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Iran Tokamak 1 (IR-T1) Pressure Simulation for Resistive and Magnetized Plasma by Vlasov Equation

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Abstract. Now a days it is universally recognized that building huge fusion reactors, everywhere in the world is far from economical view point even in the advanced countries, because the outcome of the research projects is not completely clear. In IR-T1 experimental Tokamak also some practical uncertainty and experimental limitations are found out. In order to achieve transport criteria we simulated this phenomenon through magnetohydrodynamic (MHD) model, by using Vlasov equation taking into account plasma resistivity.

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1 System Modelling

In IR-T1 Tokamak the particle density is about $n \sim 10^{20}$ m^{-3} and the magnetic field consists of a toroidal field produced from external coil around the chamber with a current about 6-7 kA and a poloidal magnetic field from the plasma current. The resistive plasma is collisional and this latter pretended as Ohm law, partially thermalized the plasma mode.

In this paper the pressure transport in the Tokamak has been taken into account in the central cross-section of the toroidal chamber.

The system equations used are as following:

$$\frac{D\rho}{Dt} = -\rho\vec{\nabla}\cdot\vec{u}, \tag{1}$$

$$\rho\frac{D\vec{u}}{Dt} = (\vec{\nabla}\times\vec{B})\times\frac{\vec{B}}{\mu_0} - \vec{\nabla}P, \tag{2}$$

$$\frac{Dp}{Dt} = -\wp p\vec{\nabla}\cdot\vec{u} + (\wp - 1)\frac{(\vec{\nabla}\times\vec{B})^2}{\sigma\mu_0^2}\,, \tag{3}$$

$$\frac{\partial \vec{B}}{\partial t} = \frac{\nabla^2 \vec{B}}{\sigma \mu_0} + \vec{\nabla} \times (\vec{u} \times \vec{B}) \tag{4}$$

Equation (1) is conservation law of mass, Eq. (2) – of motion and Eq. (3) – thermodynamics of Tokamak, the first equation in (5) is convection differentiation and in the second one, σ is the conductivity of the plasma [1] and for simplicity we have assumed that the plasma is not compressible, then

$$\frac{D}{Dt} = \left(\frac{\partial}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \right), \qquad \sigma \equiv \frac{1}{\eta} \tag{5}$$

$$\vec{\nabla} \cdot \vec{u} = 0. \tag{6}$$

From this Eq. (1) would lead to

$$\frac{D\rho}{Dt} = 0\,. \tag{7}$$

This means that ideally the plasma is densified and ρ is constant.

With the neutrality assumption of the plasma, we have

$$(n_e = n_i \approx n_0$$

and

$$\vec{\nabla} \cdot \vec{E} = 0.$$

It is assumed that the plasma is homogenous and the electric field is locally cancelled.

The total magnetic field in the reactor is

$$(B_{\text{Total}} = B_\varphi + B_\theta). \tag{8}$$

IR-T1 is a low β Tokamak and high aspect ratio one, with toroidal circular cross section, that is,

$$B_\varphi = B_0 \left(\frac{R_0}{R} (1 + O(\varepsilon^2)) \right),$$

where O denotes higher order of 2,

$$\varepsilon = \frac{a}{R} \tag{9}$$

is the inverse of the aspect ratio and

$$B_\theta \approx \varepsilon B_0. \tag{10}$$

Here a is the minor radius, R is the major radius, B_Φ is the toroidal magnetic field, B_θ is the poloidal magnetic field [2].

In the central cross-section of chamber $R = R_0$ in the first approximation in the central cross section of toroidal chamber

$$\frac{\partial B}{\partial t} = 0.$$

For evolution of the magnetic field, we have:

$$\vec{B} = B_\psi \hat{\psi} + B_\varphi \hat{\varphi} \tag{11}$$

B_ψ is the poloidal field and $\hat{\psi}$ is the unit vector in this direction; B_φ is the toroidal field with $\hat{\varphi}$ – the unit vector in this direction.

The simulation is done in toroidal system and translations of Cartesian axis into toroidal system are

$$\hat{\psi} = \sin\psi(-\cos\varphi\hat{x} - \sin\varphi\hat{y}) + \cos\psi\hat{z} \tag{12}$$

$$\hat{\rho} = \cos\varphi\hat{x} + \sin\varphi\hat{y}\,. \tag{13}$$

In the central cross-section of toroidal chamber

$$\sin\psi = \frac{z}{a}, \qquad \cos\psi = \sqrt{1-\sin^2\psi} = \sqrt{1-\frac{z^2}{a^2}}\,. \tag{14}$$

In the limit $z \ll a$ and replacing it into the above equation, we have

$$\hat{\psi} = \frac{-z}{a}\hat{\rho} + \sqrt{1-\frac{z^2}{a^2}}\;\hat{z} \tag{15}$$

Then for magnetic field we have:

$$\vec{B} = B_\psi\hat{\psi} + B_\varphi\hat{\varphi} = B_\psi\left(\frac{-z}{a}\right)\hat{\rho} + B_\psi\sqrt{1-\frac{z^2}{a^2}}\;\hat{z} + B_\varphi\hat{\varphi}\,. \tag{16}$$

In the first approximation

$$B_\varphi = B_0, \tag{17}$$

$$B_\psi = B_1 = \frac{a}{R}B_0. \tag{18}$$

Here B_0 is the toroidal magnetic field in the major radius of Tokamak chamber.

From the above results, we would get

$$\vec{B} = \frac{-B_1 z}{a}\,\hat{\rho} + B_0\hat{\varphi} + \frac{B_1\sqrt{a^2-z^2}}{a}\hat{z} \tag{19}$$

with the values

$$\begin{cases} R-a < \rho < R+a \\ z < a \end{cases}$$

For achieving the pressure evolution we have assumed that at first ($t = 0$) the pressure of Tokamak is constant about the 0.5 torr. Then for pressure we have

$$\frac{Dp}{Dt} = -\wp p \vec{\nabla} \cdot \vec{u} + (\wp - 1)\frac{(\vec{\nabla} \times \vec{B})^2}{\sigma \mu_0^2}\,, \tag{20}$$

$$\begin{aligned}\frac{Dp}{Dt} &= \frac{\partial p}{\partial t} + (\vec{u} \cdot \vec{\nabla})p = \frac{\partial p}{\partial t} + \left(u_\rho \frac{\partial}{\partial \rho} + u_z \frac{\partial}{\partial z}\right) p \\ &= \frac{\partial p}{\partial t} + u_\rho \frac{\partial p}{\partial \rho} + u_z \frac{\partial p}{\partial z}\,.\end{aligned} \tag{21}$$

For calculating the right side of Eq. (20), we proceed as follows:

$$\vec{\nabla} \cdot \vec{u} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho u_\rho) + \frac{1}{\rho}\frac{\partial u_\varphi}{\partial \varphi} + \frac{\partial u_z}{\partial z} \tag{22}$$

Because of azimuthal symmetry in Tokamak [3], we have

$$\frac{\partial u_\varphi}{\partial \varphi} = 0\,. \tag{23}$$

Then

$$\vec{\nabla} \cdot \vec{u} = \frac{u_\rho}{\rho} + \frac{\partial u_\rho}{\partial \rho} + \frac{\partial u_z}{\partial z}\,, \tag{24}$$

$$(\vec{\nabla} \times \vec{B})^2 = \left[\frac{-B_1}{a}\hat{\varphi} + \frac{B_0}{\rho}\hat{z}\right]^2. \tag{25}$$

Inserting the above achievements into Eq. (20) would lead to pressure evolution equation in a reactor

$$\frac{\partial p}{\partial t} = -u_\rho \frac{\partial p}{\partial \rho} - u_z \frac{\partial p}{\partial z} - \wp p\left[\frac{u_\rho}{\rho} + \frac{\partial u_\rho}{\partial \rho} + \frac{\partial u_z}{\partial z}\right] + \frac{(\wp - 1)}{\sigma \mu_0^2}\left[\frac{-B_1}{a}\hat{\varphi} + \frac{B_0}{\rho}\hat{z}\right]^2. \tag{26}$$

In IR-T1 $R = 45$ cm $a = 12.5$ cm, $B_T = 0.6$–0.9 T, $\eta_p = 0.86 \times 10^{-6}$ Ωm, $\gamma = 1.4$ [4].

Simulations for the pressure transport in IR-T1 refer to Eq. (26) and are shown in Figures 1–5.

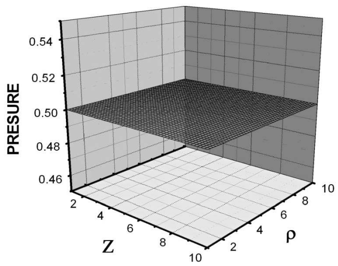

Figure 1. Initial constant pressure of 0.5 Torr in central cross section of IR-T1.

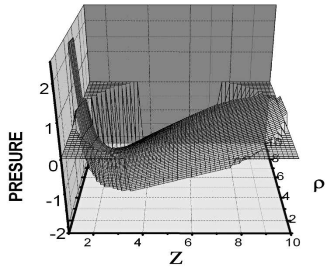

Figure 2. Evolutionary pressure in the next steps of time and B values.

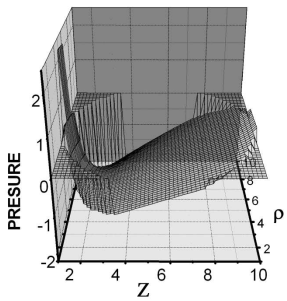

Figure 3. Evolutionary pressure in the next steps of time, B and plasma velocity.

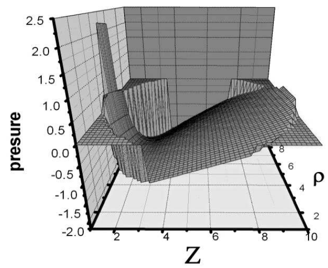

Figure 4. Evolution of pressure in IR-T1 refers to the time steps and evolutionary values of B and plasma velocities.

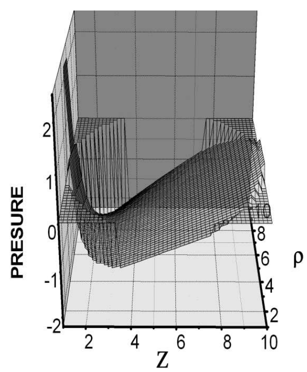

Figure 5. Evolutionary pressure in the last steps of time proceeding with latest values of magnetic field B and plasma velocities.

2 Conclusion Remarks

From all these Figures it is clear that:

1. Although the pressure started at about 0.6-0.9 torr in reactor, after its evolution during special time range the plasma shows a tendency toward stability at the end of the process at about the beginning values.

2. The integral value of iteration of the simulation to reach the stability is an indication of the self consistency of the method used for a simulation of the pressure transport in the reactor.

3. The inversion of the pressure around the wall of the reactor is because of the scattering of the plasma particles around the walls.

4. The above mentioned consideration is the reason for a need to insert diverter in the reactor.

5. The pressure profile in this research started from a uniform primary value of the plasma p_0 inside the IR-T1 at the beginning of the plasma shot.

   For fitting this pressure to our simulation code it scales relevantly as shown in Figures 1–5.

6. The transport mechanism of the plasma under such condition of the pressure profile is introduced in another paper.

7. Obviously in this approach the evolutionary transition of the pressure in the cross section of the IR-T1 Tokamak simulated after 100000 time steps repetition of the equation as seen in Figures 1-5.

References

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