (1951) *Ann. Math.* **53** 472.

[4] D. Kramer *et. al*. (1980) "*Exact Solutions of Einstein's Field Equations*", Cambridge University, Cambridge, England, 155 and 158.

[5] N. Rosen (1973) *Gen. Rel. Grav.* **4** 435.

[6] P.S. Letelier (1983) *Phys. Rev. D* **28** 2414.

[7] Li Jian-zeng, Liang Can-bin (1985) *Chinese Phys. Lett*. **2** 23.