Bulg. J. Phys. **38** (2011) 155–163

An Analytical Approach to the Prism Coupling Problem in Otto Configuration in the Presence of Parabolic Metal Surface

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Received *20 March 2011*

Abstract. Prism coupling in the Otto configuration is a well-known method for excitation of surface plasmon polaritons in a metal film in the classical planar three-layered geometry consisting of optical prism, intermediate medium (air) and metal surface (thick film). The analysis of the reflectance in the upper half-space (prism) is based on the well-known Fresnel's formula. The prism-metal separation variation causes specific distribution of the amplitude and the phase of the p- and s-components of the reflected field [1]. The Fresnel complex coefficients are no longer constant across the aperture of the beam so further analysis is needed to be done. This phenomenon can be studied by a polarization interferometric approach [2], which allows us to separate these two components and to let them interfere. Nevertheless the nonplanar geometry of the metal attenuator additionally complicates the problem. The paper represents an analytical approach to the reflection from a three-layer system with parabolic one-dimensional metal surface.

PACS codes: 42.25Fx, 42.25Gy, 73.20Mf

1 Introduction

The surface plasmon polaritons (SPP's) represent electromagnetic excitations which are bound to metal–dielectric interface. Their properties have been the subject of extensive studies especially those involved in the SPP scattering on a surface characterized with periodic or random roughness [3,4]. In recent years the focus of investigations has been shifted from rough surface scattering towards exploring the behavior of artificially created layered structures with flat boundaries, for example, chemically assembled nanomaterials. These new structures can be modeled successfully supposing a planar multilayer system consisting of isotropic homogeneous media [5,6]. Unfortunately the technology of vacuum or chemical deposition very often leads to the presence of curvature in one or more boundary surfaces. In such a situation the Fresnel reflection coefficients are no longer valid. For a three-layer system one can apply the reduced Rayleigh equation approach [7] to derive the reflectance coefficients.

The present paper is devoted to an analytical approach considering the reflection from a three-layer system (prism, vacuum, metal) with parabolic one-dimensional metal surface. The ambition is to clarify the limits of applicability of the reduced Rayleigh equation by comparison with the experimental data.

2 Exact Theory

The system considered here consists of three media (Figure 1). The medium 0 is supposed to be ideal dielectric (optical glass prism) and occupies the space $z > h$. As a second medium 1 we will consider vacuum but it can be also a perfect dielectric or a medium with some amount of absorption. It occupies the space $\varsigma(x) < z < h$, where $\varsigma(x)$ describes the surface contour line, separating media 1 and 2. For simplicity we assume a cylindrical form of medium 2 with axes parallel to y (one-dimensional case).

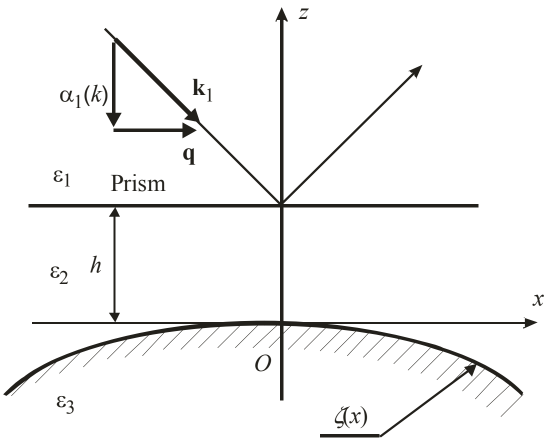

Figure 1. Sketch of the optical three-layered system.

We make the assumptions about the incident p-polarized (TM, Neumann problem) electromagnetic field

$$\mathbf{E}^{\text{in}} = \mathbf{E}\text{e}^{-i\omega t}$$

with $H_y \neq 0$, $E_x \neq 0$, $E_z \neq 0$, $H_x = H_z = E_y = 0$, so that (see Figure 1) $\mathbf{E} = E(E_x,\, 0,\, E_z)$ and $\mathbf{H} = H(0,\, H_y,\, 0)$. From the second Maxwell equation

$$\text{rot}\,\mathbf{H} = \frac{\partial}{\partial t}\mathbf{D}, \qquad \text{where} \quad \mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E},$$

assuming the components of **H** are given, we can deduce the components of the electrical vector in the j-th medium

$$E_x^{(j)} = -\frac{i}{k_0\varepsilon_j}\,\frac{\partial H_y}{\partial z} \qquad \text{and} \qquad E_z^{(j)} = \frac{i}{k_0\varepsilon_j}\,\frac{\partial H_y}{\partial x}\,. \tag{1}$$

The field in the first medium (the glass prism) can be written as a superposition in the form

$$H_y^{(1)}(x,z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{2\pi}\left[H_{\text{in}}(q)\mathrm{e}^{iqx-i\sqrt{k_1^2-q^2}(z-h)} + H_{\text{r}}(q)\mathrm{e}^{iqx+i\sqrt{k_1^2-q^2}(z-h)}\right], \tag{2}$$

where $k_1 = k_0 n_1$ is the wave number in the prism (medium 1) and $H_{\text{in}}(q)$, $H_{\text{r}}(q)$ are the incident and reflected fields, respectively. Let us now substitute Eq. (2) in (1) and introduce the forward $T_-(q)$ and backward $T_+(q)$ running fields in the curvilinear gap filled with the medium 2, Figure 2. From the boundary conditions on the interface 1–2

$$\mathbf{n}\times\mathbf{E}^{(1)} - \mathbf{n}\times\mathbf{E}^{(1)} = 0,$$
$$\mathbf{n}\times\mathbf{H}^{(1)} - \mathbf{n}\times\mathbf{H}^{(1)} = \mathbf{J}_s,$$

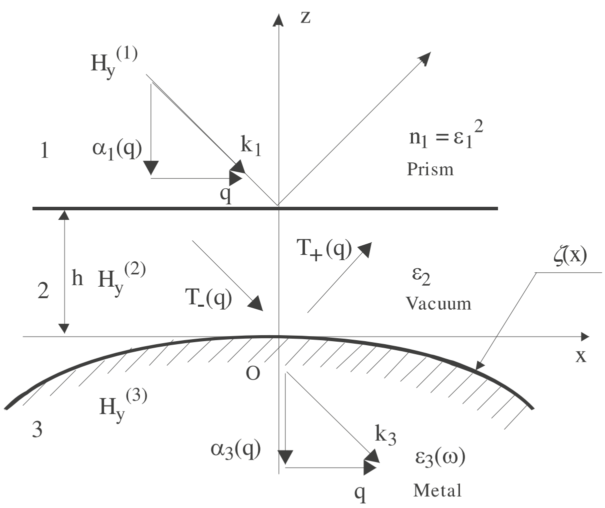

Figure 2. The wave vectors in each medium.

follows that

$$H_{\rm in}(q) + H_{\rm r}(q) = T_-(q)\mathrm{e}^{-i\sqrt{k_0^2-q^2}h} + T_+(q)\mathrm{e}^{i\sqrt{k_0^2-q^2}h}, \tag{3}$$

$$\frac{\sqrt{k_0^2-q^2}}{n_p^2}\Big(-H_{\rm in}(q) + H_{\rm r}(q)\Big) = \sqrt{k_0^2-q^2}\Big(-T_-(q)\mathrm{e}^{-i\sqrt{k_0^2-q^2}h} + T_+(q)\mathrm{e}^{i\sqrt{k_0^2-q^2}h}\Big). \tag{4}$$

Rearranging these equations we will get the forward $T_-(q)$ and backward $T_+(q)$ fields in the medium 2 (see Figure 2)

$$T_-(q) = \frac{1}{2}\exp[i\sqrt{k_0^2-q^2}h][(1+\rho)H_{\rm in}(q) + (1-\rho)H_{\rm r}(q)], \tag{5}$$

$$T_+(q) = \frac{1}{2}\exp[-i\sqrt{k_0^2-q^2}h][(1-\rho)H_{\rm in}(q) + (1+\rho)H_{\rm r}(q)]. \tag{6}$$

Here we denoted by

$$\rho(q) = \frac{\sqrt{k_1^2-q^2}}{n_1^2\sqrt{k_0^2-q^2}}\,.$$

The field in the third medium can be formally written in the form

$$H_y^{(3)}(x,z) = \int_{-\infty}^{\infty}\frac{\mathrm{d}q}{2\pi}S(q)\mathrm{e}^{iqx-i\alpha(q)z}, \tag{7}$$

where the normal component of the penetrated in the metal wave vector is $\alpha(q) = \sqrt{k_0^2\varepsilon(\omega)-q^2}$ and Im$[\alpha(q)] > 0$, Re$[\alpha(q)] > 0$, $-\infty < z < \zeta(x)$.

From the boundary conditions on the interface 2-3 follows that

$$H_y^{(2)}(x,\zeta(x)) = H_y^{(3)}(x,\zeta(x)) \quad \text{and} \quad \frac{\partial H_y^{(2)}}{\partial n}\Big|_{z=\zeta(x)} = \frac{1}{\varepsilon(\omega)}\frac{\partial H_y^{(3)}}{\partial n}\Big|_{z=\zeta(x)}, \tag{8}$$

where

$$\frac{\partial f}{\partial n} = [1+\zeta'^2(x)]^{-1/2}\Big[\frac{\partial f}{\partial z} - \zeta'(x)\frac{\partial f}{\partial x}\Big]. \tag{9}$$

Equations (5), (6) and (8), (9) tie together the field in the metal $H_y^{(3)}(x,z)$, the forward $T_-(q)$ and backward $T_+(q)$ fields in the medium 2 with the incident and reflected fields $H_{\rm in}(q)$, $H_{\rm r}(q)$, respectively. After a lengthy rearrangement [7,8], we come to the final relation

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}q}{2\pi}T_-(q)\frac{I(p-q|\alpha_3(p)+\alpha_2(q))}{\alpha_3(p)+\alpha_2(q)} = \int_{-\infty}^{\infty}\frac{\mathrm{d}q}{2\pi}T_+(q)\frac{I(p-q|\alpha_3(p)-\alpha_2(q))}{\alpha_3(p)-\alpha_2(q)}[pq+\alpha_3(p)\alpha_2(q)]. \tag{10}$$

This relation governs the balance between both fields in the second medium – the forward propagating $T_-(q)$ and backward one $T_+(q)$. Here is introduced the compact notation for the kernel function

$$I(Q|\alpha) = \int_{-\infty}^{\infty} \mathrm{e}^{-iQx - i\alpha\zeta(x)} \mathrm{d}x. \tag{11}$$

Everywhere we use the relationships between the normal and the tangential components of the wave vectors together with the condition for surface localization of the waves

$$\begin{aligned}
\alpha_1(q) &= (k_0^2\varepsilon_1 - q^2)^{1/2}, \quad \mathrm{Re}[\alpha_1(q)],\ \mathrm{Im}[\alpha_1(q)] > 0, \quad \text{for } z > D,\\
\alpha_2(q) &= (k_0^2\varepsilon_2 - q^2)^{1/2}, \quad \mathrm{Re}[\alpha_2(q)],\ \mathrm{Im}[\alpha_2(q)] > 0, \quad \text{for } \varsigma(x) < z < D,\\
\alpha_3(q) &= (k_0^2\varepsilon_3 - q^2)^{1/2}, \quad \mathrm{Re}[\alpha_3(q)],\ \mathrm{Im}[\alpha_3(q)] > 0, \quad \text{for } z < \varsigma(x).
\end{aligned}$$

From equations (10) and (11), we can calculate the desired amplitude reflection coefficient as a function of the incidence (k) and observation (p)

$$R(p|k) = \frac{H_\mathrm{r}}{H_\mathrm{in}}\,.$$

Substituting in (10) one obtains the reduced Rayleigh equation [7], which contains the normalized reflected field $R(p|k)$ rather than the intermediate fields $T_-(q)$ and $T_+(q)$

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{\mathrm{d}q}{2\pi} \frac{R(q|k)}{t_2(q)} &\{\exp[i2\alpha_2(k)h] r_1(q) M(p|q) + N(p|q)\}\\
&= -\Big[\frac{\exp[i\alpha_2(k)h]}{t_2(k)} M(p|q) + \frac{r_1(k)}{t_2(k)} N(p|q)\Big],
\end{aligned} \tag{12}$$

where we denoted by

$$\begin{aligned}
M(p|q) &= \frac{I[p-q|\alpha_3(p)+\alpha_2(q)]}{\alpha_3(p)+\alpha_2(q)} [pq - \alpha_3(p)\alpha_2(q)],\\
N(p|q) &= \frac{I[p-q|\alpha_3(p)-\alpha_2(q)]}{\alpha_3(p)-\alpha_2(q)} [pq + \alpha_3(p)\alpha_2(q)].
\end{aligned} \tag{13}$$

The tangential wave vector component q defines the direction of observation of the reflected light and the integral summation of all contributions is over wide range of this vector. The reflection and the transmission coefficients in Eq. (12) are respectively

$$\begin{aligned}
r_1(q) &= \frac{\varepsilon_1\alpha_2(q) - \varepsilon_2\alpha_1(q)}{\varepsilon_1\alpha_2(q) + \varepsilon_2\alpha_1(q)},\\
t_2(q) &= \frac{2\varepsilon_1\alpha_2(q)}{\varepsilon_1\alpha_2(q) + \varepsilon_2\alpha_1(q)} \exp[i\alpha_2(q)h].
\end{aligned} \tag{14}$$

Formulas like (12) are not new in the physical literature and all they stem from the pioneering works of McGurn and Maradudin [8-10], Simonsen [11] and perhaps firstly from the original work of Marvin, Celli and Toigo [12,13].

3 The Kernel Function for Parabolic Attenuator

The one-dimensional parabolic cylindrical interface with axes parallel to y, Figure 3, can be represented in the form

$$z = \begin{cases} \zeta(x) = t(1 - L^2x^2), & x < L^{-1} \\ \zeta(x) = 0, & x > L^{-1}, \end{cases} \tag{15}$$

where t and L are appropriate constants. Substituting from (21) to (18), we arrive to

$$I(Q|\alpha) = \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{2\pi} \exp(-iQx - i\alpha t + i\alpha L^2x^2t)$$

$$= \frac{\exp(-i\alpha t)}{2\pi L(\alpha t)^{1/2}} \int_{-\infty}^{\infty} \mathrm{d}\tau \exp\Big(-i\tau^2 - i\frac{2Q\tau}{2(\alpha h)^{1/2}L} + i\frac{Q^2}{4L^2\alpha t} - i\frac{Q^2}{4L^2\alpha t}\Big). \tag{16}$$

Here we simplified the equation (16) by denoting $\tau^2 = \alpha t L^2 x^2$ or $\tau = (\alpha t)^{1/2} Lx$. Then

$$I(Q|\alpha) = \frac{\mathrm{e}^{-i\alpha t}}{2\pi L(\alpha t)^{1/2}} \exp\Big(-i\frac{Q^2}{4L^2\alpha t}\Big) \int_{-\infty}^{\infty} \mathrm{d}\tau \exp\Big[i\Big(\tau - \frac{Q\tau}{(\alpha t)^{1/2}L}\Big)^2\Big]$$

$$= \frac{\exp(-i\alpha t)}{2\pi L(\alpha t)^{1/2}} \exp\Big(-i\frac{Q^2}{4L^2\alpha t}\Big)\Big[\sqrt{\frac{\pi}{2}}(1+i)\Big].$$

The kernel of the integral equation (11) for a parabolic boundary surface has the form

$$I(Q|\alpha) = (1+i)\Big(\frac{1}{8\pi L^2\alpha t}\Big)^{1/2} \exp\Big(-i\frac{Q^2}{4L^2\alpha t}\Big) \exp(-i\alpha t). \tag{17}$$

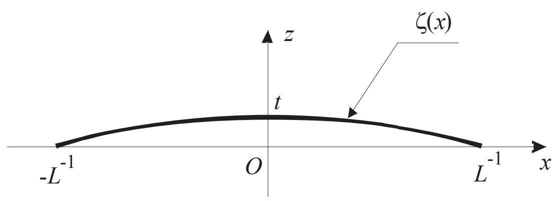

Figure 3. Sketch of the parabolic surface.

Substituting back in Eq. (13) and Eq. (12) one obtains an equation for the reflection coefficient $R(p|k)$. Applying some iterative techniques this equation can be solved numerically.

4 Numerical Results for Some Metal Attenuators

In Figures 4–7 are shown the results obtained by numerical modeling of Eq. (12). The attenuator was chosen four widely used metals: Tantalum, Chromium, Aluminum, and Silver. In all of them a symmetrical distribution of R_p has been observed. This fact plays an important role in the processes of SPP excitations.

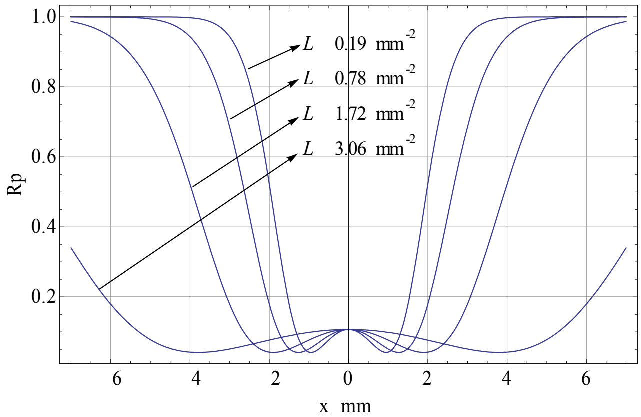

Figure 4. R_p distribution across the beam for Tantalum, $\hat{n} = 2.10 - i2.23$ for $\lambda = 632.89$ nm [14].

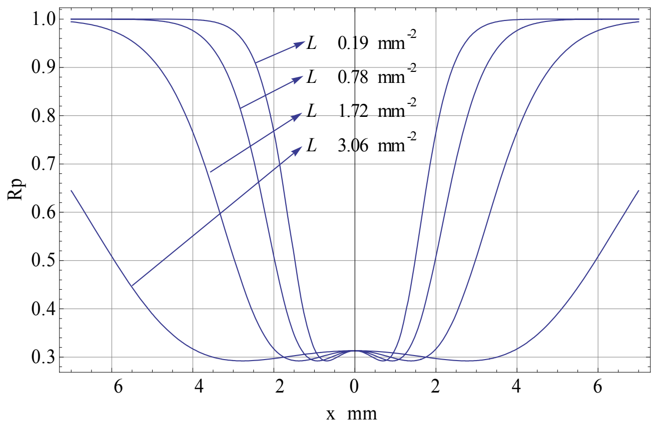

Figure 5. R_p R_p distribution across the beam for Chromium, $\hat{n} = 3.58 - i4.36$ for $\lambda = 632.89$ nm [14].

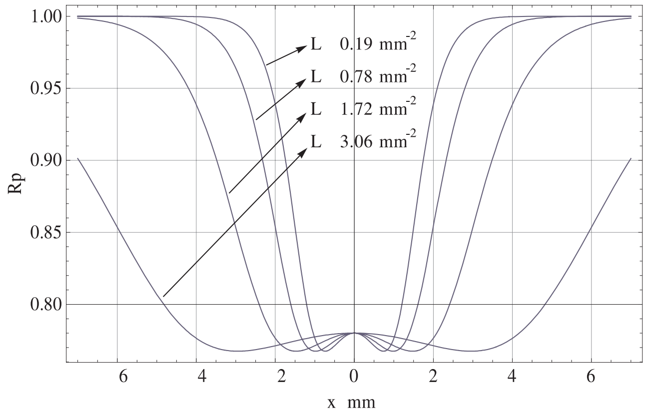

Figure 6. R_p distribution across the beam for Aluminum, $\hat{n} = 1.21 - i6.92$ for $\lambda = 632.89$ nm [14].

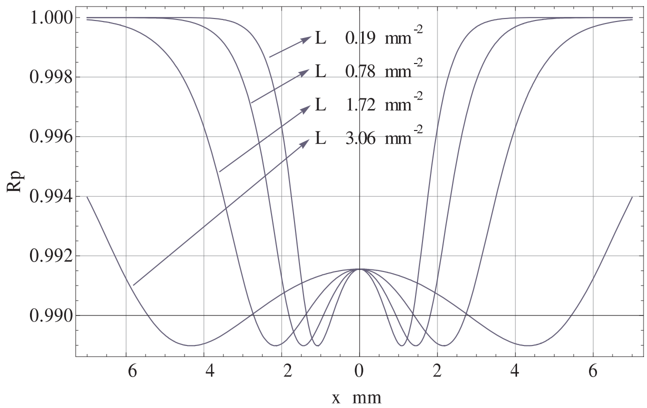

Figure 7. R_p distribution across the beam for Silver, $\hat{n} = 0.066 - i4.001$ for $\lambda = 632.89$ nm [15].

5 Conclusion

In this paper we describe in its full thoroughness the process of creation of the reflected field in the case of attenuation of the total internal reflection by a curved metallic surface. The main body of the paper is devoted to the derivation of new analytical formulae for the amplitude reflection coefficient $R(p|k)$ as a function of the direction of incidence and the direction of observation, represented by the wave vector k and the wave vector p, respectively.

Acknowledgements

This work was supported by the National Science Fund of Bulgaria, grant No. DO-02-0114-2008 and grant No. DRNF-02/8-2009.

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