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Bianchi Type-VI Bulk Viscous Fluid String Cosmological Model in General Relativity

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Abstract. We investigate the integrability of cosmic string of Bianchi type-VI space-time in presence of bulk viscous fluid by applying a new technique. The behavior of the model is reduced to the solution of single second order nonlinear differential equation. We show that this equation admits an infinite family of solutions. The physical implications of these results are also discussed.

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1 Introduction

In recent years there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe (Kibble [1]). Cosmic string plays an important role in the study of the early universe. This arises during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (Zel'dovich *et al.* [2]; Kibble [1,3]; Everett [4]; Vilenkin [5]). It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies (Zel'dovich [6]). These cosmic strings have stress energy and coupled to the gravitational field. There it is interesting to study the gravitational effects that arise from strings.

The general relativistic treatment of strings was initiated by Letelier [7,8] and Stachel [9]. Letelier [7] has obtained the solution of Einstein's field equations for a cloud of strings with spherical, plane and cylindrical symmetry. Then in 1983, he solved Einstein's field equations for a cloud of massive strings and obtained cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Banerjee *et al*. [10] have investigated an axially symmetric Bianchi type-I string dust cosmological models with a magnetic field discussed also by Chakraborty

[11], Tikekar and Patel [12]. Patel and Maharaj [13] investigated stationary rotating world model with magnetic field. Ram and Singh [14] obtained some new exact solutions of string cosmology with and without a source free magnetic field for a Bianchi type-I space-time in different basic form considered by Carmaniti and McIntosh [15]. Exact solutions of string cosmology for Bianchi type-II, VI_0, VIII and IX space-time have been studied by Krori *et al.* [16] and Wang [17].

On the other hand, the matter distribution is satisfactorily described by perfect fluids due to the large scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. It is well-known that when neutrino decoupling occurs, the matter behaves as a viscous fluid in an early stage of universe. Viscous fluid cosmological models of early universe have been widely discussed in the literature.

Recently Yadav *et al.* [18] have studied some Bianchi type string cosmological model with bulk viscosity. Motivated by the situation discussed above, in this paper we focus on the problem establishing formalism for studying the new integrability of cosmic strings in Bianchi type-VI space-time in the presence of bulk viscous fluid by applying new technique.

2 Metric and Field Equations

We consider the space-time of general Bianchi type-VI with the metric

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 \exp(-2qx) dy^2 + C^2 \exp(2qx) dz^2, \tag{1}$$

where q is a constant. A, B, C are functions of t.

The energy momentum tensor for a cloud of string dust with a bulk viscous fluid of string is given by Letelier and Landau–Lifchitz

$$T_i^j = \rho u_i u^j - \lambda x_i x^j - \xi u^i_{;i}(g_i^j + u_i u^j), \tag{2}$$

where u_i and x_i satisfy condition

$$u^i u_i = -x^i x_i = -1, \qquad u^i x_i = 0. \tag{3}$$

In (2) ρ is the proper energy density for a cloud of string with particles attached to them, λ is the string tension density, u^i is the four velocities of the particles and x^i is a unit space-like vector representing the direction of string. If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda. \tag{4}$$

The Einstein field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j, \tag{5}$$

where R_i^j is the Ricci tensor; $R = g^{ij}R_{ij}$ is the Ricci scalar. In a co-moving coordinate system, we have

$$u^i = (0,\, 0,\, 0,\, 1). \tag{6}$$

The field equations (5) with (2) subsequently lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{q^2}{A^2} = 8\pi\xi\theta, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{q^2}{A^2} = 8\pi\xi\theta, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{q^2}{A^2} = 8\pi(\xi\theta + \lambda), \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{q^2}{A^2} = 8\pi\rho, \tag{10}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \tag{11}$$

where $(\cdot)$ over the symbols A, B, C denotes ordinary differentiation with respect to t.

The particle density ρ_p is given by

$$8\pi\rho_p = \frac{\dot{B}\dot{C}}{BC} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{q^2}{A^2} \tag{12}$$

in accordance with equation (4).

The velocity field u^i specified by (6) is irrotational, the scalar expansion θ and components of shear σ_{ij} are given by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\,, \tag{13}$$

$$\sigma_{11} = \frac{A^2}{3}\Big(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\Big), \tag{14}$$

$$\sigma_{22} = \frac{B^2\exp(-2qx)}{3}\Big(2\frac{\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C}\Big), \tag{15}$$

$$\sigma_{33} = \frac{C^2\exp(2qx)}{3}\Big(2\frac{\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\Big), \tag{16}$$

$$\sigma_{44} = 0. \tag{17}$$

Therefore,

$$\sigma^2 = \frac{1}{3}\Big(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC}\Big). \tag{18}$$

3 Solutions of the Field Equations

The field equations (7-11) are a system of five equations with six unknown parameters A, B, C, ρ, λ and ξ. One additional constraint relating to these parameters is required to obtain explicit solutions of the system. We assume that the expansion θ in the model is proportional to the eigenvalue σ_3^3 of the shear tensor σ_i^j. This condition leads to

$$C = \alpha(AB)^{\beta}, \tag{19}$$

where α and β are arbitrary constants. Equation (11) leads to

$$B = mC, \tag{20}$$

where m is an integrating constant.

From (19) and (20), we obtain

$$C = MA^{N}, \tag{21}$$

where

$$M = \alpha^{1/(1-\beta)} m^{\beta/(1-\beta)}, \quad N = \frac{\beta}{1-\beta}\,. \tag{22}$$

By the use of (20) field equations (9-10) reduce to

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{q^2}{A^2} = 8\pi(\lambda + \xi\theta). \tag{23}$$

$$2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}^2}{C} - \frac{q^2}{A^2} = 8\pi\rho. \tag{24}$$

Equations (7) and (8), together with the use of (13) and (21), lead to

$$(3N+1)\frac{\ddot{A}}{A} + (5N^2 - 2N)\frac{\dot{A}^2}{A^2} = 16\pi\xi(2N+1)\frac{\dot{A}}{A}\,. \tag{25}$$

Let us consider

$$\dot{A} = f(A). \tag{26}$$

Using (26) in (25), we get

$$\frac{df}{dA} + \Big\{\Big(\frac{5N^2 - 2N}{3N+1}\Big)\frac{1}{A}\Big\}f = 16\pi\xi\Big(\frac{2N+1}{3N+1}\Big). \tag{27}$$

After integration, (27) reduces to

$$f = 16\pi\xi\Big(\frac{2N+1}{5N^2+N+1}\Big)A + \frac{P}{A^{\left(\frac{5N^2-2N}{3N+1}\right)}}, \tag{28}$$

where P is an integrating constant. Integrating (28), we obtain

$$A = \frac{1}{\xi^{k_4}}[k_1 + k_2\xi \exp(k_3\xi t)]^{k_4}, \tag{29}$$

where S is an integrating constant. Therefore

$$C = \frac{M}{\xi^{k_5}}[k_1 + k_2\xi \exp(k_3\xi t)]^{k_5}, \tag{30}$$

$$B = \frac{mM}{\xi^{k_5}}[k_1 + k_2\xi \exp(k_3\xi t)]^{k_5}, \tag{31}$$

where

$$\begin{aligned} k_1 &= -\frac{P(5N^2+5N+1)}{16\pi(2N+1)}, \\ k_2 &= S, \\ k_3 &= \frac{16\pi(2N+1)}{(3N+1)}, \\ k_4 &= \frac{(3N+1)}{(5N^2+5N+1)}. \end{aligned} \tag{32}$$

Hence the metric (1) reduces to the form

$$\begin{aligned} ds^2 = -dt^2 + \Big(\frac{k_1 + k_2\xi \exp(k_3\xi t)}{\xi}\Big)^{2k_4} dx^2 \\ + \exp(-2qx) m^2 M^2 \Big(\frac{k_1 + k_2\xi \exp(k_3\xi t)}{\xi}\Big)^{2k_5} dy^2 \\ + \exp(2qx) M^2 \Big(\frac{k_1 + k_2\xi \exp(k_3\xi t)}{\xi}\Big)^{2k_5} dz^2. \end{aligned} \tag{33}$$

Using the suitable transformation

$$\begin{aligned} &\Big(\frac{k_1 + k_2\xi \exp(k_3\xi t)}{\xi}\Big) = \frac{L\sin(\xi\tau)}{\xi}, \\ &L^{k_4} x = X, \\ &mML^{k_5} y = Y, \\ &ML^{k_5} z = Z, \end{aligned} \tag{34}$$

the metric (33) reduces to

$$\begin{aligned} ds^2 = -\Big(\frac{L\cos(\xi\tau)}{k_3(k_1 - L\sin(\xi\tau))}\Big)^2 d\tau^2 + \Big(\frac{\sin(\xi\tau)}{\xi}\Big)^{2k_4} dX^2 \\ + \exp\Big(-\frac{2qX}{L^{k_4}}\Big)\Big(\frac{\sin(\xi\tau)}{\xi}\Big)^{2k_5} dY^2 + \exp\Big(\frac{2qX}{L^{k_4}}\Big)\Big(\frac{\sin(\xi\tau)}{\xi}\Big)^{2k_5} dZ^2. \end{aligned} \tag{35}$$

The rest energy (ρ), the string tension density (λ), the particle density (ρ_p), expansion (θ) and shear (σ) for the model (35) are given by

$$8\pi\rho = k_3^2 k_5(2k_4+k_5)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big)^2 - q^2\Big(\frac{\xi}{L\sin(\xi\tau)}\Big)^{2k_4}. \quad (36)$$

$$\begin{aligned} 8\pi\lambda &= -8\pi\xi k_3(k_4+2k_5)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big) \\ &\quad + k_3^2(k_4^2+k_4k_5+k_5^2)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big)^2 \\ &\quad + k_3^2 k_1(k_4+k_5)\Big(\frac{\xi^2}{L\sin(\xi\tau)} - \frac{k_1\xi^2}{L\sin^2(\xi\tau)}\Big) - q^2\Big(\frac{\xi}{L\sin(\xi\tau)}\Big)^{2k_4}, \end{aligned} \quad (37)$$

$$\begin{aligned} 8\pi\rho_p &= k_3^2 k_4(k_5-k_4)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big)^2 \\ &\quad + \Big[8\pi\xi k_3(k_4+2k_5) - k_3^2 k_1(k_4+k_5)\frac{\xi}{L\sin(\xi\tau)}\Big]\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big), \end{aligned} \quad (38)$$

$$\sigma_1^1 = \frac{2}{3}k_3(k_4-k_5)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big), \quad (39)$$

$$\sigma_2^2 = \sigma_3^3 = \frac{1}{3}k_3(k_5-k_4)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big), \quad (40)$$

$$\sigma_4^4 = 0, \quad (41)$$

$$\sigma^2 = \frac{1}{3}\Big[k_3(k_5-k_4)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big)\Big]^2, \quad (42)$$

$$\theta = k_3(k_4+2k_5)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big). \quad (43)$$

From (36) and (38), we observe the energy conditions $\rho \geqslant 0$ and $\rho_p \geqslant 0$ are fulfilled provided

$$k_5(2k_4+k_5)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big)^2 \geqslant \Big(\frac{q}{k_3}\Big)^2\Big(\frac{\xi}{L\sin(\xi\tau)}\Big)^{2k_4}$$

and

$$\begin{aligned} & k_3^2 k_4(k_5-k_4)\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big)^2 \\ &\qquad \geqslant \Big[k_3^2 k_1(k_4+k_5)\frac{\xi}{L\sin(\xi\tau)} - 8\pi\xi k_3(k_4+2k_5)\Big]\Big(\xi - \frac{k_1\xi}{L\sin(\xi\tau)}\Big), \end{aligned}$$

respectively.

From (37), we observe that the string tension density $\lambda > 0$ provided

$$k_3^2(k_4^2+k_4k_5+k_5^2)\Big(\xi-\frac{k_1\xi}{L\sin(\xi\tau)}\Big)^2 + k_3^2k_1(k_4+k_5)\Big(\frac{\xi^2}{L\sin(\xi\tau)}-\frac{k_1\xi^2}{L\sin^2(\xi\tau)}\Big) > q^2\Big(\frac{\xi}{L\sin(\xi\tau)}\Big)^{2k_4} + 8\pi\xi k_3(k_4+2k_5)\Big(\xi-\frac{k_1\xi}{L\sin(\xi\tau)}\Big).$$

The model (35) represents an expanding universe when $\sin(\xi\tau) > k_1/L$. When $\sin(\xi\tau) < k_1/L$, then θ decreases with time. Therefore the model describes a shearing non-rotating expanding universe without big bang start. We can see from above discussion that the bulk viscosity plays a significant role in the evolution of the universe.

Furthermore, since $\lim_{T\to\infty}\dfrac{\sigma}{\theta}\neq 0$, the model does not approach isotropy for large value of τ. However, if $\sin(\xi\tau)=k_1/L$, the model (35) represents an isotropic model in presence of bulk viscosity.

In absence of bulk viscosity, when $\xi\to 0$, the metric (35) reduces to

$$ds^2 = -\Big(\frac{L}{k_1k_3}\Big)^2 d\tau^2 + \tau^{2k_4}dX^2 + \exp\Big(-\frac{2qX}{L^{k_4}}\Big)\tau^{2k_5}dY^2 + \exp\Big(\frac{2qX}{L^{k_4}}\Big)\tau^{2k_5}dZ^2. \quad (44)$$

The physical parameters ρ, λ, ρ_p and the kinematical parameters θ, σ^2 for this model are respectively given by

$$8\pi\rho = \Big(\frac{k_1k_3}{L\tau}\Big)^2 k_5(2k_4+k_5) - \frac{q^2}{(L\tau)^{2k_4}}\,, \quad (45)$$

$$8\pi\lambda = \Big(\frac{k_1k_3}{L\tau}\Big)^2 (k_4^2+k_4k_5+k_4+k_5^2+k_5) - \frac{q^2}{(L\tau)^{2k_4}}\,, \quad (46)$$

$$8\pi\rho_p = \Big(\frac{k_1k_3}{L\tau}\Big)^2 k_4(k_5-k_4+1)+k_5\,, \quad (47)$$

$$\sigma_1^1 = \frac{2}{3}\frac{k_1k_3(k_5-k_4)}{L\tau}\,, \quad (48)$$

$$\sigma_2^2 = \sigma_3^3 = \frac{k_1k_3(k_4-k_5)}{3L\tau}\,, \quad (49)$$

$$.\sigma_4^4 = 0\,, \quad (50)$$

$$\sigma^2 = \frac{1}{3}\Big[\frac{k_1k_3(k_4-k_5)}{L\tau}\Big]^2\,, \quad (51)$$

$$\theta = -\frac{k_1k_3(k_4+2k_5)}{L\tau}\,. \quad (52)$$

From (45) and (47), we observe that the energy conditions $\rho\geqslant 0$, and $\rho_p\geqslant 0$, are fulfilled provided

$$k_5(2k_4 + k_5)(L\tau)^{2(k_4-1)} \geqslant \frac{1}{(k_1k_3)^2}\,, \qquad\qquad k_5 \geqslant (k_4 - 1).$$

Respectively, from (46), we observe that the string tension density $\lambda \geqslant 0$, provided

$$(k_4^2 + k_4k_5 + k_4 + k_5^2 + k_5)(L\tau)^{2(k_4-1)} \geqslant \left(\frac{q}{k_1k_3}\right)^2.$$

In absence of bulk viscosity, the model (44) starts expanding with a big bang at $\tau = 0$ and the expansion in the model decreases as time increases. When $\tau \to \infty$ then shear is zero. Near the singularity $\tau = 0$, the physical parameters ρ, λ, ρ_p are infinite, if $k_4 < 0$. Also, since $\lim\limits_{T\to\infty} \dfrac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large value of τ.

4 Another Model

In general ξ is not constant throughout the fluid so that ξ cannot be taken always constant, especially when the universe is expanding. Since in general, ξ depends on temperature (T) and pressure (p) it is reasonable to consider ξ as a function of t.

In this case (25) after integration, leads to

$$A = \left[b_0 + k_4^{-1} \int h(t)dt\right]^{k_4}, \tag{53}$$

where

$$h(t) = c_0 \exp\left(k_3 \int \xi(t)\right)dt. \tag{54}$$

And b_0, c_0 are constants of integration. Therefore, we obtain

$$C = M\left[b_0 + k_4^{-1} \int h(t)dt\right]^{k_5}, \tag{55}$$

$$B = mM\left[b_0 + k_4^{-1} \int h(t)dt\right]^{k_5}. \tag{56}$$

Hence, in this case, the metric (1) reduces to

$$\begin{aligned} ds^2 = -dt^2 &+ \left[b_0 + k_4^{-1} \int h(t)dt\right]^{2k_4} dx^2 \\ &+ \exp(-2qx)(mM)^2 \left[b_0 + k_4^{-1} \int h(t)dt\right]^{2k_5} dy^2 \\ &+ M^2 \left[b_0 + k_4^{-1} \int h(t)dt\right]^{2k_5} \exp(2qx)dz^2. \end{aligned} \tag{57}$$

The physical parameters ρ, λ, ρ_p and the kinematical parameters θ, σ^2 for this model are respectively given by

$$8\pi\rho = \frac{k_5}{k_4}\Big(2+\frac{k_5}{k_4}\Big)\Big[\frac{h(t)}{b_0 + k_4^{-1}\int h(t)dt}\Big]^2 - q^2\Big[b_0 + k_4^{-1}\int h(t)dt\Big]^{2k_4}, \quad (58)$$

$$\begin{aligned} 8\pi\lambda = &-q^2\Big[b_0+k_4^{-1}\int h(t)dt\Big]^{-2k_4} - 8\pi\xi(t)\Big(1+\frac{2k_5}{k_4}\Big)\Big[\frac{h(t)}{b_0+k_4^{-1}\int h(t)dt}\Big] \\ &+ \Big[1+\frac{k_5}{k_4}+\frac{k_5^2}{k_4^2}\Big]\Big[\frac{h(t)}{b_0+k_4^{-1}\int h(t)dt}\Big] \\ &+ \Big(1+\frac{k_5}{k_4}\Big)\Big[\frac{\Big(b_0+k_4^{-1}\int h(t)dt\Big)k_3\xi(t)h(t) - k_4^{-1}h^2(t)}{\Big(b_0+k_4^{-1}\int h(t)dt\Big)^2}\Big], \end{aligned} \quad (59)$$

$$\begin{aligned} 8\pi\rho_p = &\Big[8\pi\xi(t)\Big(1+\frac{2k_5}{k_4}\Big) - 1 - \frac{k_5^2}{k_4^2} - \frac{k_5}{k_4}\Big]\Big[\frac{h(t)}{b_0+k_4^{-1}\int h(t)dt}\Big] \\ &+ \frac{k_5}{k_4}\Big(2+\frac{k_5}{k_4}\Big)\Big[\frac{h(t)}{b_0+k_4^{-1}\int h(t)dt}\Big]^2 \\ &- \Big(1+\frac{k_5}{k_4}\Big)\Big[\frac{\Big(b_0+k_4^{-1}\int h(t)dt\Big)k_3\xi(t)h(t) - k_4^{-1}h^2(t)}{\Big(b_0+k_4^{-1}\int h(t)dt\Big)^2}\Big], \end{aligned} \quad (60)$$

$$\theta = \Big(1+\frac{2k_5}{k_4}\Big)\Big[\frac{h(t)}{b_0+k_4^{-1}\int h(t)dt}\Big], \quad (61)$$

$$\sigma_1^1 = \frac{2}{3}\Big(1-\frac{k_5}{k_4}\Big)\Big[\frac{h(t)}{b_0+k_4^{-1}\int h(t)dt}\Big], \quad (62)$$

$$\sigma_2^2 = \sigma_3^3 = \frac{1}{3}\Big(\frac{k_5}{k_4}-1\Big)\Big[\frac{h(t)}{b_0+k_4^{-1}\int h(t)dt}\Big], \quad (63)$$

$$\sigma_4^4 = 0, \quad (64)$$

$$\sigma^2 = \frac{1}{3}\Big[1+\frac{k_5^2}{k_4^2}-\frac{2k_5}{k_4}\Big]\Big[\frac{h^2(t)}{\Big(b_0+k_4^{-1}\int h(t)dt\Big)^2}\Big]. \quad (65)$$

5 Conclusion

We have presented a new class of Bianchi type-VI string cosmological models in the presence and absence of bulk viscosity. In our solution, we have obtained a relation between metric coefficients from our field equation in a natural way. In Section 4, we have obtained a general solution that has a rich structure and admits many number of solutions by suitable choice of function $\xi(t)$. Here the choice of $\xi(t)$ is quit arbitrary but since we look for physically viable models of the universe, one can choose $\xi(t)$, such that (54) is integrable.

It is observed that the bulk viscosity plays significant role in the evolution of the universe. In presence of bulk viscosity the model represents an expanding, shearing and non-rotating universe without the big bang start. But, in absence of bulk viscosity, the model starts expanding with a big bang at $\tau = 0$.

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