Bulg. J. Phys. 37 (2010) 255–265

Higher Dimensional Bianchi Type-I Universe with Perfect Fluid and Dark Energy

K.S. Adhav, A.S. Bansod, R.P. Wankhade, M.S. Desale

Department of Mathematics, Sant Gadge Baba Amravati University, Amravati, India

Received *26 July 2010*

Abstract. We have studied the Bianchi type-I cosmological model with a binary mixture of perfect fluid and dark energy in higher dimensions. The perfect fluid is obeying the equation of state $p = \gamma\rho$, where $\gamma \in [0, 1]$. Whereas, the dark energy is considered to be either the quintessence or the Chaplygin gas. The exact solutions of Einstein's field equations corresponding to five dimensions are obtained.

PACS number: 98.80 Jk, 04.20-q

1 Introduction

The recent most remarkable observational discoveries have shown that our universe is currently accelerating. This was first observed from high red shift supernova Ia [1–7] and confirmed later by cross checks from the cosmic microwave background radiation [8, 9] and large scale structure [10–15]. In Einstein's general relativity, in order to have such acceleration, one needs to introduce a component to the matter distribution of the universe with a large negative pressure. This component is usually referred as dark energy [DE]. The astronomical observations indicate that our universe is flat and currently consists of approximately 2/3 dark energy and 1/3 dark matter. The nature of dark energy as well as dark matter is unknown, and many radically different models have been proposed, such as, a tiny positive cosmological constant, quintessence [16–18], DGP branes [19, 20], the non-linear F(R) models [21–23], and dark energy in brane worlds, among many others [24–41]; see also the review articles [42, 43], and references therein. As mentioned before, the existence of dark energy fluids comes from the observations of the accelerated expansion of the universe and the isotropic pressure cosmological models give the best fitting of the observations. Although some authors [44] have suggested cosmological model with anisotropic and viscous dark energy in order to explain an anomalous cosmological observation in the cosmic microwave background (CMB) at the largest angles.The binary mixture of perfect fluid and dark energy was studied by Bijan Saha [45] for Bianchi type-I and Singh and Chaubey [46] for Bianchi type-V.

Bianchi models have been studied by several authors in an attempt to achieve better understanding of the observed small amount of anisotropy in the Universe. The same models have also been used to examine the role of certain anisotropic sources during the formation of the large-scale structure we see in the Universe today. Some Bianchi cosmologies, for example, are natural hosts of large-scale magnetic fields and therefore their study can shed light on the implications of cosmic magnetism for galaxy formation. The simplest Bianchi family that contains the flat FLRW universe as a special case is the type-I space-times.

The study of higher dimensional physics is important because of several prominent results obtained in the development of the super-string theory. In the latest study of super-strings and super-gravity theories, Weinberg [47] studied the unification of the fundamental forces with gravity, which reveals that the space-time should be different from four. Since the concept of higher dimensions is not unphysical, the string theories are discussed in 10-dimensions or 26-dimensions of space-time. Because of this, studies in higher dimensions inspired many researchers to enter into such a field of study to explore the hidden knowledge of the universe. Chodos and Detweller [48], Lorentz-Petzold [49], Ibanez and Verdaguer [50], Gleiser and Diaz [51], Banerjee and Bhui [52], Reddy and Venkateswara [53], Khadekar and Gaikwad [54], Adhav *et al*. have studied the multi-dimensional cosmological models in general relativity and in other alternative theories of gravitation.

In the present paper, we have considered a spatially homogeneous and anisotropic Bianchi type-I cosmological model in five dimensions when the universe is filled with perfect fluid and dark energy. The work of Bijan Saha [45] has been extended in higher dimensions.

2 Metric and Field Equations

The five-dimensional Bianchi type-I line element can be written as

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2 - a_4^2 du^2, \tag{1}$$

where a_1, a_2, a_3 and a_4 are functions of t only.

Here the extra coordinate is taken to be space-like.

The energy momentum tensor of the source is given by

$$T_i^j = (\rho + p) u_i u^j - p \delta_i^j, \tag{2}$$

where u^j is the flow vector satisfying $g_{ij} u^i u^j = 1$. Here ρ is the energy density of a perfect fluid and/or dark energy density, while p is the corresponding pressure.

The Einstein's field equations for the metric (1) with the help of Eq. (2) are given by

$$\frac{\dot a_1\dot a_2}{a_1a_2}+\frac{\dot a_1\dot a_3}{a_1a_3}+\frac{\dot a_1\dot a_4}{a_1a_4}+\frac{\dot a_2\dot a_3}{a_2a_3}+\frac{\dot a_2\dot a_4}{a_2a_4}+\frac{\dot a_3\dot a_4}{a_3a_4}=k\rho\,, \tag{3}$$

$$\frac{\ddot a_2}{a_2}+\frac{\ddot a_3}{a_3}+\frac{\ddot a_4}{a_4}+\frac{\dot a_2\dot a_3}{a_2a_3}+\frac{\dot a_2\dot a_4}{a_2a_4}+\frac{\dot a_3\dot a_4}{a_3a_4}=-kp, \tag{4}$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_3}{a_3}+\frac{\ddot a_4}{a_4}+\frac{\dot a_1\dot a_3}{a_1a_3}+\frac{\dot a_1\dot a_4}{a_1a_4}+\frac{\dot a_3\dot a_4}{a_3a_4}=-kp, \tag{5}$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_2}{a_2}+\frac{\ddot a_4}{a_4}+\frac{\dot a_1\dot a_2}{a_1a_2}+\frac{\dot a_1\dot a_4}{a_1a_4}+\frac{\dot a_2\dot a_4}{a_2a_4}=-kp, \tag{6}$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_2}{a_2}+\frac{\ddot a_3}{a_3}+\frac{\dot a_1\dot a_2}{a_1a_2}+\frac{\dot a_1\dot a_3}{a_1a_3}+\frac{\dot a_2\dot a_3}{a_2a_3}=-kp. \tag{7}$$

Here k is the Einstein gravitational constant.

We define function V as

$$V=a_1a_2a_3a_4. \tag{8}$$

Subtracting Eq. (4) from Eq. (5), we get

$$\frac{d}{dt}\left(\frac{\dot a_1}{a_1}-\frac{\dot a_2}{a_2}\right)+\left(\frac{\dot a_1}{a_1}-\frac{\dot a_2}{a_2}\right)\left(\frac{\dot a_1}{a_1}+\frac{\dot a_2}{a_2}+\frac{\dot a_3}{a_3}+\frac{\dot a_4}{a_4}\right)=0. \tag{9}$$

Now, from Eqs. (8) and (9), we get

$$\frac{d}{dt}\left(\frac{\dot a_1}{a_1}-\frac{\dot a_2}{a_2}\right)+\left(\frac{\dot a_1}{a_1}-\frac{\dot a_2}{a_2}\right)\frac{\dot V}{V}=0.$$

Integrating, which gives

$$\frac{a_1}{a_2}=d_1\exp\left(x_1\int\frac{dt}{V}\right),\quad d_1=\text{const.},\ x_1=\text{const.} \tag{10}$$

Subtracting Eq. (5) from Eq. (6), we get

$$\frac{d}{dt}\left(\frac{\dot a_2}{a_2}-\frac{\dot a_3}{a_3}\right)+\left(\frac{\dot a_2}{a_2}-\frac{\dot a_3}{a_3}\right)\frac{\dot V}{V}=0.$$

Integrating, we get

$$\frac{a_2}{a_3}=d_2\exp\left(x_2\int\frac{dt}{V}\right),\quad d_2=\text{const.},\ x_2=\text{const.} \tag{11}$$

Subtracting Eq. (6) from Eq. (7), we get

$$\frac{d}{dt}\left(\frac{\dot a_3}{a_3}-\frac{\dot a_4}{a_4}\right)+\left(\frac{\dot a_3}{a_3}-\frac{\dot a_4}{a_4}\right)\frac{\dot V}{V}=0,$$

which on integration gives

$$\frac{a_3}{a_4} = d_3 \exp\left(x_3 \int \frac{dt}{V}\right), \quad d_3 = \text{const.}, \ x_3 = \text{const.} \tag{12}$$

Subtracting Eq. (4) from Eq. (7), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_4}{a_4}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_4}{a_4}\right)\frac{\dot{V}}{V} = 0$$

Integrating, we get

$$\frac{a_1}{a_4} = d_4 \exp\left(x_4 \int \frac{dt}{V}\right), \quad d_4 = \text{const.}, \ x_4 = \text{const.}, \tag{13}$$

where $d_4 = d_1 d_2 d_3$, $x_4 = x_1 + x_2 + x_3$ and $V = a_1 a_2 a_3 a_4$.

Using Eqs. (10), (11), (12), and (13), the values of $a_1(t)$, $a_2(t)$, $a_3(t)$, and $a_4(t)$ can be written explicitly as

$$a_1(t) = D_1 V^{1/4} \exp\left(X_1 \int \frac{dt}{V}\right), \tag{14}$$

$$a_2(t) = D_2 V^{1/4} \exp\left(X_2 \int \frac{dt}{V}\right), \tag{15}$$

$$a_3(t) = D_3 V^{1/4} \exp\left(X_3 \int \frac{dt}{V}\right), \tag{16}$$

$$a_4(t) = D_4 V^{1/4} \exp\left(X_4 \int \frac{dt}{V}\right), \tag{17}$$

where the relations $D_1 D_2 D_3 D_4 = 1$ and $X_1 + X_2 + X_3 + X_4 = 0$ are satisfied by $D_i (i = 1, 2, 3, 4)$ and $X_i (i = 1, 2, 3, 4)$.

Adding Eqs. (4), (5), (6), (7) and four times Eq. (3), we get

$$\begin{aligned}\frac{4}{3}k(\rho - p) &= \Big(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_4}{a_4}\Big) \\ &\quad + 2\Big(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} + \frac{\dot{a}_1\dot{a}_4}{a_1a_4} + \frac{\dot{a}_2\dot{a}_4}{a_2a_4} + \frac{\dot{a}_3\dot{a}_4}{a_3a_4}\Big).\end{aligned} \tag{18}$$

From Eq. (8), we have

$$\begin{aligned}\frac{\ddot{V}}{V} &= \Big(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_4}{a_4}\Big) \\ &\quad + 2\Big(\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} + \frac{\dot{a}_1\dot{a}_4}{a_1a_4} + \frac{\dot{a}_2\dot{a}_4}{a_2a_4} + \frac{\dot{a}_3\dot{a}_4}{a_3a_4}\Big)\end{aligned} \tag{19}$$

From Eqs. (18) and (19), we get

$$\frac{\ddot{V}}{V} = \frac{4}{3}k(\rho - p). \tag{20}$$

The energy conservation equation is given by

$$\dot{\rho} = -\frac{\dot{V}}{V}(\rho + p). \tag{21}$$

Using Eqs. (20) and (21), we get

$$\dot{V} = \pm\sqrt{2\Big(\frac{4k}{3}\rho V^2 + C_1\Big)}\,, \tag{22}$$

where C_1 is the constant of integration.

Now, rewriting Eq. (21) in the form

$$\frac{\dot{\rho}}{(\rho + p)} = -\frac{\dot{V}}{V}\,, \tag{23}$$

and as the pressure (p) and density (ρ) is related by an equation of state of type $p = f(\rho)$, we conclude that p and ρ, hence the right hand side of the Eq. (20) is a function of V only, *i.e.*,

$$\ddot{V} = \frac{4}{3}k(\rho - p)V \equiv F(V). \tag{24}$$

From the mechanical point of view Eq. (24) can be interpreted as an equation of motion of a single particle with unit mass under the force $F(V)$. Then by using Landau and Lifshitz technique [55], the following integration exists:

$$\dot{V} = \pm\sqrt{2(\varepsilon - U(V))}\,. \tag{25}$$

Here ε can be viewed as energy and $U(V)$ is the potential of the force F. Comparing Eqs. (22) and (25), we get $\varepsilon = C_1$ and

$$U(V) = -\frac{4k}{3}\rho V^2. \tag{26}$$

Finally, rearranging Eq. (22), we can write the solution to the Eq. (20) in quadrature form as

$$\int \frac{dV}{\sqrt{(8k/3)\rho V^2 + 2C_1}} = t + t_0, \tag{27}$$

where t_0 is the constant of integration and can be taken to be zero, since it only gives shift in time.

Now we examine the Eqs. (20) and (21) for perfect fluid with or without dark energy for different equations of state with different conditions.

3 Universe as a Binary Mixture of Perfect Fluid and Dark Energy

We consider the evolution of the five dimensional Bianchi type-I universe filled with perfect fluid and dark energy. Taking this into account, the energy density ρ and pressure p in this case comprise those of perfect fluid and dark energy

$$\rho = \rho_{PF} + \rho_{DE}, \qquad p = p_{PF} + p_{DE}. \tag{28}$$

The energy momentum tensor can be decomposed as

$$T_i{}^j = (\rho_{DE} + \rho_{PF} + p_{DE} + p_{PF})u_i u^j - (p_{DE} + p_{PF})\delta_i{}^j. \tag{29}$$

In the above equation ρ_{DE} is the dark energy density and p_{DE} is its pressure. We also use the notations ρ_{DE} and p_{DE} to denote the energy density and the pressure of the perfect fluid, respectively. Here we consider the case when the perfect fluid obeys the following equation of state:

$$p_{PF} = \gamma\rho_{DE}, \tag{30}$$

where γ is the constant and lies in the interval $\gamma \in [0, 1]$.

Depending on its numerical value, the γ describes the following types of universe.

$$\gamma = 0 \qquad \text{(Dust universe)}, \tag{31a}$$

$$\gamma = {}^1\!/_3 \qquad \text{(Radiation universe)}, \tag{31b}$$

$$\gamma \in ({}^1\!/_3, 1) \qquad \text{(Hard universe)}, \tag{31c}$$

$$\gamma = 1 \qquad \text{(Zel'dovich universe or stiff matter)}. \tag{31d}$$

In a co-moving frame the conservation law of energy momentum tensor leads to the balance equation for the energy density

$$\dot{\rho}_{DE} + \dot{\rho}_{PF} = -\frac{\dot{V}}{V}(\rho_{DE} + \rho_{PF} + p_{DE} + p_{PF}). \tag{32}$$

The dark energy supposed to interact with itself only and it is minimally coupled to the gravitational field. As a result the evolution equation for the energy density decouples from that of the perfect fluid and from (32) we obtain two balance equations

$$\dot{\rho}_{DE} + \frac{\dot{V}}{V}(\rho_{DE} + p_{DE}) = 0, \tag{33}$$

$$\dot{\rho}_{PF} + \frac{\dot{V}}{V}(\rho_{PF} + p_{PF}) = 0. \tag{34}$$

From Eq. (30) and Eq. (34), we get

$$\rho_{PF} = \frac{\rho_0}{V^{1+\gamma}}, \qquad p_{PF} = \frac{\rho_0\gamma}{V^{1+\gamma}}. \tag{35}$$

In the absence of dark energy,

$$\int \frac{dV}{\sqrt{2\Big(\frac{4k}{3}\rho_0 V^{1-\gamma} + C_1\Big)}} = t. \tag{36}$$

3.1 Quintessence Case

Consider the case when the dark energy is given by a quintessence obeying the equation of state

$$p_q = \omega_q \, \rho_q, \tag{37}$$

where the constant ω_q varies between -1 and zero, *i.e.*, $\omega_q \in [-1, 0]$.

The case $\omega_q = -1$ is nothing but the case of cosmological constant (Λ).

From Eq. (37) and Eq. (33), we get

$$\rho_q = \frac{\rho_{0q}}{V^{1+\omega_q}} \quad \text{and} \quad p_q = \frac{\omega_q \rho_{0q}}{V^{1+\omega_q}}\,, \tag{38}$$

where ρ_{0q} is an integration constant.

Now, the evolution equation (20) for V can be written as

$$\ddot{V} = \frac{4k}{3}\left[\frac{(1-\gamma)\,\rho_0}{V^{\gamma}} + \frac{(1-\omega_q)\,\rho_{0q}}{V^{\omega_q}}\right]. \tag{39}$$

The equation (39) admits exact solution that can be written in quadrature as

$$\int \frac{dV}{\sqrt{2\Big[\frac{4k}{3}(\rho_0 V^{1-\gamma} + \rho_{0q} V^{1-\omega_q}) + C_1\Big]}} = t + t_0, \tag{40}$$

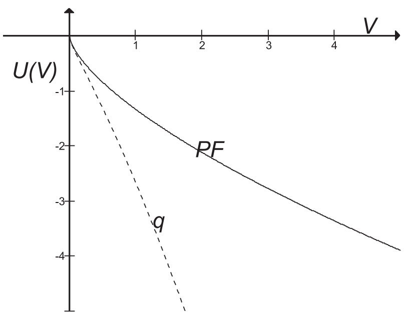

Figure 1. View of potentials when the universe is filled with perfect fluid (PF) and perfect fluid plus quintessence (q) for $\gamma = {}^1/_3$ and $\omega_q = -{}^1/_2$.

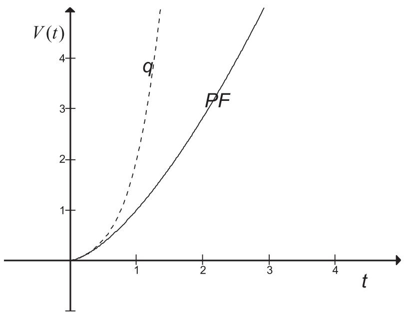

Figure 2. Evolution of volume (V) when the universe is filled with perfect fluid (PF) and perfect fluid plus quintessence (q) for $\gamma = {}^1/_3$ and $\omega_q = -{}^1/_2$.

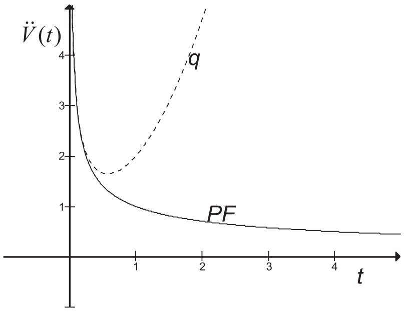

Figure 3. Acceleration ($\ddot{V}$) of universe when it is filled with perfect fluid (PF) and perfect fluid plus quintessence (q) for $\gamma = {}^1/_3$ and $\omega_q = -{}^1/_2$.

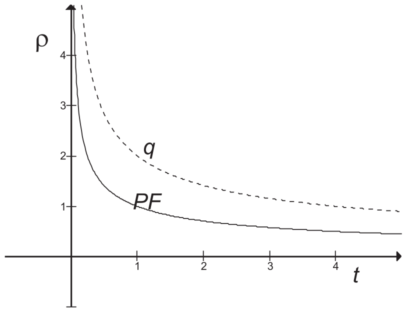

Figure 4. Evolution of density (ρ) when the universe is filled with perfect fluid (PF) and perfect fluid plus quintessence (q) for $\gamma = {}^1/_3$ and $\omega_q = -{}^1/_2$.

where t_0 is a constant of integration that can be taken to be zero. In the limit of high matter densities $(\gamma = 1)$ the general solution of the gravitational equations for a higher dimensional Bianchi type-I geometry cannot be expressed in an exact analytic form.

3.2 Chaplygin Gas Case

Now we consider the case when the dark energy is represented by Chaplygin gas obeying the exotic equation of state as

$$p_c = -\frac{A}{\rho_c} \tag{41}$$

with A being a positive constant.

Now, from Eq. (33) and Eq. (41), we get

$$\rho_c = \sqrt{\frac{\rho_{0c}}{V^2} + A}\,, \qquad p_c = \frac{-A}{\sqrt{\frac{\rho_{0c}}{V^2} + A}} \tag{42}$$

with ρ_{0c} being an integration constant.

Now, the evolution equation (20) for V can be written as

$$\ddot{V} = \frac{4k}{3}\left[\frac{(1-\gamma)\,\rho_0}{V^\gamma} + \sqrt{\rho_{0c} + AV^2} + \frac{AV^2}{\sqrt{\rho_{0c} + AV^2}}\right]. \tag{43}$$

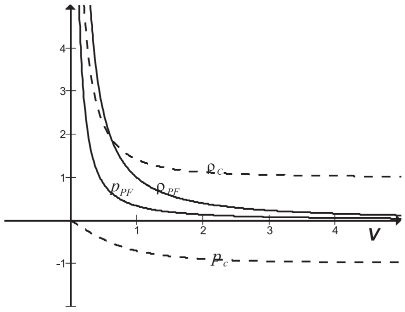

Figure 5. View of energy density and corresponding pressure when the universe is filled with perfect fluid and Chaplygin gas for $\gamma = {}^1/_3$ and $A = \rho_0 = 1$.

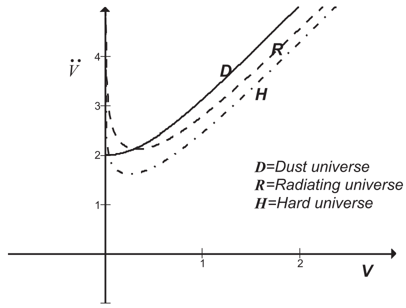

Figure 6. Acceleration ($\ddot{V}$) of the universe when it is filled with perfect fluid (PF) and perfect fluid plus quintessence (q) for $\gamma = 0,\ {}^1/_3, {}^2/_3, k = {}^3/_4$ and $A = \rho_0 = 1$.

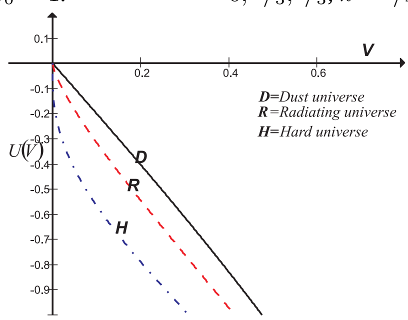

Figure 7. View of potential of the universe when it is filled with perfect fluid (PF) and perfect fluid plus quintessence (q) for $\gamma = 0,\ {}^1/_3, {}^2/_3, k = {}^3/_4$ and $A = \rho_0 = 1$.

The corresponding solution in quadrature form as

$$\int \frac{dV}{\sqrt{2\Big[C_1 + \frac{4k}{3}\Big(\rho_0 V^{1-\gamma} + \sqrt{\rho_{0c}V^2 + AV^4}\Big)\Big]}} = t, \tag{44}$$

where the second integration has been taken to be zero.

4 Conclusion

In this paper we have considered the space-time geometry corresponding to Bianchi type-I filled with perfect fluid and dark energy in five dimensions. The exact solutions of the corresponding Einstein's field equations are obtained. The inclusion of the dark energy into the system gives rise to an accelerated expansion of the model with initial singularities in five dimensions also. Bianchi type-I universe filled with perfect fluid and dark energy has been investigated by Bijan Saha [45] whose work has been extended and studied in five dimensions. An attempt has been made to retain Bijan Saha's [45] form of the various quantities. We have noted that all the results of Bijan Saha (2005) can be obtained from our results by assigning appropriate values to the functions concerned.

References

[1] A.G. Riess *et al*. (1998) *Astron. J.* **116** 1009.
[2] S. Perlmutter *et al*. (1999) *Astrophys. J.* **517** 565.
[3] A.G. Riess *et al*. (2004) *Astrophys. J.* **607** 665.
[4] P. Astier *et al*. (2006) *Astron. and Astrophys.* **447** 31.
[5] D.N. Spergel *et al*. (2006) arXiv : astro-ph/0603449.
[6] W.H. Wood-Vasey *et al*. (2007) arXiv: astro-ph/0701041.
[7] T.M. Davis *et al*. (2007) arXiv: astro-ph/0701510.
[8] C.L. Bennett *et al*. (2003) *Astrophys. J. Suppl.* **148** 1.
[9] D.N. Spergel *et al*. [WMAP Collaboration] (2003) *Astrophys. J. Suppl.* **148** 175.
[10] M. Tegmark*et al.* [SDSS Collaboration] (2004) *Phys. Rev. D* **69** 103501.
[11] K. Abazajian *et al.* (2004) arXiv:astro-ph/0410239.
[12] K. Abazajian *et al.* (2004) *Astron. J.* **128** 502; [arXiv:astro-ph/0403325].
[13] K. Abazajian *et al*. (2003) *Astron. J.* **126** 2081; [arXiv:astro-ph/0305492].
[14] E. Hawkins *et al*. (2003) *Mon. Not. Roy. Astron. Soc.* **346** 78.
[15] L. Verde *et al.* (2002) *Mon. Not. Roy. Astron. Soc*. **335** 432; [arXiv:astro-ph/0112161].
[16] R. Caldwell, R. Dave, P.J. Steinhardt (1998) *Phys. Rev. Lett*. **80** 1582.
[17] A.R. Liddle, R.J. Scherrer (1999) *Phys. Rev. D* **59** 023509.
[18] P.J. Steinhardt, L.M. Wang, I. Zlatev (1999) *Phys. Rev. D* **59** 123504.
[19] G.R. Dvali, G. Gabadadze, M. Porrati (2000) *Phys. Lett.* **B484** 112.
[20] C. Deffayet (2001) *Phys. Lett*. **B502** 199.
[21] S. Capozziello, S. Carloni, A. Troisi (2003) arXiv:astro-ph/0303041.
[22] S.M. Carroll *et al.* (2003) *Phys. Rev. D* **70** 043528.
[23] S. Nojiri, S.D. Odintsov (2003) *Phys. Rev. D* **68** 123512.
[24] P.K. Townsend, N.R. Wohlfarth (2003) *Phys. Rev. Lett*. **91** 061302.
[25] G.W. Gibbons (1985) in "*Supersymmetry, Supergravity and Related Topics*", ed. F. del Aguila, J.A. de Azcarraga, L.E. Ibanez, World. Scientific, pp. 123-146.
[26] N. Ohta (2003) *Phys. Rev. Lett*. **91** 061303.
[27] N.R. Wohlfarth, (2003) *Phys. Lett.* **B563** 1.
[28] S. Roy (2003) *Phys. Lett*. **B567** 322.
[29] J.K. Webb *et al*. (2001) *Phys. Rev. Lett*. **87** 091301.
[30] J.M. Cline, J. Vinet, (2003) *Phys. Rev. D* **68** 025015.
[31] N. Ohta (2003) *Prog. Theor. Phys*. **110** 269.
[32] N. Ohta (2005) *Int. J. Mod. Phys*. **A20** 1.
[33] C.M. Chen *et al*. (2003) *JHEP* **10** 058.

[34] E. Bergshoeff (2004) *Class. Quant. Grav*. **21** 1947.
[35] Y. Gong, A. Wang (2006) *Class. Quant. Grav*. **23** 3419.
[36] I.P. Neupane, D.L. Wiltshire (2005) *Phys. Rev. D* **72** 083509.
[37] K. Maeda, N. Ohta (2005) *Phys. Rev. D* **71** 063520.
[38] I.P. Neupane (2007) *Phys. Rev. Lett*. **98** 061301.
[39] Y. Gong, A. Wang, Q. Wu (2007) arXiv:gr-qc/0711.1597.
[40] P.R.C.T. Pereira, M.F.A. de Silva, R. Chan (2006) *Int. J. Mod. Phys*. **D15** 991.
[41] C. Brandt, R. Chan, M. de Silva, J.F. Villas (2007) *Gen. Relativ. Gravitation* **39** 1675.
[42] E.J. Copeland, M. Sami, S. Tsujikawa (2008) *Int. J. Mod. Phys*. **D15** 1753.
[43] T. Padmanabhan (2007) arXiv:gr-qc/0705.2533.
[44] T. Koivisto, D.F. Mota (2008) *J. Cosmol. Astropart. Phys*. **06** 018.
[45] S. Bijan (2005) *Chin. J. Phys.* **43** 1035.
[46] T. Singh, R. Chaubey (2009) *Astrophys. Space Sci.* **319** 149.
[47] S. Weinberg (1986) "*Physics in Higher Dimensions*", World Scientific, Singapore.
[48] A. Chodos, S. Detweller (1980) *Phys. Rev. D* **21** 2167.
[49] K. Lorentz-Petzold (1985) *Gen. Relativ. Gravitation* **17** 1189.
[50] J. Ibanez, E. Verdaguer (1986) *Phys. Rev. D* **34** 1202.
[51] R.J. Gleiser, M.C. Diaz (1988) *Phys. Rev. D* **37** 3761.
[52] S. Banerjee, B. Bhui (1990) *Mon. Not. Roy. Astron. Soc*. **247** 57.
[53] D.R.K. Reddy, N. Venkateswara Rao (2001) *Astrophys. Space Sci*. **277** 461.
[54] K. Khadekar, M. Gaikwad (2001) *Proc. Einstein Found Int.* **11** 95.
[55] L.D. Landau, E.M. Lifshitz (1976) "*Series: Course of Theoretical Physics*, v.1, 3ed., Pergamon.