

# Plane Symmetric Cosmological Model with Thick Domain Walls in Brans-Dicke Theory of Gravitation

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**Abstract.** We have investigated plane symmetric cosmological model in presence of thick domain walls in Brans-Dicke theory of gravitation, some geometrical and physical behavior of the model are discussed.

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## 1 Introduction

In recent years there has been a lot of interest in several alternative theories of gravitation; the most important among them are scalar-tensor theories of gravitation formulated by Brans-Dicke [1]. In this theory the gravity is mediated by a scalar field  $\phi$  in addition to the usual metric tensor field  $g_{ij}$  present in Einstein's theory the long range scalar field  $\phi$  is generated by the whole of matter in the universe according to Mach's principle (Dicke [2]) and has the dimension of the universe of gravitational constant  $G$ . The work of Singh and Rai [3] gives detailed discussion of Brans-Dicke cosmological model. S. Yazadjiev [4,5] studied plane symmetric inhomogeneous Brans-Dicke cosmology with an equation of state  $p = \gamma\rho$  and generating  $G_2$ -cosmologies with perfect fluid in dilation gravity. Reddy *et al.* [6] have investigated axially symmetric perfect fluid cosmological model in Brans-Dicke theory. Berman [7] considered a special law of variation for Hubble's parameter in involutory models with perfect fluid as material source, which leads to a constant value of deceleration parameter. Reddy *et al.* [8] presented models of universe with negative constant deceleration parameter in Seaz-Ballester and Brans-Dicke scalar theories. Recently,

Reddy *et al.* [9,10] studied an axially symmetric Bianchi type-I cosmological model with negative constant deceleration parameter with the help of Hubble's special law of variation proposed by Berman [7]. A. Pradhan *et al.* [11] obtained plane symmetric inhomogeneous cosmological models with a perfect fluid in general relativity. This motivates us to study the plane symmetric cosmological model with thick domain walls in Brans-Dicke theory of gravitation. The domain walls are formed when the universe undergoes a series of phase transitions with discrete symmetry being spontaneously broken (Vilenkin [12,13]). After symmetry breaking different regions of the universe can be settling into different parts of the vacuum with domain walls forming boundaries between their regions. In the recent years many of the researchers interested in the study of large scale structure of the universe because of the fact that the origin of the structure in the universe is one of the greatest cosmological mysteries even today. The light domain walls of large thickness may have produced during the late time phase transitions such as those occurring after the decoupling of the matter and radiations Hill and Schramm [14], D.R.K. Reddy [15]. Zel'dovich [16] pointed out the stress energy of the domain walls composed of the surface energy density and string tension into spatial directions with magnitude of tension equal to the surface energy density; this is of interest because there are several indications that tension acts repulsive source of gravity in general relativity.

In this paper, we consider a plane symmetric space-time in Brans-Dicke theory of gravitation in presence of thick domain walls. To get the determinate exact solution of Brans-Dicke field equations, we have used the equation of state in the form  $3\rho = p$  which is analogous to  $\rho = 3p$  in general relativity of matter with disorder radiation and some physical properties of the model are also discussed.

## 2 Field Equations

We consider the plane symmetric space-time

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2, \quad (1)$$

where  $A$  and  $B$  are the function of  $t$  only.

The field equation in Brans-Dicke theory is as follows:

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{i,j} - g_{ij}\phi^2) \quad (2)$$

and

$$\phi \equiv \phi^k_{;k} = 8\pi\phi^{-1}T(3 + 2\omega)^{-1}, \quad (3)$$

where

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R. \quad (4)$$

Here  $G_{ij}$  is the Einstein tensor,  $T_{ij}$  is the energy momentum tensor,  $R_{ij}$  is the Ricci tensor,  $R$  is the Ricci scalar,  $\phi$  is the Brans-Dicke scalar field and  $\omega$  is the dimensionless coupling constant. Also comma (,) and semicolon (;) denote partial and covariant differentiation, respectively.

The conditions of motion,

$$T_{ij}^{;j} = 0, \quad (5)$$

are consequences of the field equations (2) and (3).

The energy momentum tensor for thick domain walls is given by

$$T_{ij} = \rho (g_{ij} + \omega_i \omega_j) + p (\omega_i \omega_j) \quad (6)$$

together with

$$\omega_i \omega^i = -1, \quad (7)$$

where  $\rho$  is the energy density of the domain walls,  $p$  is the pressure in the direction normal to the plane of the domain walls and  $\omega_i$  is the unit space-like vector in the same direction, *i.e.* along the  $z$ -axis.

Using co-moving coordinate system, the non-vanishing component of  $T_{ij}$  can be obtained as

$$T_1^1 = T_2^2 = T_4^4 = \rho, \quad T_3^3 = -p. \quad (8)$$

Using (8), the field equation (2) and (3) for the metric (1) can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\phi^{-1}\rho - \frac{\omega}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{\dot{A}}{A} \frac{\dot{\phi}}{\phi} - \phi^{-1} \phi_{;k}^k \quad (9)$$

$$\frac{2\ddot{A}}{A} + \left( \frac{\dot{A}}{A} \right)^2 = -8\pi\phi^{-1}p - \frac{\omega}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{\dot{B}}{B} \frac{\dot{\phi}}{\phi} - \phi^{-1} \phi_{;k}^k \quad (10)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left( \frac{\dot{A}}{A} \right)^2 = -8\pi\phi^{-1}\rho + \frac{\omega}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{\ddot{\phi}}{\phi} - \phi^{-1} \phi_{;k}^k \quad (11)$$

$$\phi_{;k}^k \equiv \ddot{\phi} + \left( 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \dot{\phi} = \frac{8\pi^{-1}}{\phi} 3 + 2\omega (3\rho - p). \quad (12)$$

Equation (5), which is a consequence of the field equations, can be written as

$$\dot{\rho} + \rho \frac{\dot{B}}{B} + p \frac{\dot{B}}{B} = 0. \quad (13)$$

### 3 Solutions of the Field Equations

The set of field equations (9) to (12) contains four equations with five unknowns  $A$ ,  $B$ ,  $\rho$ ,  $\phi$ ,  $p$ ; to find the exact solution, one extra condition is needed, so we consider equation of state in the form

$$3\rho = p, \quad (14)$$

which is analogous to  $\rho = 3p$  in general relativity for matter with disorder radiation.

Using (14), Eq. (12) becomes

$$\phi_{;k}^k \equiv \ddot{\phi} + \left( \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \right) \dot{\phi} = 0. \quad (15)$$

Using Eqs. (14) and (15), the set of equations from (9) to (13) reduces to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\phi^{-1}\rho - \frac{\omega}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{\dot{A}}{A} \frac{\dot{\phi}}{\phi} \quad (16)$$

$$\frac{2\ddot{A}}{A} + \left( \frac{\dot{A}}{A} \right)^2 = -8\pi\phi^{-1}p - \frac{\omega}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{\dot{B}}{B} \frac{\dot{\phi}}{\phi} \quad (17)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left( \frac{\dot{A}}{A} \right)^2 = -8\pi\phi^{-1}\rho + \frac{\omega}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{\ddot{\phi}}{\phi} \quad (18)$$

$$\ddot{\phi} + \left( \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \right) \dot{\phi} = 0 \quad (19)$$

$$\ddot{\rho} + 4\rho \frac{\dot{B}}{B} = 0. \quad (20)$$

The set of equations (16)-(20) is highly nonlinear, we assume a relation between metric coefficient given by

$$A = nB, \quad (21)$$

where  $n$  is constant.

We solve the above set of highly-nonlinear equation with the help of special law of variation for Hubble's parameter proposed by Berman [5] that yield constant deceleration parameter models of the universe. Now consider constant deceleration parameter model defined by

$$q = - \left[ \frac{RR_{44}}{(R_4)^2} \right] = \text{const}, \quad (22)$$

where  $R = (A^2 B)^{\frac{1}{3}}$  and  $R$  is the overall scalar factor.

The constant is taken as negative (*i.e.* it is an accelerating model of the universe). The solution of Eq. (22) is given by

$$R = (at + b)^{\frac{1}{1+q}}, \quad a \neq 0, \quad b \neq 0, \quad (23)$$

where  $a$  and  $b$  are constants of integration.

This equation implies that the condition of the expansion is  $1 + q > 0$ . From (23) and (24), we get

$$(A^2 B)^{\frac{1}{3}} = (at + b)^{\frac{1}{1+q}}. \quad (24)$$

Using (21), Eq. (24) becomes

$$(n^2 B^3)^{\frac{1}{3}} = (at + b)^{\frac{1}{1+q}} \quad (25)$$

and by solving Eq. (25), we obtain

$$A = C_1 (at + b)^{\frac{1}{1+q}}, \quad (26)$$

where  $C_1 = (n)^{\frac{1}{3}}$ ,

$$B = C_2 (at + b)^{\frac{1}{1+q}}, \quad (27)$$

where  $C_2 = (n)^{-\frac{2}{3}}$ .

Using Eqs. (26) and (27), the line element (1) becomes

$$ds^2 = dt^2 - C_1 (at + b)^{\frac{2}{1+q}} (dx^2 + dy^2) - C_2^2 (at + b)^{\frac{2}{1+q}} dz^2. \quad (28)$$

Using the suitable transformation, the above equation (28) reduces to

$$ds^2 = \frac{dT^2}{a^2} - C_1^2 T^{\frac{2}{1+q}} (dx^2 + dy^2) - C_2^2 T^{\frac{2}{1+q}} dz^2. \quad (29)$$

The model (30) represents an exact radiating cosmological model with negative constant deceleration parameter in the frame work of Brans-Dicke scalar-tensor theory of gravitation with cosmic domain walls.

#### 4 Some Physical and Geometrical Behavior of the Model

Using the Brans-Dicke theory of gravitation, some physical and kinematical properties of cosmological model (29) are obtained as follows.

The proper volume is obtained as

$$V = \sqrt{-g} = C_3 (T)^{\frac{3}{1+q}}, \quad (30)$$

where  $C_3 = C_1 C_2 = n^{-\frac{1}{3}}$

Scalar expansion ( $\theta$ ) :

$$\theta \equiv \frac{1}{3} u^i_{;j} = \frac{a}{T(1+q)}. \quad (31)$$

Shear scalar ( $\sigma^2$ ) :

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{7}{18} \frac{1}{T^2 (1+q)^2}. \quad (32)$$

Hubble's parameter ( $H$ ):

$$H = \frac{R_4}{R} = \frac{1}{(1+q)T}. \quad (33)$$

From Eqs. (30) to (33), it may be observed that at the initial moment, *i.e.*, when  $T = 0$ , the proper volume will be zero. When  $T$  tends to zero, then the expansion scalar  $\theta$ , shear scalar  $\sigma^2$ , and Hubble's parameter  $H$  tend to infinity and for large value of  $T$ , we observe that expansion scalar  $\theta$ , shear scalar  $\sigma^2$ , and Hubble's parameter ( $H$ ) become zero; and also

$$\lim_{T \rightarrow \infty} \left( \frac{\sigma}{\theta} \right)^2 \neq 0. \quad (34)$$

Hence the model does not approach isotropy for large value of  $T$ . Further to obtain the scalar field  $\phi$  for Brans-Dicke theory, Eq. (19) can be written as

$$\frac{2\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\ddot{\phi}}{\phi} = 0. \quad (35)$$

Solving equation (35), we get

$$A^2 B d\phi = K_1 dt, \quad (36)$$

where  $K_1$  is constant of integration.

Using Eqs. (26) and (27) in Eq. (35), it becomes

$$(at + b)^{\frac{3}{1+q}} d\phi = K_1 dt, \quad (37)$$

$$d\phi = K_1 (at + b)^{-\frac{3}{1+q}} dt. \quad (38)$$

On integrating, we obtained

$$\phi = \frac{K_1}{a} \left( \frac{1+q}{q-2} \right) (at+b)^{\frac{q-2}{1+q}} + K_2, \quad (39)$$

but  $(at+b) = T$ . This leads to

$$\phi = \frac{K_1}{a} \left( \frac{1+q}{q-2} \right) T^{\frac{q-2}{1+q}} + K_2. \quad (40)$$

Using (26) and (27) in equation (17), the expression for the model can be written as

$$8\pi p = \frac{aK_1}{(1+q)(q-2)} T^{-\left(\frac{q+4}{1+q}\right)} \left[ 3q - 3 - \frac{\omega}{2} (q^2 - 4q + 4) \right]. \quad (41)$$

Using (14) and (38) the energy density for the model can be obtained as

$$8\pi\rho = \frac{aK_1}{3(1+q)(q-2)} T^{-\left(\frac{q+4}{1+q}\right)} \left[ 3q - 3 - \frac{\omega}{2} (q^2 - 4q + 4) \right]. \quad (42)$$

From (41) and (42) we observe that at the initial point  $T = 0$ , the energy density  $\rho$  and pressure  $p$  diverges. For large values of  $T$  the energy density  $\rho$  and pressure  $p$  becomes zero.

## 5 Conclusion

In this paper we have investigated plane symmetric cosmological model with thick domain walls for solving the field equation we have used the special law of variation of Hubble's parameter proposed by Berman [5]. Thus forsaid model obtained represent a radiating universe in Brans-Dicke theory of gravitation.

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