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Comparison of Two Results for Heating of the Solar Corona through MHD Waves

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Abstract. For a set of MHD equations for the solar atmosphere, Kumar *et al.* [1] have obtained a fifth degree polynomial in ω for the dispersion relation. On the other side, for the same set of equations, Dwivedi & Pandey [2,3], and Pandey & Dwivedi [4,5] have obtained a sixth degree polynomial in ω for the dispersion relation. Each of the two groups, tried to say that the results of the others are erroneous. Their main concern was that because of the difference in the degree of the polynomial, the roots in the two cases would be different and consequently, their results for the slow-mode and fast-mode magnetoacoustic waves were different. In fact, they obtained different results.

Recently, Chandra *et al.* [6] have shown analytically that five roots of the two polynomials (fifth degree as well as sixth degree) are common and these roots pertain to the slow-mode and fast-mode magnetoacoustic waves. When the roots, pertaining to the slow-mode and fast-mode waves are common in the two polynomials, a good question arises why the two groups are getting different results and claiming that the results of the others are erroneous. In the present communication, we have reinvestigated the work and found that the results of Kumar *et al*. [1] are reliable. However, we could not ascertain the cause for the error in the results of Dwivedi & Pandey [2,3], and Pandey & Dwivedi [4,5].

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1 Introduction

As the temperature of solar corona (within 1–$2R_\odot$) is being maintained at the value of the order of 10^6 K, the problem of solar coronal heating is yet unsettled. Kumar *et al*. [1], Dwivedi & Pandey [2,3], and Pandey & Dwivedi [4,5] (these four papers of Dwivedi & Pandey, and Pandey & Dwivedi will be referred to as DP hereinafter) have investigated the problem of solar coronal heating through

MHD waves by using the same set of MHD equations. They got different results for the damping rate and wavelength of slow-mode and fast-mode magnetoacoustic waves. Their main doubt about the results of each other has been the difference in the degree of polynomial for the dispersion relation, they derived. Let us first look into their dispersion relations.

For the propagation of MHD waves in a homogeneous, magnetically structured, compressible, and low-β plasma, the basic equations are [6]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot\ (\rho \vec{v}) = 0, \tag{1}$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v} \cdot \nabla)\vec{v} = -\nabla p + \frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B} - \nabla \cdot \mathbf{\Pi}, \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}), \tag{3}$$

$$\frac{Dp}{Dt} + \gamma p(\nabla \cdot \vec{v}) = (\gamma - 1)[Q_{\text{th}} + Q_{\text{vis}} - Q_{\text{rad}}], \tag{4}$$

$$p = \frac{2\rho k_B T}{m_p}\,, \tag{5}$$

where symbols have their usual meanings and the quantities Q_{th}, Q_{vis} and Q_{rad} are [6]

$$Q_{\text{th}} = \kappa_{\|}\left(\frac{\partial T}{\partial z}\right)^2 T^{-1}, \quad Q_{\text{vis}} = \frac{\eta_0}{3}(\nabla \cdot \vec{v})^2, \quad Q_{\text{rad}} = n_e n_H Q(T),$$

where $\kappa_{\|}$ represents the conductivity along the magnetic field and is expressed by $\kappa_{\|} \approx 10^{-6} T^{5/2}$. For this set of equations (1) – (5), Kumar *et al.* [1], and Chandra & Kumthekar [7] derived the dispersion relation

$$\omega^5 + \mathrm{i}A\omega^4 - B\omega^3 - \mathrm{i}C\omega^2 + D\omega + \mathrm{i}E = 0, \tag{6}$$

where

$$A = c_0 + \frac{\eta_0}{3\rho_0}(k_x^2 + 4k_z^2),$$

$$B = \frac{c_0\eta_0}{3\rho_0}(k_x^2 + 4k_z^2) + (c_s^2 + v_A^2)k^2,$$

$$C = \frac{3\eta_0}{\rho_0}c_s^2 k_x^2 k_z^2 + \frac{c_0 p_0 k^2}{\rho_0} + v_A^2 c_0 k^2 + \frac{4\eta_0 v_A^2 k_z^2 k^2}{3\rho_0}\,,$$

$$D = \frac{3c_0 p_0 \eta_0 k_x^2 k_z^2}{\rho_0^2} + \frac{4\eta_0 c_0 v_A^2 k_z^2 k^2}{3\rho_0} + v_A^2 c_s^2 k_z^2 k^2,$$

$$E = \frac{v_A^2 c_0 p_0 k_z^2 k^2}{\rho_0}\,.$$

For the same set of MHD equations (1) – (5), DP derived the dispersion relation

$$\omega^6 + \mathrm{i}A'\omega^5 - B'\omega^4 - \mathrm{i}C'\omega^3 + D'\omega^2 + \mathrm{i}E'\omega - F' = 0, \tag{7}$$

where

$$
\begin{aligned}
A' &= 2c_0 + c_1, \\
B' &= (c_s^2 + v_A^2)k^2 + c_0(2c_1 + c_0), \\
C' &= c_2 + c_0(k^2(c_s^2 + 2v_A^2 + \frac{p_0}{\rho_0}) + c_0 c_1)\,, \\
D' &= c_s^2 c_6 + c_0(c_3 + c_0 c_4), \\
E' &= c_0\Big[c_0 c_5 + c_6\Big(c_s^2 + \frac{p_0}{\rho_0}\Big)\Big], \\
F' &= c_0^2 c_6 p_0/\rho_0
\end{aligned}
$$

and

$$
\begin{aligned}
c_0 &= (\gamma - 1)k_\parallel k_z^2 T_0/p_0, \\
c_1 &= \eta_0(k_x^2 + 4k_z^2)/3\rho_0, \\
c_2 &= \eta_0 k_z^2(4v_A^2 k^2 + 9c_s^2 k_x^2)/3\rho_0, \\
c_3 &= \frac{\eta_0 k_z^2}{3\rho_0}\Big(8v_A^2 k^2 + 9\Big(c_s^2 + \frac{p_0}{\rho_0}\Big)k_x^2\Big), \\
c_4 &= \Big(v_A^2 + \frac{p_0}{\rho_0}\Big)k^2, \\
c_5 &= \frac{\eta_0 k_z^2}{3\rho_0}\Big(4v_A^2 k^2 + \frac{9p_0 k_x^2}{\rho_0}\Big), \\
c_6 &= v_A^2 k^2 k_z^2.
\end{aligned}
$$

After getting the dispersion relation of degree five in ω (Eq. 6), Kumar *et al*. [1] have tried to say that the results of Dwivedi & Pandey [2] seem to be erroneous as they obtained a dispersion relation of degree six in ω. Dwivedi & Pandey [3] have claimed that the results of Kumar *et al*. [1] are erroneous as the degree of polynomial should be six and not five. Chandra & Kumthekar [7] tried to short out the discrepancy by showing that the degree of polynomial should be five. Dwivedi & Pandey [4] reacted very strongly saying that even the work of Chandra & Kumthekar [7] was also erroneous. Once again Dwivedi & Pandey [5] claimed that the degree of the polynomial should be six and consequently the results of Kumar *et al*. [1] as well as those of Chandra & Kumthekar [7] were wrong.

Instead of going in further controversy, Chandra *et al*. [6] derived successfully the following relation:

$$
\begin{aligned}
\omega^6 + \mathrm{i}A'\omega^5 - B'\omega^4 - \mathrm{i}C'\omega^3 + D'\omega^2 + \mathrm{i}E'\omega - F' \\
= (\omega + \mathrm{i}c_0) \times (\omega^5 + \mathrm{i}A\omega^4 - B\omega^3 - \mathrm{i}C\omega^2 + D\omega + \mathrm{i}E).
\end{aligned}
$$

After this derivation, we have been astonished that such a simple relation was found. This algebraic relation shows categorically that five roots of both polynomials (6) and (7) are common. Chandra *et al*. [6] have also shown that the sixth ($-\mathrm{i}c_0$) has been unnecessarily introduced by DP. The five common roots are of the form $\mathrm{i}\alpha_{1i}$, $\alpha_{2r} \pm \mathrm{i}\alpha_{2i}$, $\alpha_{3r} \pm \mathrm{i}\alpha_{3i}$. Moreover, the characteristics of slow-mode and fast-mode magnetoacoustic waves are expressed in terms of these roots. When it is so, the question arises why the two groups are getting different results and claiming that the results of others are wrong.

Here, we have made an attempt to find out the answer to this controversy.

2 Calculations and Results

After finding out analytically that five roots of the two dispersion relations (6) and (7) are common, it does not matter if we solved Eq. (6) or (7), numerically.

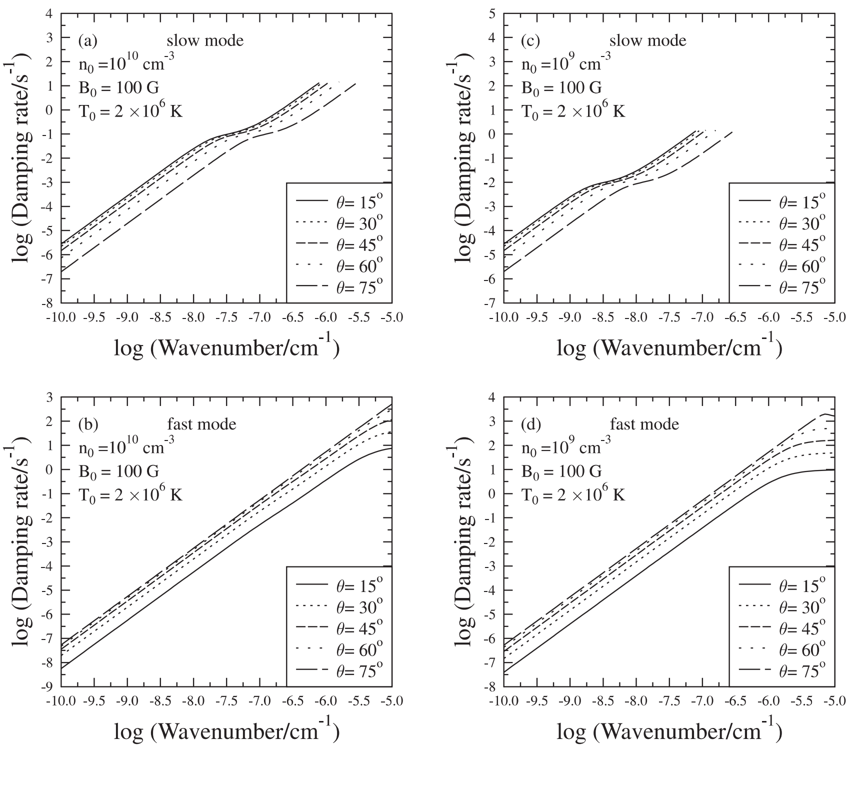

Figure 1. Variation of damping rate as a function of wavenumber for: (a) slow-mode wave and (b) fast-mode wave. Values of parameters are written there.

We have solved both of them and as expected, the five roots in both results have been common. For the investigation of slow-mode and fast-mode waves, values of physical parameters, *i.e.*, plasma density, temperature, magnetic field strength, and angle of propagation relative to the magnetic field, are varied over a range so that the possible conditions could be accounted for.

In Figure 1, we have plotted the damping rate versus wavenumber: (a) and (b) are, respectively, for slow-mode wave and fast-mode wave for $n_0 = 1 \times 10^{10}$ cm^{-3}, $B_0 = 100$ G and $T_0 = 2 \times 10^6$ K; (c) and (d) are, respectively, for slow-mode wave and fast-mode waves for $n_0 = 1 \times 10^9$ cm^{-3}, $B_0 = 100$ G and $T_0 = 2 \times 10^6$ K. In the figures, we have taken $\theta = 15°$, 30^{c}irc, 45^{c}irc, 60°, 75°. For the slow-mode waves, we have found that as the value of k increases, after a certain value, called k_c, the slow mode disappears. For $k > k_c$, the roots are of the form $i\alpha_{1i}$, $i\alpha_{2i}$, $i\alpha_{3i}$, $\alpha_{4r} \pm \mathrm{i}\alpha_{4i}$. This feature is different from that shown by Kumar *et al.* [1]. For the fast mode waves the graph is similar as shown by Kumar *et al.* [1] in their Figures 1(b) and 2 (b).

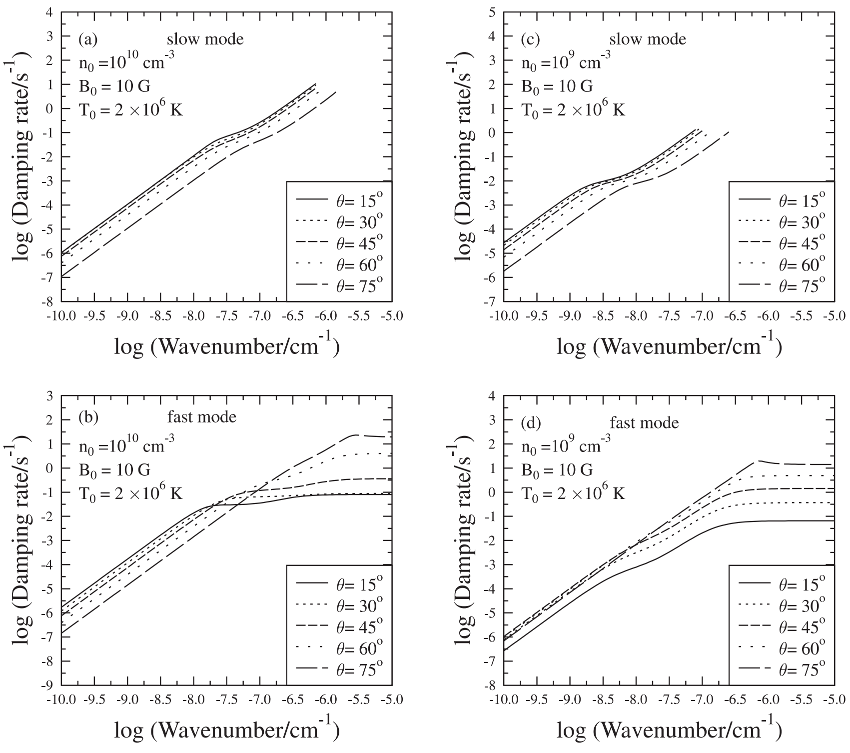

Figure 2. Variation of damping rate as a function of wavenumber for: (a) slow-mode wave and (b) fast-mode wave. Values of parameters are written there.

Figure 2 is similar to Figure 1, where the magnetic is taken 10 G. One can compare Figures 1(a) and 1(c) with Figures 2(a) and 2(c). It gives, the change in magnetic field does not affect the damping rates of slow mode waves. Because the slow mode waves can propagate with velocity of sound c_s, which is independent of magnetic field (Narain & Agarwal [8]). For the fast mode waves the graph is similar in qualitative nature as shown by Kumar *et al.* [1] in their Figures 4(a) and 4(b).

In Figure 3, we have plotted k_c *versus* density for slow-mode waves for: (a) $T_0 = 2 \times 10^6$ K and $B_0 = 100$ G, (b) $T_0 = 3 \times 10^6$ K and $B_0 = 100$ G, (c) $T_0 = 2 \times 10^6$ K and $B_0 = 10$ G and (d) $T_0 = 3 \times 10^6$ K and $B_0 = 10$ G. In the figures, we have taken $\theta = 15°$, 30°, 45°, 60°, 75°. At larger values of number density ($n_0 \sim 10^{10}$ cm^{-3}) and low values of magnetic field ($B = 10$ G) the values of k_c become equal for angle less than or equal to 60° (*i.e.*, $\theta = 15°$, 30°, 45°, 60°) which gives different values for angle greater than 60° (*i.e.*, $\theta = 75°$).

In Figure 4, we have damping rate as a function of wavenumber for: (a) slow-

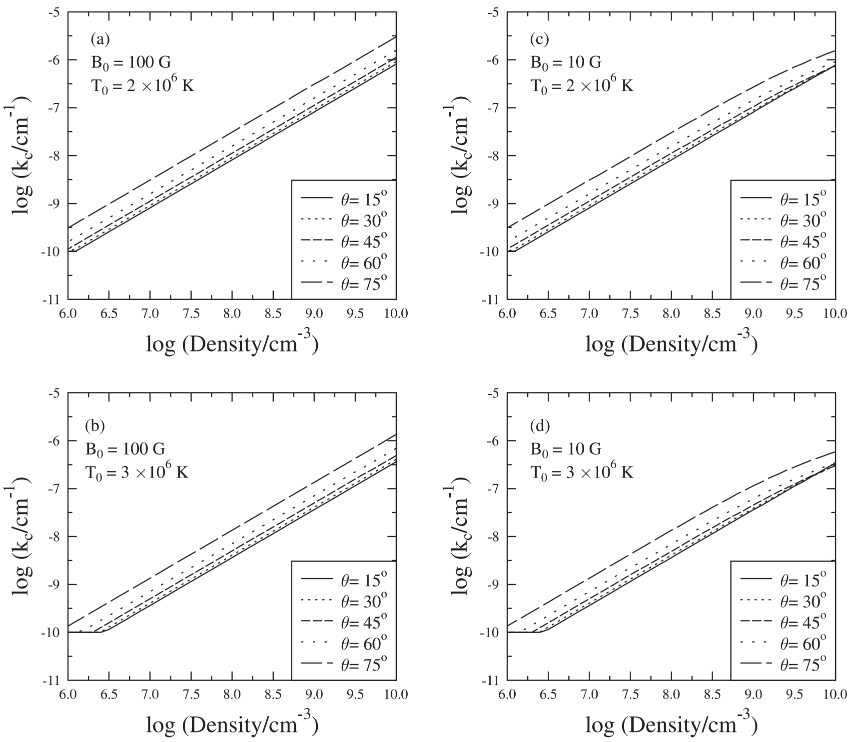

Figure 3. Variation of k_c versus density for slow-mode waves. Values of parameters are written there.

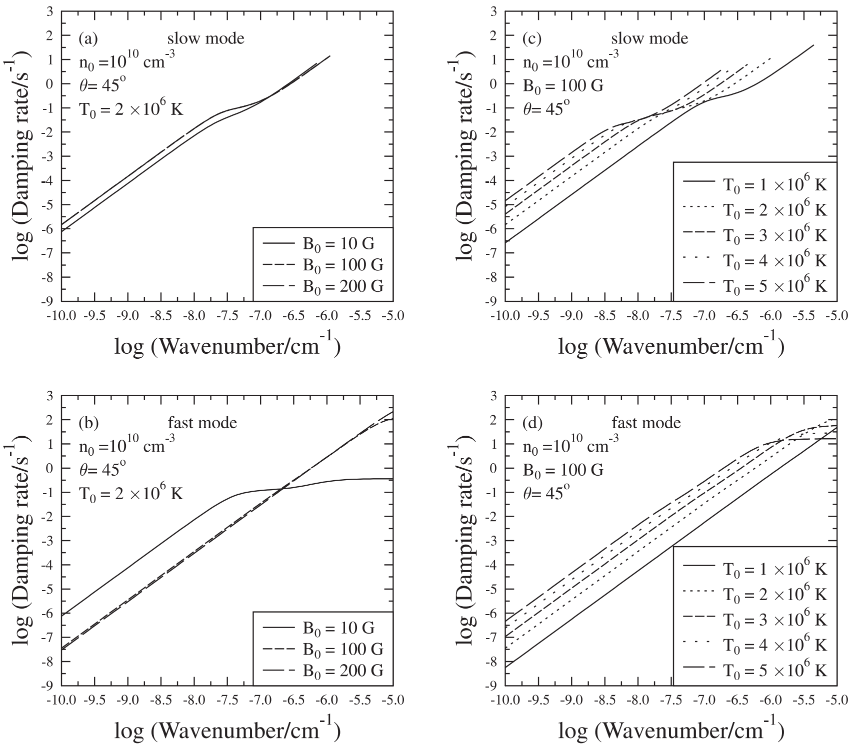

Figure 4. Variation of damping rate as a function of wavenumber for: (a) slow-mode wave and (b) fast-mode wave. Values of parameters are written there.

mode wave and (b) fast-mode wave, for $n_0 = 1 \times 10^{10}$ cm^{-3}, $\theta = 45°$, $T_0 = 2 \times 10^6$ K for three values of magnetic field $B_0 = 10$ G, 100 G and 200 G; (c) slow-mode wave and (d) fast-mode wave, for $n_0 = 1 \times 10^{10}$ cm^{-3}, $\theta = 45°$, $B_0 = 100$ G for five values of temperature $T_0 = 1 \times 10^6$, 2×10^6, 3×10^6, 4×10^6, 5×10^6 K. From Figure 4(c) at low temperature (*i.e.*, 1×10^6 K) the response of damping rate gives larger value of k. As the temperature increases the response decreases.

3 Discussion

In Figures 1 – 4, we have plotted our results for damping rate of slow-mode as well as fast-mode waves. We find that except for $k > k_c$, our results are the same as those of Kumar *et al*. [1]. In order to find out the cause of the controversy between the results of DP and Kumar *et al*. [1], we have attempted to contact Dwivedi and Pandey, separately, so that we could know about the complete set of physical parameters used in their calculations, but they have never shown courtesy to reply the queries. Under such circumstances, we cannot ascertain the cause why the results of DP are not the same as we have obtained the same as obtained by Kumar *et al*. [1]. We hope our present investigation will provide an idea to DP to look into their work again. For the difference in the results, there are two possibilities: (i) either there is a mismatching in the units of physical parameters used and/or (ii) there are some errors in the computer program.

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