

Relevance of Density Dependent Correlated Hopping Interaction in High- T_c Superconductivity within Extended Hubbard Model

Papiya Pal, Piyali Ghosh, Nanda Kumar Ghosh 

Department of Physics, University of Kalyani, Kalyani-741235, West Bengal, India

Received 18 August 2026

Abstract. Using Lanczos exact diagonalization (ED) technique, we study the relevance of correlated hopping interaction term on some principal characteristics of high- T_c superconductors (HTSC) for doping concentration of holes as $\langle h \rangle = 0.25$. The interaction breaks the electron-hole symmetry and facilitates the transition to non-BCS type superconductivity state from BCS type. A short range antiferromagnetic (AF) order exists in the system up to next-nearest-neighbour (NNN) sites. Specific heat curves show correlated hopping induced transition at low temperature. The interaction also enhances local moment destroying double occupancy due to electron-hole symmetry breaking property of the interaction.

KEY WORDS: High- T_c superconductivity, Hubbard model, Correlated hopping, Exact diagonalization method.

1 Introduction

One of the most studied topics in strongly correlated electron system is Cu based high- T_c superconductivity [1]. A large number of investigations [2–5] have been done to figure out the mechanism which makes some compounds superconductor at comparatively high temperatures than the conventional BCS-superconductors after doping. The largely discussed models are Hubbard [6] and t - J [7,8] models to explain the properties of copper oxide (CuO_2) plane, responsible for different characteristics of cuprate superconductors. Two-dimensional Hubbard model manifests antiferromagnetism [9–11], superconductivity [12, 13], metal-insulator transition [14–16] and pseudo gap characteristics near to half-filling. Variational calculations [17] of t - J model on square lattice indicate

a very close-range competition within different stripe phases and consistent d -wave superconducting phase. Using two-dimensional t - J model, single particle spectral properties have been investigated [18] on high- T_c cuprates.

Double occupancy is prohibited for the wave function within the t - J model and antiferromagnetic (AF) super exchange term emerges. Double occupancy suppressed by on-site Coulomb repulsion (U) is permitted in the Hubbard model. So, to know about the pair formation responsible for high- T_c superconductivity, a refined model is necessary. In Ref. [19], authors have proposed t - J - U model containing both the features of the t - J and Hubbard model with some particular limiting condition. Ground state properties of high- T_c superconductors, nematic phase, charge density wave, non-BCS kinetic energy gaining etc. and dynamic properties like Fermi velocity, Fermi wave-vector, enhancement of effective mass are discussed [20] extensively using t - J - U model with Gutzwiller wave function (GWF) method [21]. This model is simple, consisting ordinary Fermionic operator. The t - J - U model reproduces t - J model with $U \rightarrow \infty$ limit and Hubbard model with $J = 0$ limit. However, since there are limited number of works on this model, it requires further investigations.

Photoemission spectra of lightly doped BSCCO crystal shows [22] that next-nearest-neighbour (NNN) hopping interaction term (t') influences pseudogap energy more than that of super-exchange interaction (J). Using single-band t - J model, the materials having round Fermi surface required to be shaped with major t' and third neighbour hopping (t'') value while the La-materials with low critical temperature have small t' and t'' value [23]. The coexistence of antiferromagnetism and superconductivity in the frame of t - t' - J model with honeycomb lattice is proved within a wide range of doping [24]. An exact diagonalization and density matrix renormalization group (DMRG) study on Hubbard model shows enhancement of pair-binding energy with positive t' and suppression with negative t' [25, 26]. NNN hopping plays an important role in the formation of tightly bound electronic bi-polarons which unitedly results in superconductivity at lower temperature [27].

Correlated hopping is a density dependent electron hopping amplitude term [5] having electron-hole symmetry breaking property [28, 29]. Antiferromagnetic exchange between ions is reduced by correlated hopping term [5] and it has an effect mainly on s -wave pairing [28]. In the presence of correlated hopping in Hubbard model, a covalent bond is formed between intra-site and inter-site pairing due to on-site and inter-site repulsion [30]. In Ref. [29], authors have discussed the case where singly occupied site happened with on-site Coulomb repulsion tends to infinity. For correlated hopping strength less than unity, superconducting correlation is dominant for low doping concentrations [31]. Transition from superconducting state to Mott insulator state occurs directly in the half-filled band as shown by Bulka using slave boson mean field study performed on correlated hopping induced Hubbard model [32]. Farakasovsky showed [33] that stable ferromagnetic state occurs in the correlated hopping motivated Hub-

bard model for a wide range of on-site Coulomb repulsion U and electron concentration greater than unity. The interplay between correlated hopping term, kinetic energy term and on-site Coulomb repulsion term leads to the formation of stripe phases in the strongly correlated electron systems [34]. In [35], the doping level dependent fluctuation of the critical temperature has been analysed properly by using correlated hopping induced Hubbard model. The specific heat power law is also justified by this model in the absence of any adaptable parameter. Correlated hopping term increases the probability of pair formation in electron-doped system and narrows the non-BCS regime [21]. This hopping term has an important effect [36] on many body systems to form ferromagnetic state (half-metallic) by means of ED method. Correlated hopping interaction within extended Falicov-Kimball model successfully reproduced [37] many experimental observations of mixed-valent systems.

Motivated by earlier discussion on different experimental and theoretical outcomes, in the present work, we have incorporated correlated hopping interaction in the t - J model with a little admixture of on-site repulsive Coulomb interaction. NNN hopping term is included here to fabricate the model Hamiltonian more practical. In strongly correlated systems, perturbation theory is inapplicable. Moreover, the calculations based on different approximations give diverse results. Therefore, we have applied exact diagonalization (ED) method on 8-site tilted square cluster, and finite temperature and ground state properties are drawn varying other parameters. The effect of correlated hopping interaction on the pairing mechanism of the system is also discussed. There is a conclusion of the overall results at the end.

Superconducting state is observed above a critical value ($U = U/|t| \sim 5$) [20] for d -wave superconductivity and it exists for a wide value of U . In this communication, we have described different properties of HTSC with threshold value of $U/t = 5.0$. Here, NNN hopping amplitude is of the order of $|t'|/|t| \approx 0.3$ and the exchange interaction have value $J/|t| \approx 0.3$ as taken in Ref. [21].

In our calculation, we have employed approximation free ED technique to obtain eigenvectors and eigenvalues of the ground state & other excited states. However, due to limited memory size of the computers, the size of the cluster is restricted to $\sqrt{8} \times \sqrt{8}$. As a result, finite size effect is obvious in our observation and attention should be given to keep it as minimum as possible. Earlier, it has been shown qualitatively that the ground state energies of 16-site & 8-site clusters are similar up to a variation of $\leq 7\%$ [38]. Apart from finite size effect, cluster geometry is also responsible for it. Finite size effect is largely reduced when we consider the system temperature $T > 0$ as thermodynamic average taken over all the eigenstates reduces the effect appreciably.

2 Hamiltonian and Formulation

In this communication, we have used t - J - U model Hamiltonian extended with NNN hopping interaction term and correlated hopping term on a 2D 8-site tilted square cluster within periodic boundary conditions:

$$\begin{aligned}
 H_{\text{total}} = & -t \sum_{\langle i,j \rangle, \sigma} \left(\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \text{H.c.} \right) - t' \sum_{\langle p,q \rangle, \sigma} \left(\hat{c}_{p\sigma}^\dagger \hat{c}_{q\sigma} + \text{H.c.} \right) \\
 & + J \sum_{\langle i,j \rangle} \left[\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right] + U \sum_i n_{i\uparrow} n_{i\downarrow} \\
 & + K \sum_{\langle i,j \rangle, \sigma} (\hat{n}_{i\bar{\sigma}} + \hat{n}_{j\bar{\sigma}}) \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma}. \quad (1)
 \end{aligned}$$

where, $\langle i, j \rangle$ represents nearest-neighbour sites, $\langle p, q \rangle$ represents next-nearest-neighbour sites. σ ($\bar{\sigma}$) is the spin (opposite spin) of electrons. $c_{i\sigma}^\dagger$ ($c_{i\sigma}$) is the fermionic creation (annihilation) operator at site i . t and t' are the hopping amplitudes respectively. J is the near-neighbour antiferromagnetic (AF) interaction. U is the on-site Coulomb repulsion term and K is the density dependent correlated hopping interaction strength. $\vec{S}_i$ is the spin operator at site i in the form

$$S_i^\alpha = \sum_{\sigma, \bar{\sigma}} C_{i,\sigma}^\dagger \tau_{\sigma, \bar{\sigma}}^\alpha C_{i,\bar{\sigma}},$$

where τ^α is the Pauli matrices and $\alpha = x, y, z$. Here, n_i is the total number density operator expressed as

$$n_i = \sum_{\sigma} C_{i,\sigma}^\dagger C_{i,\sigma}. \quad (2)$$

To determine the magnetic tendency of the system at different distances, we have calculated spin-spin correlation function as

$$C(|i-j|) = \langle (n_{i\uparrow} - n_{i\downarrow}) (n_{j\uparrow} - n_{j\downarrow}) \rangle. \quad (3)$$

Considering lattice constant as unity, $|i-j|$ is the distance between two sites. At a particular distance, $C(|i-j|)$ is the average over all the sites i . In our cluster, we have used five different neighbour distances like $|i-j| = 1$ (NN sites), $|i-j| = \sqrt{2}$ (NNN sites), $|i-j| = 2$ (next to NNN sites) and so on.

To study the spatial arrangement of mobile holes and nature of pairing mechanism in cuprates, we have used the following relation to calculate the hole-hole correlation function:

$$P_{hh}(|i-j|) = \frac{1}{N_h N_E(|i-j|)} \sum_{\langle i,j \rangle} \langle (1 - n_i)(1 - n_j) \rangle, \quad (4)$$

N_h stands for total hole number and $N_E(|i - j|)$ denotes the equivalent site number at a distance $|i - j|$ from a standard reference point.

To study the effect of correlated hopping on bare electron mobility, we have calculated effective hopping amplitude defined as

$$t_{\text{eff}} = \frac{\sum_{\langle i,j \rangle, \sigma} \langle \psi_0 | (c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}) | \psi_0 \rangle}{\sum_{\langle i,j \rangle, \sigma} \langle \phi_0 | (c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}) | \phi_0 \rangle}, \quad (5)$$

where, $|\psi_0\rangle$ is the wave function of the ground state of total Hamiltonian and $|\phi_0\rangle$ is the wave function of the ground state of t - J - U model Hamiltonian. To understand the effect of correlated hopping on AF alignment of the system, we have studied effective antiferromagnetic exchange term (J_{eff}) like

$$J_{\text{eff}} = \frac{\sum_{\langle i,j \rangle, \sigma} \langle \psi_0 | (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j) | \psi_0 \rangle}{\sum_{\langle i,j \rangle, \sigma} \langle \phi_0 | (\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j) | \phi_0 \rangle}. \quad (6)$$

We have also investigated some finite temperature properties. Formulas of the parameters are given below:

If $\hat{O}$ is any physical observable, its thermal average can be calculated using the relation

$$\hat{O} = \frac{\text{Tr}(\hat{O} e^{-\beta H})}{\text{Tr}(e^{-\beta H})}. \quad (7)$$

Here, $\beta = \frac{1}{k_B T}$; k_B is Boltzmann constant which is taken as unity for simplification.

We have calculated low temperature specific heat like

$$C = k_B \beta^2 \frac{\partial^2}{\partial \beta^2} \ln Z, \quad (8)$$

where

$$Z = \sum_{\alpha} e^{-\beta E_{\alpha}},$$

the summation is over all the eigenstates and the eigenvalue for the i -th state is E_{α} .

The entropy associated with a lattice site is

$$S = \frac{1}{N} \left(\ln Z + \frac{\langle H \rangle}{T} \right), \quad (9)$$

where N is the number of lattice sites. We have calculated the local moment $\langle m_z^2 \rangle$ per site, which is the on-site spin-spin correlation function given as

$$\langle m_z^2 \rangle = \langle (n_{i\uparrow} - n_{i\downarrow})^2 \rangle = \langle n \rangle - 2\delta, \quad (10)$$

where $\delta = \langle n_{i\uparrow}n_{i\downarrow} \rangle$ indicates the probability of doubly occupied state. Our calculations are performed based on an 8-site tilted square cluster following the ED method with hole doping concentration $\langle h \rangle = 0.25$, which is appropriate to explain some important features [26] about HTSC. We have considered here the $S_Z^{\text{tot}} = 0$ subspace.

3 Results and Discussions

In our analysis, we have followed the numerical values of t' and J as prescribed in Ref. [19] as they are in good agreement with some specified experimental data. Specifically, we set $t' = -0.3|t|$, $J = 0.3|t|$. We fix $t = 1$ as unit of energy and all the energies are stated in terms of $|t|$. For hole doped cuprates, sign of t' is opposite to that of t . An estimated values for cuprates give $U = 3\text{--}5\text{ eV}$ and $K = 0.1\text{--}0.3\text{ eV}$ [32] and in the large- U limit correlated hopping becomes unfruitful [28]. We have taken $U/|t| = 5.0$ which is small in value and justified in Ref. [20]. Namely, d -wave superconductivity is observed for $U/|t| > 5.0$ demonstrating a threshold value at $U/|t| = 5.0$. Hubbard proposed [39] the value of correlated hopping like $K = 0.5\text{ eV}$. In this communication, we have worked with K values limited within 0 and $0.5|t|$ as BCS to non-BCS superconducting transition is well explained [21] in this regime. 8-site tilted square cluster has been chosen with hole doping concentration $\langle h \rangle = 0.25$, appropriate for HTSC.

3.1 Ground state properties

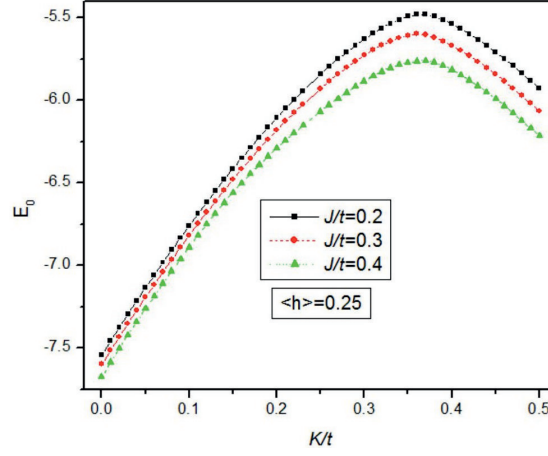
In this section, we discuss about various properties of the system based on ground state. Figure 1 shows the variation of ground state energy (E_0) as a function of (K/t) for different AF exchange interaction J/t .

3.1.1 Ground state energy

Figure 1 shows the variation of ground state energy (E_0) as a function of (K/t) for different AF exchange interaction J/t .

Here E_0 increases with K/t up to $K/t \approx 0.35$. With further increase of K/t , E_0 decreases then. So, a peak appears at $K/t \approx 0.35$. This is due to the presence of a discontinuity in the correlated gap at the phase separation between BCS like and non-BCS superconducting phase [21]. Initially, E_0 increases with K/t due to the electron-hole symmetry breaking property of K [29]. Also, non-BCS phase have been carried out within the under-doped region for hole doping [21], which is verified by our result for hole doping concentration $\langle h \rangle = 0.25$ (under-doped case).

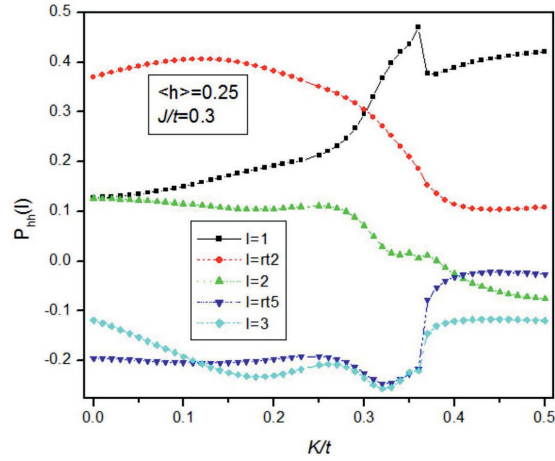
In addition, with the increasing value of AF exchange interaction, more spins are aligned in AF order, resulting a more stable system with lower ground state

Figure 1. Variation of E_0 with K/t for different J/t .

energy. Specifically, ground state energy will decrease with the increment of J , which is justified by our graph.

3.1.2 Hole-hole correlation

Hole-hole (h-h) correlation ($P_{hh}(l)$) is an important parameter to study the formation of two-hole bound state. In Figure 2, the h - h correlation for $l = 1, \sqrt{2}$

Figure 2. Schematic diagram to study the effect of correlated hopping on $P_{hh}(l)$ for different lattice spacing.

is dominant at smaller values of correlated hopping. For $\langle h \rangle = 0.25$, characterization of $P_{hh}(\sqrt{2})$ shows that the correlation increases initially attaining a maximum at $K/t \sim 0.125$ due to delocalization of spin polarons [26], and then diminishes with K/t to a lower value at about $K/t \approx 0.35$. BCS like to non-BCS like superconducting transition occurs at about $K/t \approx 0.35$ which may be assigned due to the sharp fall of P_{hh} for $l = 1, \sqrt{2}$ and sharp rise of P_{hh} for $l = \sqrt{5}, 3$. Moreover, negative value of hole-hole correlation means the electron-electron correlation. For larger distances (i.e. $l = \sqrt{5}, 3$ for our 8-site tilted square cluster) negative value of h - h correlation assign that electron-electron correlation comes into play at higher distances due to delocalization of spin-polarons.

3.1.3 Spin-spin correlation

Spin-spin correlation function ($C(l)$) has been plotted with correlated hopping K/t for various lattice distances to observe the magnetic alignment present in the system. $C(l)$ is negative for $K/t \approx 0.35$ for nearest-neighbour (NN) site i.e. for $l = 1$ distance and a sharp increase is observed at $K/t \approx 0.35$ and then it becomes positive. $C(l)$ is positive up to $K/t \approx 0.1$ for next-nearest-neighbour (NNN) sites i.e. for $l = \sqrt{2}$ distance. Value of $C(l)$ for other sites are negative except $l = 2$ lattice spacing. So, the existence of AF order is observed for nearest diagonal lattice spacing ($l = \sqrt{2}$) up to $K/t \cong 0.1$ and AF order between NN spacing (short range) emerges [39] for $K/t \approx 0.35$. A sharp change of spin-spin correlation at $K/t \approx 0.35$ signifies BCS to non-BCS superconducting phase transition.

The presence of long-range AF order mediated by exchange interaction has been

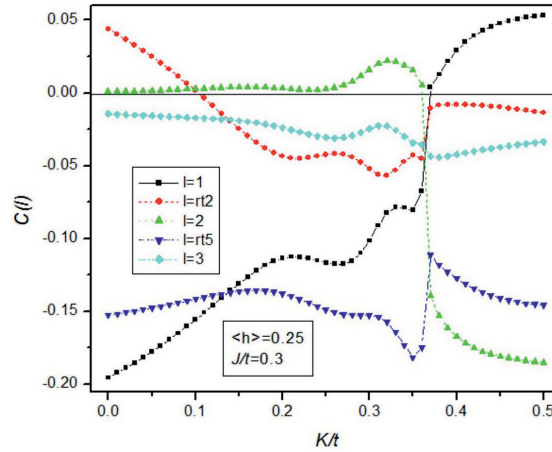


Figure 3. Plot of the $C(l)$ as a function of K for different lattice separations.

observed in the undoped parent compound. This AF order is destroyed upon doping and a short-range AF order emerges [40].

3.1.4 Effective hopping amplitude

Nearest-neighbour effective hopping amplitude $t_{\text{eff}}^{\text{NN}}$ has been plotted with K/t in Figure 4. We have seen that $t_{\text{eff}}^{\text{NN}}$ decreases with increase of K/t for a fixed value of t'/t . Electron mobility decreases with correlated hopping and the electrons become further localized. At lower K/t , t_{eff} drops slowly but for higher values of it, diminution is rapid. Electrons become localized in the presence of correlated hopping. Also, with the enhancement of correlated hopping, electron-hole symmetry configuration of the system breaks which decreases the movement of electrons between NN sites. As a result, $t_{\text{eff}}^{\text{NN}}$ decreases with K/t . Besides this, with the increasing value of exchange interaction, a large number of spins are aligned in AF order.

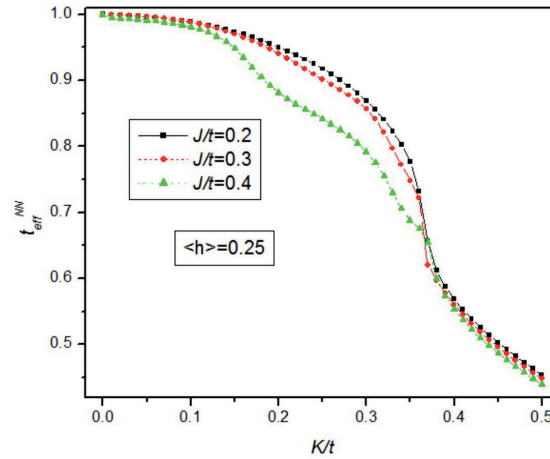
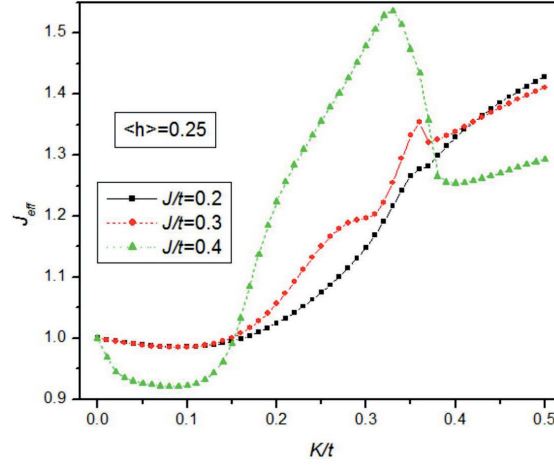


Figure 4. Dependence of effective hopping amplitude ($t_{\text{eff}}^{\text{NN}}$) on K for various J/t values.

So, mobility of electrons decreases with the increased value of J/t . As a result, t_{eff} decreases with J/t , which is clearly shown in the figure.

3.1.5 Effective exchange interaction

Effective exchange interaction (J_{eff}) is an important probe to investigate the pairing mechanism in HTSC [41]. In Figure 5 we have plotted J_{eff} against K/t . It is observed that J_{eff} increases with correlated hopping (K/t). With the increasing value of correlated hopping, electrons become more localized, so, the probability of arrangement of spins in AF order is enhanced emerging the rise of J_{eff} value.

Figure 5. Effective exchange interaction (J_{eff}) with K/t .

J_{eff} also depends on t' and AF exchange interaction (J). Obviously, J_{eff} should increase with the increase of J , which is justified by our graph. Besides this, in Ref. [42], authors show that the essence of variation of J_{eff} closely matches with the d -wave pairing susceptibility. There is a kink near about $K/t \approx 0.35$ in the graph which probably indicates that a transition of the system occurs from s -wave pairing (BCS-type) to d -wave pairing (non-BCS type).

3.2 Thermodynamic properties

The extension of the study with finite temperature ($T > 0$) is essential as the next step. So, we have covered up some thermodynamic properties.

3.2.1 Specific heat

Specific heat versus temperature for different correlated hopping strengths has been plotted in Figure 6 to discuss about the superconducting transition induced by temperature and affected by K .

Following the curves, a single-peak structure is observed when the temperature is low. It proves that specific heat of the system obeys second degree power law behaviour at low temperature. This single peak is Schottky anomaly [43] and comes into play when spin degrees of freedom are thermally excited in the hole doped system with an additional exchange interaction term. This single peak structure also indicates non-Fermi liquid character of the system at low temperature. Further, increase of temperature diminishes the non-Fermi liquid character, and a gradual decrease of specific heat is observed. Moreover, peak height max-

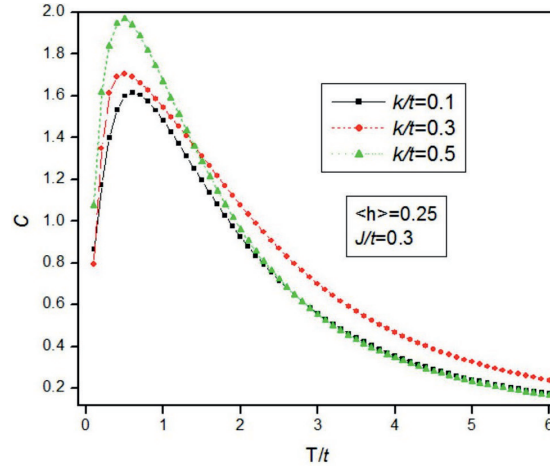


Figure 6. Temperature dependence of specific heat (C) for distinct correlated hopping strengths.

ima increase with correlated hopping interaction. This feature indicates that a superconducting transition is possible at low temperature by increasing K value for $J/t = 0.3$ and hole concentration $\langle h \rangle = 0.25$.

3.2.2 Entropy

Temperature variation of entropy (S) for different correlated hopping strength is observed in Figure 7. It is seen that a sharp increase of entropy with temperature is observed. Entropy remains constant at higher temperatures. This character-

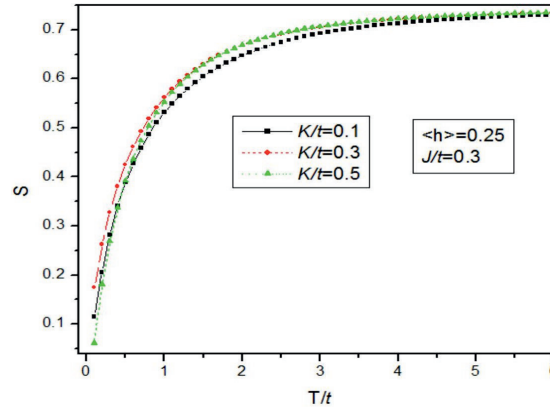


Figure 7. Entropy (S) vs. temperature (T/t) for different K/t .

istic of entropy is in conformity with Ref. [44]. Superconducting state may be considered as an ordered state and entropy is a measure of disorder. So, we can say that sharp increase at low temperature indicates a transition from superconducting (ordered) to normal (disordered) state. This transition is due to the fluctuation of energy with temperature at the ground state and also affected by K .

3.2.3 Local moment

In Figure 8, we plot local moment as a function of temperature for various correlated hopping strengths. A peak is observed in local moment ($\langle m_z^2 \rangle$) at very low temperature and with the increase of temperature $\langle m_z^2 \rangle$ decreases to a very low value at larger temperatures. Correlated hopping breaks the electron-hole symmetry which in turn diminishes the double occupancy and favours the formation of local moment. The nature of the graph establishes this fact. Moreover, the graph shows correlated hopping interaction compete with temperature. Namely, the effect of correlated hopping is dominant at low temperature and local moment is increased at that temperature regime. Besides this, drop of the local moment with temperature happens at higher temperatures due to suppression of the effect of correlated hopping by temperature dependent kinetic energy. Local moment is vanishingly small at higher temperature.

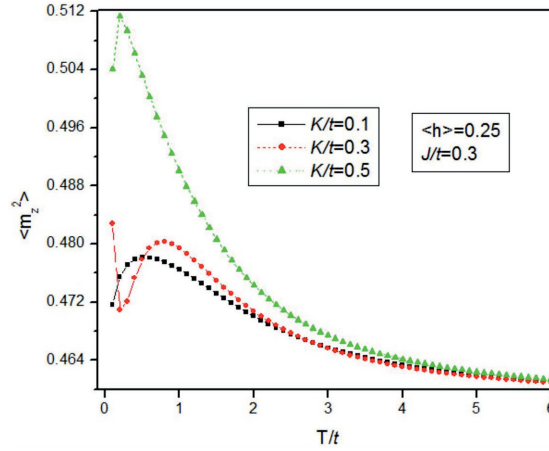


Figure 8. Local moment ($\langle m_z^2 \rangle$) as a function of temperature for discrete K values.

3.2.4 Double occupancy

In Figure 9, we have studied temperature dependence of double occupancy (δ) for different correlated hopping strengths. For $K/t = 0.5$, primarily there is a

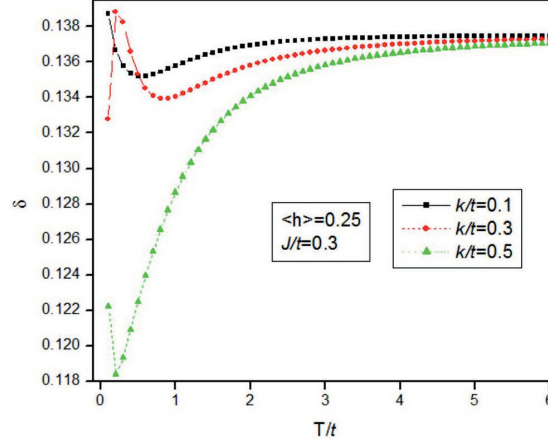


Figure 9. Dependence of double occupancy (δ) on temperature for different K/t .

drop of double occupancy from a very small value and then increases sharply reaching to a constant value at higher temperature. For $K/t = 0.3, 0.1$, initially, the graph shows a short peak at low temperature regime followed by a broad peak at higher temperature. At higher temperature, value of δ becomes constant. Double occupancy graph is like the mirror reflection of the local moment graph though magnitude differs largely. Correlated hopping term tries to break the double occupancy whereas thermal excitation favours the formation of doublons [45]. According to this fact, with the increased value of correlated hopping, double occupancy may decrease, but this competition is not clear in our study at lower values of K/t . At a relatively high temperature, correlated hopping competes with the thermal kinetic energy. Due to this reason, the curves for different K/t values merge with each other.

4 Conclusions

In this paper, we have calculated various ground state properties like ground state energy, hole-hole correlation, spin-spin correlation, effective hopping amplitude, effective antiferromagnetic exchange interaction and finite temperature properties like specific heat, entropy, local moment, double occupancy and reciprocal susceptibility using t - t' - J - U - K Hamiltonian on 8-site tilted square cluster by exact diagonalization method. The role of correlated hopping on the said parameters has been investigated. The essential remarks obtained from the study are summarized below:

- Correlated hopping interaction breaks the symmetry of electron-hole arrangement of the system depending on electron density and conduct a superconducting transition from BCS to non-BCS regime.

14 Relevance of Density Dependent Correlated Hopping Interaction in ...

- This interaction delocalizes spin polarons. From the hole-hole correlation and spin-spin correlation, it is observed that a transition from BCS to non-BCS like superconductivity occurs at critical $K/t \approx 0.35$.
- A short-range AF order exists in the system up to NNN sites.
- The effective hopping amplitude study shows that electron mobility decreases with correlated hopping.
- The specific heat curves show that the system behaves like non-Fermi liquid at low temperature and a superconducting transition is possible with higher correlated hopping interaction at low temperature. This fact is also observed in entropy graphs. Moreover, the system becomes more disordered due to fluctuation of correlated hopping induced energy at ground state.
- Correlated hopping approves the development of local moment and destroys double occupancy due to its electron-hole symmetry breaking property. Also, existence of competition between correlated hopping and temperature has been observed.

References

- [1] J.G. Bednorz, K.A. Müller (1986) Possible high Tc superconductivity in the Ba-La-Cu-O system. *Z. Phys. B* **64** 189-193.
- [2] S.R. White, D.J. Scalapino (1997) Hole and pair structures in the t-J model. *Phys. Rev. B* **55** 6504-6517.
- [3] T.A. Mayer et al. (2005) Systemic Study of d-Wave Superconductivity in the 2D Repulsive Hubbard Model. *Phys. Rev. Lett.* **95** 237001.
- [4] K. Roy, S. Nath, N.K. Ghosh (2019) Hole-polarons and bipolarons in the Holstein t-J model: Relevance of inter-site hole-phonon interaction. *Phys. Lett. A* **383** 1510-1515.
- [5] J. Matysiak, R. Lemański (2021) Description of molecular nanomagnets by the multi-orbital Hubbard model with correlated hopping. *Phys. Rev. B* **104** 014431.
- [6] J. Hubbard (1964) Electron correlations in narrow energy bands III. An improved solution. *Proc. R. Soc. Lond. Ser. A* **281** 401-419.
- [7] P.W. Anderson (1987) The Resonating Valence Bond State in La_2CuO_4 and Superconductivity. *Science* **235** 1196.
- [8] F.C. Zhang, T.M. Rice (1988) Effective Hamiltonian for the superconducting Cu oxides. *Phys. Rev. B* **37** 3759-3761.
- [9] P. Korbel et al. (2003) Antiferromagnetism of almost localized fermions: Evolution from Slater-type to Mott-Hubbard gap. *Eur. Phys. J. B* **32** 315-322.
- [10] S. Sanna et al. (2004) Nanoscopic Coexistence of Magnetism and Superconductivity in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ Detected by Muon Spin Rotation. *Phys. Rev. Lett.* **93** 207001.
- [11] Q. Yuan, F. Yuan, C.S. Ting (2005) Doping dependence of the electron-doped cuprate superconductors from the antiferromagnetic properties of the Hubbard model. *Phys. Rev. B* **72** 054504.
- [12] I. Martin, G. Ortiz, A.V. Balatsky, A.R. Bishop (2001) A minimal model of striped superconductors. *Europhys. Lett.* **56** 849-855.

- [13] K. McElory et al. (2004) Homogeneous nodal superconductivity coexisting with inhomogeneous charge order in strongly underdoped Bi-2212. Preprint Cond-mat/0404005v1.
- [14] M. Imada, A. Fujimori, Y. Tokura (1998) Metal-insulator transitions. *Rev. Mod. Phys.* **70** 1039-1263.
- [15] Y. Ono et al. (2001) Phase diagram of the Mott transition in a two-band Hubbard model in infinite dimensions. *Eur. J. Phys. B* **19** 375-384.
- [16] F. Mancini (2004) The Mott-Hubbard transition and the paramagnetic insulating state in the two-dimensional Hubbard model. *Europhys. Lett.* **50** 229-235.
- [17] P. Corboz, T.M. Rice, M. Troyer (2014) Competing States in the t-J Model: Uniform d-wave State versus Stripe State. *Phys. Rev. Lett.* **113** 046402.
- [18] M. Kohono (2015) Spectral properties near the Mott transition in the two-dimensional t-J model. *Phys. Rev. B* **92** 085128.
- [19] J. Spalek, M. Zegrodnik, J. Kaczmarczyk (2017) Universal properties of high-temperature superconductors from real space pairing: t-J-U model and its quantitative comparison with experiment. *Phys. Rev. B* **95** 024506.
- [20] J. Spalek, M. Fidrysiak, M. Zegrodnik (2022) Superconductivity in high-Tc and related strongly correlated systems from variational perspective: Beyond mean field theory. *Phys. Rep.* **959** 1-117.
- [21] M. Zegrodnik, J. Spalek (2017) Universal properties of high-temperature superconductors from real space pairing: Role of correlated hopping and intersite Coulomb interaction within the t-J-U model. *Phys. Rev. B* **96** 054511.
- [22] K. Tanaka et. al. (2004) Effects of next-nearest-neighbor hopping t' on the electronic structure of cuprate superconductors. *Phys. Rev. B* **70** 092503.
- [23] D.J. Scalapino (2007) In: *Handbook of high temperature superconductivity*, eds. J.R. Schrieffer, J.B. Brooks; Springer, New York, (Chap. 13).
- [24] Y. Zhong, L. Zhang, H.T. Lee, H.G. Luo (2015) Coexistence of antiferromagnetism and superconductivity of t - t' - J model on honeycomb lattice. *Physica B* **462** 1-7.
- [25] G. Wachtel, S. Baruch, D. Orgad (2017) Optimal inhomogeneity for pairing in Hubbard systems with next-nearest-neighbor hopping. *Phys. Rev. B* **96** 064527.
- [26] K. Roy, P. Pal, S. Nath, N.K. Ghosh (2018) Hole pairing and ground state properties of high-Tc superconductivity within the t - t' - J - V model. *Eur. Phys. J. B* **91** 64.
- [27] M. Danilov et al. (2022) Degenerate plaquette physics as key ingredient of high-temperature superconductivity in cuprates. *npj Quantum Mater.* **7** 50.
- [28] R. Micnas, J. Ranninger, S. Robaszkiewicz (1989) Superconductivity in a narrow-band system with intersite electron pairing in two dimensions. II Effects of nearest-neighbor exchange and correlated hopping. *Phys. Rev. B* **39** 11653-11662.
- [29] B.R. Bulka, M. Robaszkiewicz (1997) Superconductivity in Hubbard model with correlated hopping. *Acta Phys. Pol. A* **91** 363-366.
- [30] G.A. Lara, G.G. Cabrera (1990) Covalent bond pairing in an extended Hubbard model with correlated hopping: Interplay of antiferromagnetism and superconductivity. *Solid State Commun.* **76** 1121-1127.
- [31] A. Schadschneider, G. Su, J. Zittartz (1997) Exact solution of the one-dimensional fermionic model with correlated hopping. *Z. Phys. B* **102** 393-397.
- [32] B.R. Bulka (1998) Superconductivity in the Hubbard model with correlated hopping: Slave-boson study. *Phys. Rev. B* **57** 10303-10306.

- [33] P. Farkašovský (2004) Ferromagnetism in the Hubbard model with long-range and correlated hopping. *Czech. J. Phys.* **54** D419-D422.
- [34] H. Čenčariková, P. Farkašovský (2004) Phase transitions induced by correlated hopping in the Falicov-Kimball model. *Int. J. Mod. Phys.* **18** 357-368.
- [35] C.G. Galván, L.A. Pérez, C. Wang (2012) d-wave superconductivity from correlated hopping interactions determined by angle-resolved photoemission spectroscopy. *Phys. Lett.* **376** 1380-1383.
- [36] T. Westerhout, M.I. Katsnelson (2022) Role of correlated hopping in the many-body physics of flat-band systems: Nagaoka ferromagnetism. *Phys. Rev. B* **106** L041104.
- [37] P. Ghosh, N.K. Ghosh, S. Nath (2026) Relevance of Correlated Hopping in Mixed Valence Phenomena within Extended Falicov-Kimball Model. *Bulg. J. Phys.* **53** 112-122.
- [38] K. Roy, S. Ghosh, S. Nath, N.K. Ghosh (2019) Specific heat, entropy and magnetic properties of high T_c superconductivity within the planar t - t' - J - V model. *Eur. Phys. J. B* **92** 270.
- [39] J. Hubbard (1963) Electron correlations in narrow energy bands. *Proc. R. Soc. Lond. A* **276** 238-257.
- [40] M.A. Kastner, R.J. Birgeneau, G. Shirane, Y. Endoh (1998) Magnetic, transport, and optical properties of monolayer copper oxides. *Rev. Mod. Phys.* **70** 897-928.
- [41] V. Barzykin, D. Pines (2009) Universal behaviour and the two-component character of magnetically underdoped cuprate superconductors. *Adv. Phys.* **58** 1-65.
- [42] K. Roy, N.K. Ghosh (2021) High- T_c Superconductivity within t - t' - J - J' Model. *Bull. Of Pure and Appl. Sc.* **40D** 34-38.
- [43] R.T. Scalettar, D.J. Scalapino, R.L. Sugar, D. Toussaint (1989) Phase diagram of the half-filled 3D Hubbard model. *Phys. Rev. B* **39** 4711-4714.
- [44] W.O. Putikka (2015) Entropy and thermopower in the 2D t - J model. *J. Phys.* **640** 012046.
- [45] F. Mancini, F.P. Mancini (2008) One-dimensional extended Hubbard model in the atomic limit. *Phys. Rev. E* **77** 061120.