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# Study of the Yrast Band Structures of the Hafnium Isotopes

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**Abstract.** In the present work we study the properties of high spin states and the alignment effects in the lighter $^{157-175}$Hf isotopes. An interesting nuclear feature emerging from this study concerns the evaluation of the moment of inertia and the yrast line yields conclusions about the nuclear shape. The nature of the back-bending phenomena in the Hf isotopes is discussed.

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## 1 Introduction

During the last decade nuclear high spin states have been the subject of experimental and theoretical studies. The fascinating progress in this new field has been made possible by the essential development in experimental on exciting and detecting the high spin states in nuclei. A part from such interesting expectations as the existence of a super-back-bending or super-deformation [1], the research in this new field can also be considered as providing a tool for testing the validity of nuclear models under extreme conditions. As an example, the back-bending phenomena discovered in 1970 [2] is now interpreted as being due to the crossing of the ground state band with an aligned quasi-particle band [3-5].

Recently A.F. Saad *et al.* [6] established a completely new level scheme for $^{157}$Hf. It was constructed up to $E_x \cong 6.5$ MeV and $J^\pi \cong 51/2$ for the first time. In the present work we study the properties of the bands at high spins and alignment effects in the lighter $^{157-175}$Hf isotopes [6-17] (Equivalent information on $^{159,161,169,173}$Hf isotopes is not available). They are located on the border of the strongly deformed region and are of interest for an investigation of the transition towards more spherical nuclei. In this mass region the study of the nuclear states offers interesting nuclear features. The evaluation of the moment of inertia and the yrast line yields conclusions about the nuclear shape.

The strong staggering in $^{157}$Hf shows this nucleus may be soft rotor. A comparison data for the $^{163-175}$Hf isotopes has been done which show that these isotopes may be hard rotor. Also the back-bending phenomena for $^{163,165,167}$Hf nuclei [10,11] show a dramatic changes through a plot of the moment of inertia $(2\Im/\hbar^2)$ versus the square of rotational frequency $(\hbar^2\omega^2)$.

## 2 Analysis of the Data and Discussion

***Alignment.*** The alignment which has been computed for the studied nuclei is always decreasing when the masses increase (Table 1 and Figure 1). The explanation is similar in all nuclei. Comparison study of the calculations in all studied isotopes is also done. It is interesting to calculate and plot the moment of inertia as a function of the square of the rotational frequency and to investigate how they change as the rotational angular velocity of the nucleus varies (Figure 1). In this figure the increase in the moment of inertia at $\hbar^2\omega^2 \cong 0.11$ MeV$^2$ is so rapid that the rotational frequency actually decreases as higher spin states are reached. The following interesting features can be observed:

(i) with decreasing mass number (decreasing deformation and decreasing moments of inertia, see Table 2, Figure 2), the back-bending becomes sharper, which reflects a decreasing interaction energy;

Table 1. Excitation energy, moment of inertia, square of rotational frequency and the inertial parameter for the Hf-isotopes

| | | $^{157}_{72}$Hf$_{85}$ | | | | $^{163}_{72}$Hf$_{91}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| $I$ | $I(I+1)$ | $E$ | $\frac{2\Im}{\hbar^2}$ | $(\hbar\omega)^2$ | $\frac{E_I-E_{I-1}}{2I}$ | $E$ | $\frac{2\Im}{\hbar^2}$ | $(\hbar\omega)^2$ | $\frac{E_I-E_{I-1}}{2I}$ |
| 13/2 | 49 | 0.954 | 29 | 0.180 | 65 | — | — | — | — |
| 17/2 | 81 | 1.590 | 50 | 0.100 | 37 | 0.255 | 126 | 0.016 | 15 |
| 21/2 | 121 | 1.860 | 148 | 0.018 | 13 | 0.690 | 92 | 0.048 | 21 |
| 25/2 | 169 | 2.706 | 57 | 0.178 | 34 | 1.254 | 85 | 0.80 | 23 |
| 29/2 | 255 | 3.292 | 96 | 0.085 | 20 | 1.912 | 85 | 0.109 | 23 |
| 33/2 | 289 | 3.986 | 151 | 0.045 | 21 | 2.642 | 88 | 0.133 | 22 |
| 37/2 | 361 | 4.225 | 301 | 0.014 | 7 | 3.405 | 94 | 0.147 | 21 |
| 41/2 | 441 | 5.020 | 305 | 0.017 | 19 | 4.120 | 112 | 0.128 | 17 |
| 43/2 | 484 | 5.416 | 212 | 0.039 | 9 | — | — | — | — |
| 45/2 | 529 | — | — | — | — | 4.813 | 127 | 0.120 | 15 |
| 47/2 | 576 | 5.615 | 462 | 0.0099 | 4 | — | — | — | — |
| 49/2 | 625 | 6.107 | 195 | 0.061 | 3 | 5.554 | 130 | 0.137 | 15 |
| 51/2 | 676 | 6.050 | 255 | 0.039 | 17 | — | — | — | — |
| 53/2 | 729 | — | — | — | — | 6.360 | 129 | 0.163 | 15 |
| 57/2 | 841 | — | — | — | — | 7.234 | 128 | 0.192 | 15 |
| 61/2 | 961 | — | — | — | — | — | — | — | — |

Table 1. Continued

| | | | | $^{165}_{72}\mathrm{Hf}_{93}$ | | | | $^{167}_{72}\mathrm{Hf}_{95}$ | |
|---|---|---|---|---|---|---|---|---|---|
| $I$ | $I(I+1)$ | $E$ | $\frac{2\Im}{\hbar^2}$ | $(\hbar\omega)^2$ | $\frac{E_I-E_{I-1}}{2I}$ | $E$ | $\frac{2\Im}{\hbar^2}$ | $(\hbar\omega)^2$ | $\frac{E_I-E_{I-1}}{2I}$ |
| 13/2 | 49 | — | — | — | — | — | — | — | — |
| 17/2 | 81 | 0.335 | 149 | 0.012 | 13 | 0.350 | 156 | 0.001 | 12 |
| 21/2 | 121 | 0.716 | 105 | 0.037 | 18 | 0.693 | 117 | 0.029 | 16 |
| 25/2 | 169 | 1.225 | 94 | 0.066 | 20 | 1.152 | 105 | 0.053 | 18 |
| 29/2 | 255 | 1.826 | 93 | 0.091 | 21 | 1.705 | 102 | 0.076 | 19 |
| 33/2 | 289 | 2.486 | 97 | 0.109 | 20 | 2.333 | 102 | 0.099 | 19 |
| 37/2 | 361 | 3.168 | 106 | 0.116 | 18 | 3.007 | 107 | 0.144 | 18 |
| 41/2 | 441 | 3.843 | 119 | 0.113 | 17 | 3.668 | 121 | 0.110 | 16 |
| 43/2 | 484 | — | — | — | — | — | — | — | — |
| 45/2 | 529 | 4.533 | 128 | 0.118 | 15 | 4.336 | 132 | 0.111 | 15 |
| 47/2 | 576 | — | — | — | — | — | — | — | — |
| 49/2 | 625 | 5.272 | 130 | 0.137 | 15 | 5.085 | 128 | 0.141 | 15 |
| 51/2 | 676 | — | — | — | — | — | — | — | — |
| 53/2 | 729 | 6.079 | 129 | 0.163 | 15 | 5.923 | 124 | 0.176 | 15 |
| 57/2 | 841 | 6.962 | 127 | 0.195 | 15 | 6.838 | 122 | 0.211 | 16 |
| 61/2 | 961 | 7.915 | 126 | 0.227 | 15 | — | — | — | — |

| | | | | $^{171}_{72}\mathrm{Hf}_{99}$ | | | | $^{175}_{72}\mathrm{Hf}_{103}$ | |
|---|---|---|---|---|---|---|---|---|---|
| $I$ | $I(I+1)$ | $E$ | $\frac{2\Im}{\hbar^2}$ | $(\hbar\omega)^2$ | $\frac{E_I-E_{I-1}}{2I}$ | $E$ | $\frac{2\Im}{\hbar^2}$ | $(\hbar\omega)^2$ | $\frac{E_I-E_{I-1}}{2I}$ |
| 13/2 | 49 | 0.245 | 131 | 0.009 | — | 0.437 | 135 | 0.008 | — |
| 17/2 | 81 | 0.513 | 119 | 0.018 | 16 | 0.713 | 116 | 0.019 | 16 |
| 21/2 | 121 | 0.867 | 113 | 0.032 | 17 | 1.078 | 110 | 0.003 | 17 |
| 25/2 | 169 | 1.307 | 109 | 0.049 | 18 | 1.550 | 102 | 0.056 | 19 |
| 29/2 | 255 | 1.828 | 108 | 0.068 | 18 | 2.006 | 123 | 0.052 | 16 |
| 33/2 | 289 | 2.426 | 107 | 0.090 | 18 | 2.579 | 112 | 0.082 | 17 |
| 37/2 | 361 | 3.093 | 108 | 0.111 | 18 | 2.952 | 193 | 0.035 | 10 |
| 41/2 | 441 | 3.820 | 110 | 0.133 | 18 | 3.804 | 94 | 0.181 | 21 |
| 43/2 | 484 | — | — | — | — | — | — | — | — |
| 45/2 | 529 | 4.599 | 114 | 0.149 | 17 | 4.727 | 117 | 0.142 | 21 |
| 47/2 | 576 | — | — | — | — | — | — | — | — |
| 49/2 | 625 | 5.378 | 122 | 0.155 | 16 | 5.531 | 119 | 0.163 | 16 |
| 51/2 | 676 | — | — | — | — | — | — | — | — |
| 53/2 | 729 | 6.178 | 130 | 0.160 | 15 | 6.371 | 124 | 0.176 | 16 |
| 57/2 | 841 | — | — | — | — | 7.455 | 103 | 0.296 | 19 |
| 61/2 | 961 | — | — | — | — | — | — | — | — |

(ii) in general, the calculated backbends $I_{\text{crit.}} \cong A^{7/12}$, where $I_{\text{crit.}}$ corresponds to the point at which the moment of inertia is changing at the maximum rate [4,5], which corresponds to the value of $\hbar^2\omega^2 \cong 0.11$ MeV$^2$; the calculations predict the back bending to occur at $I_{\text{crit.}} \cong 40/2$;

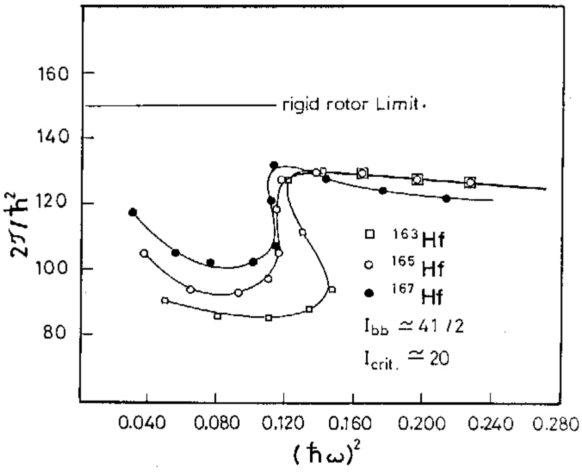

Figure 1. Moment of inertia as a function of the square of the rotational angular velocity for $^{163,165,167}$Hf isotopes.

(iii) at higher frequencies the behaviour of the three nuclei is approximately the same.

In the range between 0.138 MeV$^2$ and 0.228 MeV$^2$, $2\Im/\hbar^2$ depends linearly on $\hbar^2\omega^2$. A more revealing display the yrast band properties are giving in Figure 3, where the moment of inertia parameter as calculated from the level spacing is plotted versus $I$. In $^{157}$Hf the plot shows strong oscillations of $\hbar^2/2\Im$, which can be traced to the admixture in the ground, banding its decoupling parameters. Also this strong staggering shows that the $^{157}$Hf may be soft rotor against $\gamma$-deformation, since the clustering of levels as (17/2, 21/2), (25/2, 29/2), *etc.* is predicted in models using $\gamma$-unstable potentials [21,22]. Figures 3 and 4, Table 1 include the corresponding data for the $^{163-175}$Hf isotopes [10-17], which show that these isotopes may be hard rotor, clearly seen in Figure 7. The comparison demonstrates that both the oscillation and the compression strongly increase

Table 2. The experimental moment of inertia for the $Z = 72$ Hf isotopes

| Nuclide | $2\Im_E/\hbar^2$ (MeV)$^{-1}$ | $2\Im_R/\hbar^2$ (MeV)$^{-1}$ |
|---|---|---|
| $^{157}$Hf | $88 \pm 11$ | $73 \pm 10$ |
| $^{163}$Hf | $112 \pm 13$ | $90 \pm 8$ |
| $^{165}$Hf | $116 \pm 9$ | $96 \pm 5$ |
| $^{167}$Hf | $154 \pm 31$ | $104 \pm 11$ |
| $^{171}$Hf | $131 \pm 16$ | $111 \pm 12$ |
| $^{175}$Hf | $139 \pm 22$ | $114 \pm 16$ |

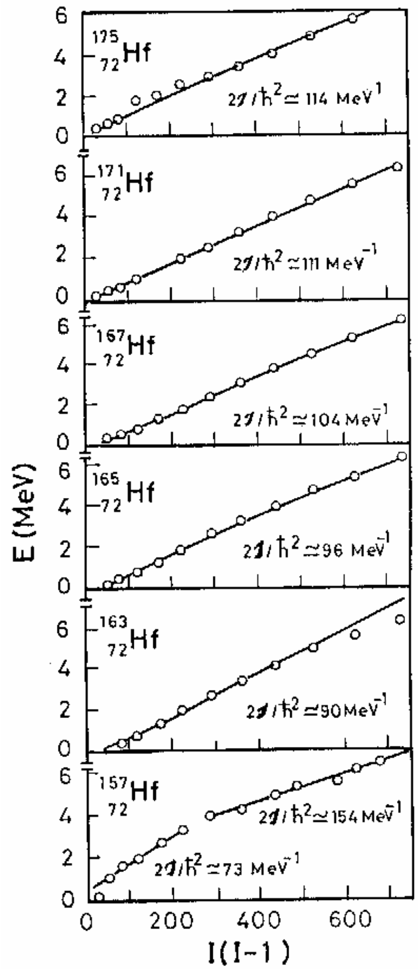

Figure 2. Excitation energies of yrast states at the $Z = 72$ Hf isotopes *versus* $I(I-1)$. The average moments of of inertia are indicated.

from $^{175}$Hf to $^{157}$Hf which in turn indicates that the coriolis mixing becomes much stronger for the isotopes with lower $N$. The energy displacement relation is given in Ref. [18] in the form

$$\delta E(I) = E(I) - [(I+1)E(I-1) + IE(I+1)]/(2I+1)$$

and plotted against the neutron number in Figure 5 for $_{72}$Hf isotopes. The $\delta E(I)$ value of the $^{157}$Hf is very much larger than that of the other isotopes and for well-deformed nuclei it approaches zero [18]. Figure 5 clearly shows how the stretching (positive values of $\delta E(I)$) evidenced in the case of soft rotors goes

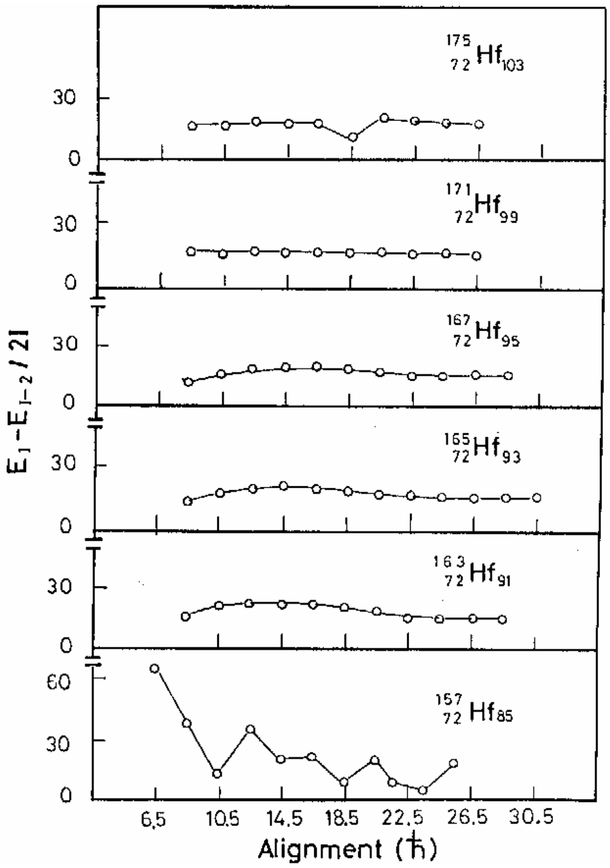

Figure 3. Plot of the moment of inertia, $\hbar^2/2\Im = {}^1\!/_2(E_I - E_{I-1})$, *versus* the spin for the yrast bands in $_{72}$Hf isotopes.

over into the shrinkage (negative values of $\delta E(I)$) for hard rotors exactly the same as in Figure 4. The large $\delta E$ (13/2, 17/2) value of the $^{157}$Hf nucleus probably reflected their $\gamma$-softness, which is exactly the same as the one we discussed before, since $^{157}$Hf lies on the border between the prolate and the oblate Hf nuclei.

## 3 Comparison Study of the Results in Studied Isotopes

***Moment of inertia.*** A comparison of the low energetic level patterns of the $Z =$ 72 Hf isotopes can be done by comparing the $E_x$ versus $I(I+1)$ plots (Figure 2) or the effective moments of inertia deduced therefrom. If the $\Im_E$ values of even–odd nuclei are compared directly and a significant discrepancy is observed which

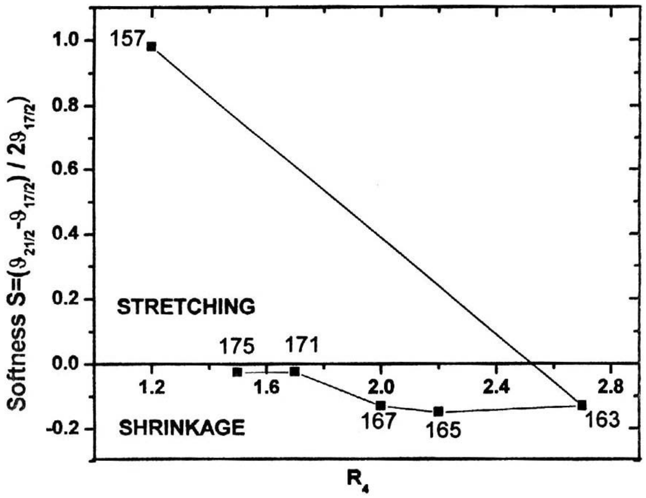

Figure 4. Softness, $S$, as a function of the energy ration $R_4 = E_{21/2}/E_{17/2}$ for the nuclides in the isotopic sequence of Hf showing the gradual transition of stretching (positive $S$) into shrinkage (negative $S$).

is caused by the different spins of the ground states. To overcome this difficulty and to enable a realistic comparison, the ground-state spin is subtracted, so that all spins are normalized to the ground-state spin. Table 2 shows the deduced moments of inertia whereby $\Im_R$ was calculated using the "reduced spin".

Also the sharp increase of the moment of inertia can be found to occur as a bend in the $E_x$ versus $I(I+1)$ plots of $^{157}$Hf (Figure 2) at a state with a reduced spin value of around (29/2)$\hbar$ which is an isomeric state in this nucleus.

***Deformation changes in gauge space.*** One of the most interesting problems in high spin physics is the question of a quenching of the static pair correlations in rapidly rotating nuclei. It has been suggested [19] that evidence for a reduction of pairing correlations can be obtained from the analysis of rotations in Gauge space by plotting the neutron number as a function of the Fermi energy, in analogy to the plots of $I_x$ versus $\hbar\omega$ which are frequently used for investigation rotational bands in ordinary space.

Since the slope of the $N(\lambda)$ curve reflects the level density at the Fermi surface, an upbend (for example) in this curve (*i.e.*, an increase of slope in a small energy region) indicates a sudden increase in level density. This tends to be energetically unfavorable and must therefore be compensated, most likely by a change in deformation, typically an increase. Thus, in which a case, the upper branch in the $N(\lambda)$ curve is called the deformed branch in Figure 5. The kink in these curves would indicate that the lighter Hf the less deformed. The Fermi energies (Table 3) are calculate from the relation

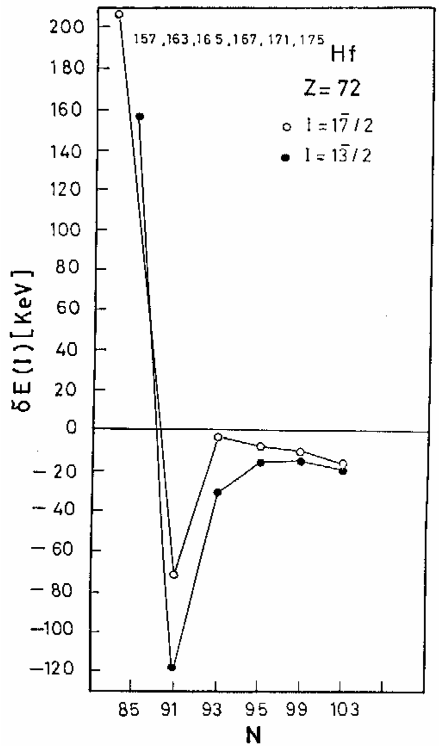

Figure 5. Energy displacement of the $I = {}^{13}/_{2}$, ${}^{17}/_{2}$ levels plotted *versus* the neutron number for the yrast band in ${}_{72}$Hf isotopes.

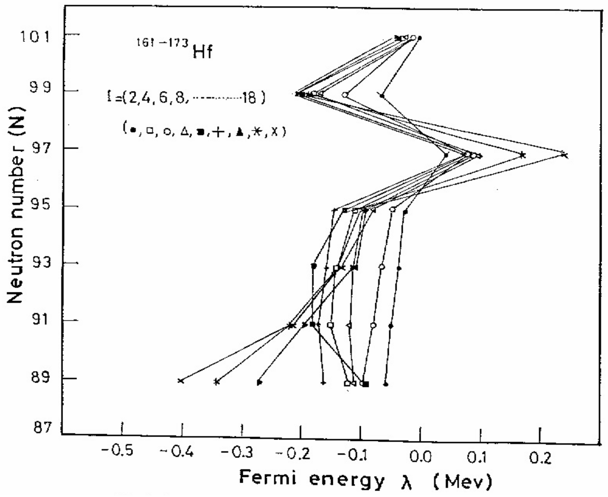

Figure 6. The neutron number, $N$, *versus* Fermi energy, $\lambda$, for ${}_{72}$Hf isotopes.

Table 3. The Fermi energy *vs.* neutron number for the Hf isotopes

| | $\lambda$ (MeV) | | | | | | |
|---|---|---|---|---|---|---|---|
| $I$ | ${}^{161}_{72}\mathrm{Hf}_{89}$ | ${}^{163}_{72}\mathrm{Hf}_{91}$ | ${}^{165}_{72}\mathrm{Hf}_{93}$ | ${}^{167}_{72}\mathrm{Hf}_{95}$ | ${}^{169}_{72}\mathrm{Hf}_{97}$ | ${}^{171}_{72}\mathrm{Hf}_{99}$ | ${}^{173}_{72}\mathrm{Hf}_{101}$ |
| 2 | -0.062 | -0.047 | -0.036 | -0.026 | 0.040 | -0.071 | -0.010 |
| 4 | -0.094 | -0.081 | -0.068 | -0.052 | 0.070 | -0.124 | -0.014 |
| 6 | -0.110 | -0.114 | -0.104 | -0.079 | 0.084 | -0.165 | -0.018 |
| 8 | -0.113 | -0.146 | -0.141 | -0.106 | 0.087 | -0.175 | -0.021 |
| 10 | -0.099 | -0.175 | -0.176 | -0.128 | 0.082 | -0.205 | -0.025 |
| 12 | -0.154 | -0.167 | -0.162 | -0.140 | 0.072 | -0.209 | -0.029 |
| 14 | -0.256 | -0.188 | -0.110 | -0.085 | 0.088 | -0.206 | -0.035 |
| 16 | -0.343 | -0.205 | -0.124 | -0.078 | 0.169 | -0.202 | — |
| 18 | -0.391 | -0.205 | -0.136 | -0.148 | 0.236 | -0.209 | — |

$$\lambda(N-1) = \frac{1}{2}\left[E(Z,N+1,I) - E(Z,N-1,I) - S_{zn}^{N+1}\right]$$

The separation energy $S_{2n}$ for each neutron number is taken from [20]. From the figure we see a pronounced irregularity around neutron number 97, which reminds very much about the back-bending or up bending behaviour (deformation change), with which we are familiar from the $I(\omega)$-plots (Figures 6 and 7).

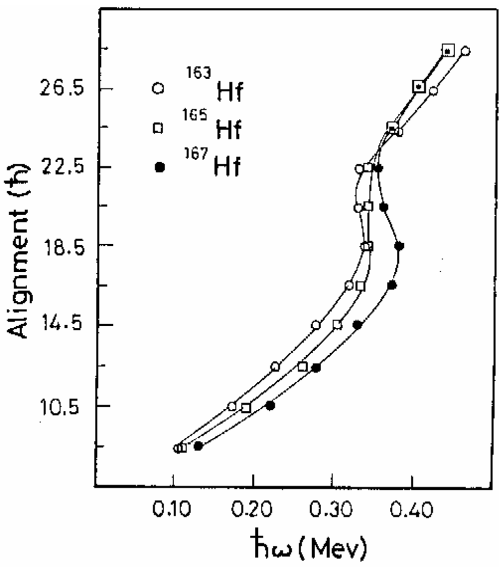

Figure 7. Experimental $I$ *versus* $\hbar\omega$ plot for some Hf isotopes.

## 4 Conclusions

A comparison study of the results in the $^{157-175}$Hf isotopes has been done. They are located on the border of the strongly deformed region, so self-consistent calculations are required to test the suggestion of the $\gamma$-deformation changes. From an analysis of the alignment which has been computed it is always decreasing when the masses increase (Figure 1). In $^{157}$Hf isotope a strong oscillation of the moment of inertia which can be traced to the admixture in the ground band its decoupling parameter, the oscillation and compression are strong by increase from $^{175}$Hf to $^{157}$Hf which means that the coriolis mixing becomes much stronger with lower $N$. The larger value of $\delta E(I)$ for $^{157}$Hf nucleus probably reflected their $\gamma$-softness. Using the evaluation of the moment of inertia we obtain a very accurate description of back-bending in $^{163,165,167}$Hf-isotopes.

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