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Higher Dimensional Bianchi Type -VIo Universe in Creation-Field Cosmology

K.S. Adhav, S.D. Katore, A.S. Bansod, P.S. Gadodia

Sant Gadge Baba Amravati University, Amravati (444602) India

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Abstract. We have studied the Hoyle-Narlikar C-field cosmology with Bianchi type-VIo space-time in higher dimensions. Using methods of Narlikar and Padmanabham (1985), the solutions have been studied when the creation field C is a function of time t only. The geometrical and physical aspects for model are also studied.

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1 Introduction

The study of higher dimensional physics is important because of several prominent results obtained in the development of the super-string theory. In the latest study of super-strings and super-gravity theories, Weinberg [1] studied the unification of the fundamental forces with gravity, which reveals that the space-time should be different from four. Since the concept of higher dimensions is not unphysical, the string theories are discussed in 10-dimensions or 26-dimensions of space-time. Because of this, studies in higher dimensions inspired many researchers to enter into such a field of study to explore the hidden knowledge of the universe. Chodos and Detweller [2], Lorentz-Petzold [3], Ibanez and Verdaguer [4], Gleiser and Diaz [5], Banerjee and Bhui [6], Reddy and Venkateswara [7], Khadekar and Gaikwad [8], Adhav et al. [9] have studied the multi-dimensional cosmological models in general relativity and in other alternative theories of gravitation.

The three important observations in astronomy viz., the phenomenon of expanding universe, primordial nucleon-synthesis and the observed isotropy of cosmic microwave background radiation (CMBR) were supposed to be successfully explained by big-bang cosmology based on Einstein's field equations. However, Smoot *et al*. [10] revealed that the earlier predictions of the Friedman-Robertson-Walker type of models do not always exactly meet our expectations. Some puzzling results regarding the red shifts from the extra galactic objects continue to contradict the theoretical explanations given from the big bang type of the model. Also, CMBR discovery did not prove it to be a out come of big bang theory. In fact, Narlikar *et al*. [11] have proved the possibility of non-relic interpretation of CMBR. To explain such phenomenon, many alternative theories have been proposed from time to time. Hoyle [12], Bondi and Gold [13] proposed steady state theory in which the universe does not have singular beginning nor an end on the cosmic time scale. Moreover, they have shown that the statistical properties of the large scale features of the universe do not change. Further, the constancy of the mass density has been accounted by continuous creation of matter going on in contrast to the one time infinite and explosive creation of matter at $t = 0$ as in the earlier standard model. But the principle of conservation of matter was violated in this formalism. To overcome this difficulty Hoyle and Narlikar [14] adopted a field theoretic approach by introducing a massless and chargeless scalar field C in the Einstein-Hilbert action to account for the matter creation. In the C-field theory introduced by Hoyle and Narlikar there is no big bag type of singularity as in the steady state theory of Bondi and Gold [13]. A solution of Einstein's field equations admitting radiation with negative energy massless scalar creation fields C was obtained by Narlikar and Padmanabhan [15]. The study of Hoyle and Narlikar theory [14, 16, 17] to the space-time of dimensions more than four was carried out by Chatterjee and Banerjee [18]. Raj Bali and Tikekar [19] studied C-field cosmology with variable G in the flat Friedmann-Robertson-Walker model. Whereas, C-field cosmological models with variable G in FRW space-time has been studied by Raj Bali and Kumawat [20]. The solutions of Einstein's field equations in the presence of creation field have been obtained for Bianchi type-VIo universe in four dimensions by Singh and Chaubey [21].

Here, we have considered a spatially homogeneous and anisotropic Bianchi type-VIo cosmological model in Hoyle and Narlikar C-field cosmology with five dimensions. We have assumed that the creation field C is a function of time t only, *i.e.*, $C(x,t) = C(t)$.

2 Hoyle and Narlikar C-field Cosmology

Introducing a massless scalar field called as creation field viz. C-field, Einstein's field equations are modified. Hoyle and Narlikar [14, 16, 17] field equations are

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi\left({}^{m}T_{ij} + {}^{c}T_{ij}\right) , \tag{1}$$

where ${}^{m}T_{ij}$ is matter tensor of Einstein theory and ${}^{c}T_{ij}$ is matter tensor due to the C-field which is given by

$${}^{c}T_{ij} = -f\left(C_iC_j - \frac{1}{2}g_{ij}C^kC_k\right) , \tag{2}$$

where $f > 0$ and $C_i = \dfrac{\partial C}{\partial x^i}$.

Because of the negative value of T^{00} $\big(T^{00} < 0\big)$, the C-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$$ {}^{m}T^{ij}_{;j} = -{}^{c}T^{ij}_{;j} = fC^{i}C^{j}_{;j} \tag{3} $$

i.e. matter creation through non-zero left hand side is possible while conserving the over all energy and momentum.

Above equation is similar to

$$ mg_{ij}\frac{dx^i}{ds} - C_j = 0. \tag{4} $$

which implies that the 4-momentum of the created particle is compensated by the 4-momentum of the C-field. In order to maintain the balance, the C-field must have negative energy. Further, the C-field satisfy the source equation $fC^{i}_{;i} = J^{i}_{;i}$ and $J^i = \rho\dfrac{dx^i}{ds} = \rho v^i$, where ρ is homogeneous mass density.

3 Metric and Field Equations

The five-dimensional Bianchi-Type-VIo line element can be written as

$$ ds^2 = dt^2 - a_1^2dx^2 - a_2^2e^{-2m^2x}dy^2 - a_3^2e^{2m^2x}dz^2 - a_4^2e^{-2mx}du^2\,, \tag{5} $$

where a_1, a_2, a_3 and a_4 are functions of t only and m is constant.

Here the extra coordinate is taken to be space like.

We have assumed that creation field C is function of time t only, *i.e.*

$$ C(x,t) = C(t) \quad \text{and} \quad {}^{m}T^{i}_{j} = diag(\rho, -p, -p, -p, -p). \tag{6} $$

We have assumed that velocity of light and gravitational constant are equal to one unit.

Now, the Hoyle-Narlikar field equations (1) for metric (5) with the help of equations (2), (3), and (6) can be written as

$$ \frac{\dot a_1\dot a_2}{a_1a_2} + \frac{\dot a_1\dot a_3}{a_1a_3} + \frac{\dot a_1\dot a_4}{a_1a_4} + \frac{\dot a_2\dot a_3}{a_2a_3} + \frac{\dot a_2\dot a_4}{a_2a_4} + \frac{\dot a_3\dot a_4}{a_3a_4} - \frac{3m^4}{a_1^2} = 8\pi\Big(\rho - \frac{1}{2}f\dot C^2\Big) \tag{7} $$

$$ \frac{\ddot a_2}{a_2} + \frac{\ddot a_3}{a_3} + \frac{\ddot a_4}{a_4} + \frac{\dot a_2\dot a_3}{a_2a_3} + \frac{\dot a_2\dot a_4}{a_2a_4} + \frac{\dot a_3\dot a_4}{a_3a_4} = 8\pi\Big(-p + \frac{1}{2}f\dot C^2\Big) \tag{8} $$

$$ \frac{\ddot a_1}{a_1} + \frac{\ddot a_3}{a_3} + \frac{\ddot a_4}{a_4} + \frac{\dot a_1\dot a_3}{a_1a_3} + \frac{\dot a_1\dot a_4}{a_1a_4} + \frac{\dot a_3\dot a_4}{a_3a_4} - \frac{2m^4}{a_1^2} = 8\pi\Big(-p + \frac{1}{2}f\dot C^2\Big) \tag{9} $$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_4}{a_4}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_1\dot{a}_4}{a_1a_4}+\frac{\dot{a}_2\dot{a}_4}{a_2a_4}-\frac{2m^4}{a_1^2}=8\pi\Big(-p+\frac{1}{2}f\dot{C}^2\Big) \quad (10)$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_1\dot{a}_3}{a_1a_3}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}-\frac{2m^4}{a_1^2}=8\pi\Big(-p+\frac{1}{2}f\dot{C}^2\Big) \quad (11)$$

$$\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_3}{a_3}=\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_4}{a_4} \quad (12)$$

$$\dot{\rho}+\Big(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}+\frac{\dot{a}_4}{a_4}\Big)(\rho+p)=f\dot{C}\Big[\ddot{C}+\Big(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}+\frac{\dot{a}_4}{a_4}\Big)\dot{C}\Big] \quad (13)$$

where dot $(\cdot)$ indicates the derivative with respect to t.

From equation (12), we get

$$a_1a_3=a_2a_4. \quad (14)$$

Assume that V is a function of time t defined by

$$V=a_1a_2a_3a_4. \quad (15)$$

From equation (14) and equation (15), we get

$$V=a_1^2a_3^2. \quad (16)$$

Above equation (13) can be written in the form

$$\frac{d}{dV}(V\rho)+p=f\dot{C}(V)\frac{d}{dV}[V\dot{C}(V)]. \quad (17)$$

In order to obtain a unique solution, one has to specify the rate of creation of matter-energy (at the expense of the negative energy of the C-field). Without loss of generality, we assume that the rate of creation of matter energy density is proportional to the strength of the existing C-field energy-density, *i.e.* the rate of creation of matter energy density per unit proper-volume is given by

$$\frac{d}{dV}(V\rho)+p=\alpha^2\dot{C}^2\equiv\alpha^2g^2(V), \quad (18)$$

where α is proportionality constant and we have defined $\dot{C}(V)\equiv g(V)$.

Substituting it in equation (17), we get

$$\frac{d}{dV}(V\rho)+p=fg(V)\frac{d}{dV}(Vg) \quad (19)$$

Comparing right hand sides of equations (18) and (19), we get

$$g(V)\frac{d}{dV}(gV)=\frac{\alpha^2}{f}g^2(V) \quad (20)$$

Integrating, which gives

$$g(V) = A_1 V^{(\alpha^2/f-1)}, \tag{21}$$

where A_1 is arbitrary constant of integration.
We consider the equation of state of matter as

$$p = \gamma\rho. \tag{22}$$

Substituting equations (21) and (22) in the equation (18), we get

$$\frac{d}{dV}(V\rho) + \gamma\rho = \alpha^2 A_1^2 V^{2(\alpha^2/f-1)}. \tag{23}$$

Which further yields

$$\rho = \frac{\alpha^2 A_1^2}{(2\alpha^2/f - 1 + \gamma)} V^{2(\alpha^2/f-1)}. \tag{24}$$

From equation (22), we get

$$p = \frac{\alpha^2 A_1^2 \gamma}{(2\alpha^2/f - 1 + \gamma)} V^{2(\alpha^2/f-1)}. \tag{25}$$

Adding equations (8), (9), (10), (11) and 4 times equation (7), we get

$$\begin{aligned}&\Big(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_4}{a_4}\Big) \\ &\quad + 2\Big(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} + \frac{\dot{a}_1\dot{a}_4}{a_1a_4} + \frac{\dot{a}_2\dot{a}_4}{a_2a_4} + \frac{\dot{a}_3\dot{a}_4}{a_3a_4}\Big) \\ &\qquad - \frac{6m^4}{a_1^2} = \frac{32}{3}\pi(\rho - p).\end{aligned} \tag{26}$$

From equation (15) we have

$$\begin{aligned}\frac{\ddot{V}}{V} &= \Big(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_4}{a_4}\Big) \\ &\quad + 2\Big(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} + \frac{\dot{a}_1\dot{a}_4}{a_1a_4} + \frac{\dot{a}_2\dot{a}_4}{a_2a_4} + \frac{\dot{a}_3\dot{a}_4}{a_3a_4}\Big).\end{aligned} \tag{27}$$

From equations (26), (27) and (22), we get

$$\frac{\ddot{V}}{V} - \frac{6m^4}{a_1^2} = \frac{32}{3}\pi(1-\gamma)\rho. \tag{28}$$

Here we discuss two cases.

Case I: When $a_1 = \sqrt{V}$

Then equation (28) reduces to

$$\frac{\ddot{V}}{V} - \frac{6m^4}{V} = \frac{32}{3}\pi(1-\gamma)\rho. \tag{29}$$

Which further gives

$$\int \frac{dV}{\sqrt{\frac{32\pi(1-\gamma)A_1^2 f}{3(2\alpha^2/f - 1 + \gamma)} V^{2\alpha^2/f} + 12m^4 V + k_1}} = t\,, \tag{30}$$

where k_1 is integration constant.

For $\gamma = 1$ (Zel'dovich fluid or Stiff fluid) and $k_1 = 0$, the above equation gives

$$V = 3m^4 t^2\,, \tag{31}$$

Substituting equation (31) in equation (21), we get

$$g = A_1(3m^4)^{(\alpha^2/f-1)} t^{2(\alpha^2/f-1)}. \tag{32}$$

Also, from equation $\dot{C}(V) = g(V)$, we get

$$C = \frac{A_1(3m^4)^{(\alpha^2/f-1)} t^{(2\alpha^2/f-1)}}{(2\alpha^2/f - 1)}\,. \tag{33}$$

Substituting equation (31) in equation (24), the homogeneous mass density becomes

$$\rho = \frac{1}{2} A_1^2 f (3m^4)^{2(\alpha^2/f-1)} t^{4(\alpha^2/f-1)}\,. \tag{34}$$

Using equation (25) and $\gamma = 1$, pressure becomes

$$p = \frac{1}{2} A_1^2 f (3m^4)^{2(\alpha^2/f-1)} t^{4(\alpha^2/f-1)}\,. \tag{35}$$

From equations (34) and (35), it is observed that for $f = \alpha^2$, there is no singularity in density and pressure.

From equation (31), we get

$$a_1(t) = \sqrt{3} m^2 t\,. \tag{36}$$

From equation (16), we get

$$a_3(t) = 1,\,. \tag{37}$$

Physical properties:

The expansion scalar θ is given by

$$\theta = 4H = \frac{2}{t}\,. \tag{38}$$

The mean anisotropy parameter is given by

$$A = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right) = 1. \tag{39}$$

The shear scalar σ^2 is given by

$$\begin{aligned}\sigma^2 &= \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 4H^2\right) = \frac{4}{2}AH^2 \\ \sigma^2 &= \frac{1}{2t^2}\,.\end{aligned} \tag{40}$$

The deceleration parameter q is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 1\,, \tag{41}$$

where $\Delta H_i = H_i - H$ and H is Hubble parameter.

For large t, the Expansion Scalar and Shear tends to zero. Further, if $f > \alpha^2$, for large t, the model reduces to the vacuum case.

Case II: When $a_3 = \sqrt{V}$

Then equation (28) reduces to

$$\frac{\ddot{V}}{V} - 6m^4 = \frac{32}{3}\pi(1-\gamma)\rho\,, \tag{42}$$

which further gives

$$\int \frac{dV}{\sqrt{\frac{32\pi(1-\gamma)A_1^2 f}{3(2\alpha^2/f - 1 + \gamma)}V^{2\alpha^2/f} + 6m^4V^2 + k_1}} = t\,, \tag{43}$$

where k_1 is integration constant.

For $\gamma = 1$ (Zel'dovich fluid or Stiff fluid) and $k_1 = 0$, the above equation gives

$$V = \exp\left(\sqrt{6}\,m^2 t\right), \tag{44}$$

Substituting equation (44) in equation (21), we get

$$g = A_1 \exp\left(\sqrt{6}\, m^2 t(\alpha^2/f - 1)\right). \tag{45}$$

Also, from equation $\dot{C}(V) = g(V)$, we get

$$C = \frac{A_1 \exp\left(\sqrt{6}\, m^2 t(\alpha^2/f - 1)\right)}{\sqrt{6}\, m^2(\alpha^2/f - 1)}. \tag{46}$$

Substituting equation (44) in equation (24), the homogeneous mass density becomes

$$\rho = \frac{1}{2} A_1^2 \exp\left(2\sqrt{6}\, m^2 t(\alpha^2/f - 1)\right). \tag{47}$$

Using equation (25) and $\gamma = 1$, pressure becomes

$$p = \frac{1}{2} A_1^2 \exp\left(2\sqrt{6}\, m^2 t(\alpha^2/f - 1)\right). \tag{48}$$

From equations (47) and (48), it is observed that for $f = \alpha^2$, there is no singularity in density and pressure.

From equation (44), we get

$$a_3(t) = \exp\frac{1}{2}\left(\sqrt{6}\, m^2 t(\alpha^2/f - 1)\right). \tag{49}$$

From equation (16), we get

$$a_1(t) = 1. \tag{50}$$

Physical properties

The expansion scalar θ is given by

$$\theta = 4H = \frac{\sqrt{6}}{2} m^2. \tag{51}$$

The mean anisotropy parameter is given by

$$A = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right) = 5. \tag{52}$$

The shear scalar σ^2 is given by

$$\begin{aligned} \sigma^2 &= \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 4H^2\right) = \frac{4}{2} A H^2, \\ \sigma^2 &= \frac{15}{16} m^4. \end{aligned} \tag{53}$$

The deceleration parameter q is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1, \tag{54}$$

where $\Delta H_i = H_i - H$ and H is the Hubble parameter.

Further, if $f > \alpha^2$, for large t, the model reduces to the vacuum case.

4 Discussion

In case I, we get positive deceleration parameter indicating that the expansion is decelerating quickly enough for the Universe eventually to collapse.

Whereas, in case II, we get negative deceleration parameter indicating that the universe is accelerating which is consistent with the present day observation.

In both the cases, $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$ and hence the model is anisotropic.

In general, we have observed that the creation field C is proportional to time t. That is, the creation of matter increases as time increases.

Also, we have observed that for $f > \alpha^2$, the matter density is inversely proportional to time t. When $t \to 0$, we get $\rho \to \infty$ and when $t \to \infty$, we get $\rho \to 0$. This is physically valid result indicating that there is a situation where our higher dimensional C-field cosmology starts from infinite mass density. Referring to Hoyle and Narlikar [17], Hawking and Ellis [22], we can interpret our result as the matter is suppose to move along the geodesic normal to the surface $t =$ const. As the matter moves further apart, it is assumed that more mass is continuously created to maintain the matter density. However, matter density tends to zero when time will be infinitely large.

5 Conclusion

In this paper we have considered the space-time geometry corresponding to Bianchi type-VIo in Hoyle-Narlikar [14, 16, 17] creation field theory of gravitation with five dimensions. Bianchi type-VIo universe in creation – field cosmology has been investigated by Singh and Chaubey [21] whose work has been extended and studied in five dimensions. An attempt has been made to retain Singh and Chaubey's forms of the various quantities [21]. We have noted that all the results of Singh and Chaubey can be obtained from our results by assigning appropriate values to the functions concerned.

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