

Diamagnetic Critical Singularity in Unconventional Ferromagnetic Superconductors

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Abstract. The scaling properties of the free energy, the diamagnetic moment, and the diamagnetic susceptibility above the phase transition from the ferromagnetic phase to the phase of coexistence of ferromagnetic order and superconductivity in unconventional ferromagnetic superconductors with spin-triplet (*p*-wave) electron pairing are considered. The crossover from weak to strong magnetic induction is described for both quasi-2D (thin films) and 3D (bulk) superconductors. The singularities of diamagnetic moment and diamagnetic susceptibility are damped for large variations of the pressure and, hence, such singularities could hardly be observed in experiments. The results are obtained within Gaussian approximation on the basis of general theory of ferromagnetic superconductors with *p*-wave electron pairing.

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1 Introduction

The discovery of coexistence of ferromagnetism and bulk superconductivity in Uranium-based intermetallic compounds, UGe₂ [1–3], URhGe [4], UCoGe [5,6] has led to renewed interest in the interrelationship between ferromagnetism and superconductivity. In these itinerant ferromagnets, the phase transition to superconductivity state occurs in the domain of stability of ferromagnetic phase, including sub-domains, where a considerable spontaneous ferromagnetic moment M is present. This seems to be a general feature of ferromagnetic superconductors with spin-triplet (*p*-wave) electron pairing [7–9] (see also reviews [10, 11]). In such situation the thermodynamic properties near the phase transition line may differ from those known for the superconducting-to-normal metal transition.

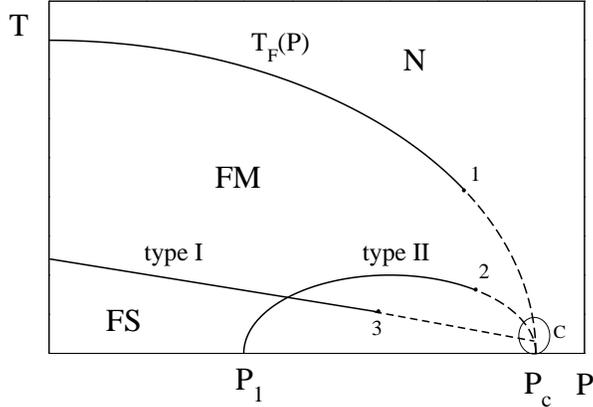


Figure 1. An illustration of $T - P$ phase diagram of p -wave ferromagnetic superconductors (details are omitted): N – normal phase, FM - ferromagnetic phase, FS – phase of coexistence of ferromagnetic order and superconductivity, $T_F(P)$ and $T_{FS}(P)$ are the respective phase transition lines: solid line corresponds to second order phase transition, dashed lines correspond to first order phase transition; 1 and 2 are tricritical points; P_c is the critical pressure, and the circle C surrounds a relatively small domain of high pressure and low temperature, where the phase diagram may have several forms depending on the particular substance. The line of the FM-FS phase transition may extend up to ambient pressure (type I ferromagnetic superconductors), or, may terminate at $T = 0$ at some high pressure $P = P_1$ (type II ferromagnetic superconductors, as indicated in the figure).

The basic thermodynamic properties of these systems are contained in their $T - P$ phase diagrams. According to the experiments [1–6] carried on the above mentioned compounds and $ZrZn_2$ [12], the $T - P$ diagrams exhibit several basic features, which are shown in Figure 1 (as claimed in Ref. [13], owing to a special treatment of sample surfaces, only a surface superconductivity has been proven in $ZrZn_2$). As seen from Figure 1, the phase transition line $T_F(P)$, corresponding to the phase transition from normal (paramagnetic) state (N) to ferromagnetic phase (FM) is substantially above the lines referring to phase transition from FM phase to the phase of coexistence of ferromagnetism and superconductivity (in short, FS phase, or, FS). The exception is only very near to the critical pressure P_c , where both ferromagnetism and superconductivity vanish and phase transition lines are close to each other. This picture reflects the real situation in the above mentioned compounds, for example, UGe_2 , where the $T_F(P)$ at ambient pressure P_a is of order 53 K, whereas the maximal T_{FS} does not exceed 1.23 K; for UGe_2 , $P_c \sim 1.6$ GPa.

Moreover, Figure 1 shows that the line $T_{FS}(P)$ of FM-FS phase transition may have two or more distinct shapes. Beginning from the maximal (critical) pressure P_c , this line may extend to all pressures $P < P_c$, including the ambient pressure P_a ; see the almost straight line containing the point 3 in Figure 1. A sec-

and possible form of this line, revealed by experiments, for example, in UGe_2 , is shown in Figure 1 by the curve which begins at $P \sim P_c$, passes through the point 2, and terminates at some pressure $P_1 > P_a$, where the superconductivity vanishes. These are two qualitatively different physical pictures: (a) when the superconductivity survives up to ambient pressure, and (b) when the superconducting states are possible only at relatively high pressure (for UGe_2 , $P_1 \sim 1$ GPa).

Within the general phenomenological theory of spin-triplet ferromagnetic superconductors [7], these different pictures are distinguished by simple mathematical conditions on the theory parameters [8, 9]. Therefore, there are both experimental and theoretical arguments to classify these superconductors in “type I” and “type II” p -wave ferromagnetic superconductors, as proposed for the first time in Ref. [8] and illustrated in Figure 1 above the respective $T_{FS}(P)$ lines. The tricritical points 1, 2 and 3 (see Figure 1), at which the order of the phase transitions changes from second order (solid lines) to first order (dashed lines) are a quite reliable experimental fact and have already been theoretically explained [7]. Firstly, the interaction between the superconducting and magnetic subsystems naturally generates a first order phase transition along the high-pressure part ($P \lesssim P_c$) of the FM-FS phase transition. Secondly, under certain circumstances, an additional \mathbf{M}^6 term in the free energy may describe the experimentally observed first order phase transition along the high-pressure part of the N-FM line [7].

In Figure 1, the circle C denotes a narrow domain around P_c at relatively low temperatures ($T \lesssim 300$ mK), where the experimental data are quite few and we may not reliably conclude about the shape of the phase transition lines in this $T - P$ domain. It could be assumed, as in the most part of the experimental papers, that $(T = 0, P = P_c)$ is the zero temperature point at which both lines $T_F(P)$ and $T_{FS}(P)$ terminate. A second possibility is that these lines may join in a single (N-FS) phase transition line at some point $(T \gtrsim 0, P'_c \lesssim P_c)$ above the absolute zero. In this second variant, a direct N-FS phase transition occurs, although this option exists along a very small piece of N-FS phase transition line: from point $(0, P_c)$ to point $(T \gtrsim 0, P'_c \lesssim P_c)$. A third variant is related with the possible splitting of point $(0, P_c)$, so that the N-FM line terminates at $(0, P_c)$, whereas the FM-FS line terminates at another zero temperature point $(0, P_{0c})$; $P_{0c} \lesssim P_c$. In this case, the p -wave ferromagnetic superconductor has three points of quantum (zero temperature) phase transitions [9].

These and other possible shapes of $T - P$ phase diagrams are described within the framework of the general theory of Ginzburg-Landau (GL) type [7] in an entire conformity with the experimental data [8, 9]; for reviews, see Refs. [10, 11]. The same theory has been confirmed by a microscopic derivation based on a microscopic Hamiltonian including a spin-generalized BCS term and an additional Heisenberg exchange term [14].

Although the theory predicts correctly the shape of FM-FS phase transition line, the possible types of phase transitions, finite temperature and quantum multi-critical points [9, 15], some important features of p -wave ferromagnetic superconductors, in particular, the properties of the FM-FS phase transition, need a further investigation. The FM-FS phase transition in p -wave superconductors is remarkable for the circumstance that there the superconductivity appears in an environment of a strong ferromagnetic moment (magnetization density) \mathbf{M} . This certainly leads to a modification of the usual phase transition to superconducting state in conventional non-magnetic superconductors.

In this paper we study the effect of the spontaneous ferromagnetic moment \mathbf{M} deeply below the ferromagnetic transition on the fluctuation properties in a close vicinity of the FM-FS phase transition above the FM-FS line. Remember that near the usual critical point of standard (conventional) superconductors, the fluctuating superconduction field $\psi(\mathbf{x})$ creates an overall diamagnetic moment $V\mathbf{M}_{dia} \equiv \mathcal{M} = \{\mathcal{M}_j; j = 1, 2, 3\}$ which, except for particular circumstances, depends on the external magnetic field \mathbf{H} . The numerous studies of the fluctuation diamagnetism in conventional superconductors, including various sample geometries [three-dimensional (3D) (bulk), quasi-2D (in short, q2D; thin films), 1D (wire), 0D (small drop)] and layered structures, are summarized in Ref. [16]; D denotes the effective dimension of the superconductor places in a three-dimensional space ($d = 3$); note, that the effective dimension of the superconductor D is often different from the space dimensionality d .

Here we consider for the first time the diamagnetic properties of 3D and q2D p -wave ferromagnetic superconductors — diamagnetic moment \mathcal{M} and diamagnetic susceptibility χ_{dia} . Some of our preliminary results for 3D (bulk) superconductors have been recently published [17]. We demonstrate that the diamagnetic susceptibility χ_{dia} and diamagnetic moment \mathcal{M} of p -wave ferromagnetic superconductors are dumped along the prevailing part of FM-FS phase transition line, in particular, at the second order phase transition line. So, our theoretical results predict that the singularities at critical points, typical for usual superconductors, do not exist at the second order phase transition line $T_{FS}(P)$ of p -wave ferromagnetic superconductors.

In Section 2 we present the GL fluctuation Hamiltonian of p -wave ferromagnetic superconductors in Gaussian approximation for the fluctuation field $\psi(\mathbf{x})$ of the superconducting order parameter ψ . We consider temperature $T \geq T_{FS}(P)$, where the statistical average $\langle\psi(\mathbf{x})\rangle$ of $\psi = \langle\psi(\mathbf{x})\rangle + \delta\psi(\mathbf{x})$ is equal to zero, namely, the field $\psi(\mathbf{x})$ is a net fluctuation: $\psi \equiv \delta\psi(\mathbf{x})$. The Gaussian approximation is the usual tool for study of basic properties of fluctuation diamagnetism [16], and here we follow this approach as well as the notations in Refs. [7, 9, 18, 19]. We shall essentially use formulations and results of the general phenomenological theory of p -wave ferromagnetic superconductors [7–11], as well as modern concepts of crossover and critical phenomena [18]. Our theoretical approach essentially generalizes preceding theoretical treatments (see,

e.g., Ref. [16] and references therein) and might be used as an advanced calculational scheme in further investigations of critical crossovers in complex superconductors.

In Section 3 we derive a general expression for the fluctuation contribution to the equilibrium free energy of system above the curve $T_{FS}(P)$ for arbitrary values of magnetic induction $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$. We show that apart of special role of magnetic induction \mathbf{B} , this expression is very similar to that for conventional nonmagnetic superconductors.

In Section 4 we calculate the diamagnetic moment $\mathcal{M}(T, \mathbf{H})$ and the diamagnetic susceptibility $\chi_{dia}(T, \mathbf{H})$. Further, we investigate the “weak- B – strong- B ” crossover in the behavior of these important quantities. For this aim, both 3D and q2D sample geometries are considered and the results are compared with known studies of conventional (s -wave) non-ferromagnetic superconductors. We predict a dumping of the singularities of the diamagnetic moment and the diamagnetic susceptibility along the FM-FS phase transition line. Moreover, we deduce a universality in the behavior of these quantities in p -wave ferromagnetic superconductors and usual (s -wave) non-magnetic superconductors. We will also briefly focus on related experimental problems. In Section 4.C and Section 5 we summarize our main results and discuss their applicability to real systems.

2 Fluctuation Hamiltonian

The fluctuation Hamiltonian of p -wave ferromagnetic superconductor can be given in the form [7]

$$\begin{aligned} \mathcal{H}(\psi) = \int d^3x & \left[\frac{\hbar^2}{4m} \sum_{j=1}^3 \left| \left(\nabla - \frac{2ie}{\hbar c} \mathbf{A} \right) \psi_j \right|^2 + a_s |\psi|^2 \right. \\ & \left. + \frac{b_s}{2} |\psi|^4 + i\gamma_0 \mathbf{M} \cdot (\psi \times \psi^*) + \delta_0 \mathbf{M}^2 |\psi|^2 \right], \end{aligned} \quad (1)$$

where $2m$ and $2e = 2|e|$ are the effective mass and the charge of the electron Cooper pairs, respectively, the superconducting order parameter $\psi = \{\psi_j(\mathbf{x}); j = 1, 2, 3\}$ is a three-component complex field, the magnetization $\mathbf{M}(\mathbf{x}) = \{M_j(\mathbf{x}); j = 1, 2, 3\}$, describing the ferromagnetic order in the FM phase is a three component real field, *i.e.*, the components $M_j(\mathbf{x})$ are real fields, $a_s = \alpha_s(T - T_s)$ is represented by the generic critical temperature T_s of hypothetical pure superconducting state ($|\psi| > 0, \mathbf{M} = 0$) and the positive material parameter α_s ; as usual, $b_s > 0$. Besides, $\gamma_0 \sim J > 0$, where $J > 0$ is an effective ferromagnetic exchange constant, and δ_0 are parameters describing the interaction between superconducting and magnetic electron subsystems. As usual, the vector potential $\mathbf{A} = \{A_j(\mathbf{x})\}$ obeys the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ and is

related to the magnetic induction \mathbf{B} . In a space of dimensionality d , in short, dD dimensional space, the relation between \mathbf{A} and \mathbf{B} is represented in different mathematical forms. Here we work in a three dimensional space ($d=3$), where the geometry of superconductor body is chosen of two types: q2D (thin film) or 3D (bulk); hence, we may use the relation $\mathbf{B} = \nabla \times \mathbf{A}$. We neglect the gradient anisotropy [20] as it has a small effect compared to the exchange interactions between the normal and superconducting electrons; this point is discussed in the Section below.

In mean field (MF) approximation [18, 19], the magnetization $M = |\mathbf{M}|$ in the pure ferromagnetic phase (FM) ($\psi = 0, \mathbf{M} \neq 0$) at zero external magnetic field ($\mathbf{H} = 0$) is given by $M_{MF} = (-a_f/b_f)^{1/2}$ as a global minimum of the magnetic fluctuation Hamiltonian

$$\mathcal{H}(M) = \int d^3x \left[c_f \sum_{j=1}^3 (\nabla M_j)^2 + a_f M^2 + \frac{b_f}{2} M^4 \right], \quad (2)$$

where $a_f = \alpha_f(T - T_F)$, T_F is the generic critical temperature of the (pure) ferromagnetic state, and α_f and b_f are positive material parameters [7]. Note that throughout this paper \mathbf{M} denotes the density of the magnetization, whereas the total magnetization of the ferromagnetic phase is given by $\int d^3x \mathbf{M}(\mathbf{x})$.

In the present investigation we ignore the fluctuations of the field $\mathbf{M}(\mathbf{x})$. This is justified by the fact, that $T_F(P) \gg T_{FS}$ for almost all variations of the pressure P , except for a very narrow domain near the critical pressure P_c , where the mean-field value $M_{MF} = (-a_f/b_f)^{1/2}$ of the magnetization tends to zero ($T_{FS} \sim T_F$). Remind, in this domain both N-FM and FM-FS phase transitions are of first order, and therefore strong fluctuations of \mathbf{M} could not be expected.

The last two terms in Eq. (1) have a key role in the description of thermodynamics and phase diagram of this type of ferromagnetic superconductors. These terms describe the interaction between the normal (non-superconducting) electron fraction and the electron Cooper pairs. While the δ_0 -term has the supporting role of ensuring a stability of phases at relatively large negative values of the parameter a_f , the γ_0 -term has a key role in the phenomenon of coexistence of superconductivity and ferromagnetism. The presence of this term ensures the theoretical description of real situation in p -wave ferromagnetic superconductors, in particular, a reliable description of the mentioned coexistence of phases. This is the term which triggers the superconductivity [7]; for a more detailed discussion, see Refs. [7, 10, 11]. This point is not trivial. Although such "trigger" terms, where one of ordering fields is present by its first power and the second order parameter interacts by its second power, are well known, for example, in the theory of certain improper ferroelectrics [21], here the symmetry of the γ_0 -term is very particular and leads to critical phenomena of essentially new universality class [15].

Note, that according to the symmetry analysis in Ref. [22], a γ_0 -term follows from the gradient anisotropy, which is typical for unconventional superconductors [20]. However, if the gradient anisotropy is the only source of creating such important term, the thermodynamics of these systems could not be described in compliance with the experimental data. This is so, because the exchange energy that creates the ferromagnetic order is of much bigger magnitude than the potential corresponding to the occurrence of FS. This argument is readily justified by the inequalities $T_F \gg T_{FS} \gg T_s \sim 0$, which follow from the experimental data. Therefore, the major contribution to the γ_0 -term and to the effective interaction parameter γ_0 , comes from the interaction between the Cooper pair fraction and the normal (non-superconducting) electrons of conduction electron bands of compounds rather than from the exchange interaction between Cooper pairs in the superconducting sub-band only. So, contrary to the consideration in Ref. [22], where only the superconducting sub-band takes part in the description and γ_0 -term is a net product of the gradient anisotropy of p -wave Cooper pairs, within the present general quasi-phenomenological approach, the interaction parameter γ_0 includes both the inter-sub-band exchange interaction between the magnetic moments of Cooper pairs and normal electrons, and the exchange interactions within the sub-band of Cooper pairs, namely, between the Cooper pairs themselves. Obviously, the second type of interaction is much weaker, and can be safely ignored in many calculations. This point of view is supported by recent microscopic theories [14, 23].

As known, for the Uranium-based compounds, all these remarkable phenomena are created by the $5f$ -electrons, whereas for $ZrZn_2$ the same phenomena might be ascribed to the behavior of the $4d$ -electrons. Furthermore, within our approach one may extend the consideration beyond the itinerant ferromagnetism and take into account exchange effects on the conduction band electrons produced by localized spins, attached to atoms at the vertices of crystal lattice.

3 Free Energy

Here our task is to calculate the fluctuation part of free energy

$$F = -\beta^{-1} \ln \int \prod_{i,j=1}^3 \prod_{\mathbf{x} \in V} \mathcal{D}\psi_j(\mathbf{x}) \mathcal{D}\delta M_i(\mathbf{x}) \exp[-\beta\mathcal{H}] \quad (3)$$

in the superconductor volume $V = L_x L_y L_z$, above the FM-FS phase transition line $T_{FS}(P) \equiv T_c(P)$; henceforth we shall use the notation T_c for the temperature T_{FS} . In Eq. (3), the functional integral is over all independent degrees of freedom: the complex components of the superconducting fluctuations, $\psi(\mathbf{x})$ and the fluctuations $\delta M_i(\mathbf{x})$ of the magnetization vector components M_i ; $\beta^{-1} = k_B T$. The functional integral is taken over both real [$\Re\psi(\mathbf{x})$] and imaginary [$\Im\psi(\mathbf{x})$] parts of the complex field $\psi(\mathbf{x})$, *i.e.*, $\mathcal{D}\psi(\mathbf{x}) \equiv d\Re\psi(\mathbf{x}) d\Im\psi(\mathbf{x})$.

Note that for temperatures near $T_c(P)$ we can always set $\beta \approx \beta_c = 1/k_B T_c$ (see, e.g., Ref. [18]). Mostly in this paper we shall consider 3D (bulk) superconductors and (q2D) thin superconducting films in a transverse magnetic field, so we shall use the 3D notations: $\mathbf{x} = (x, y, z)$, and the labels x, y, z for quantities defined along the respective Cartesian axes in 3D space ($d = 3$).

We are interested in the magnetic thermodynamics in close vicinity ($T \sim T_c$) above the temperature $T_c(P)$ of FM-FS phase transition. For $T_F \gg T_c$, the fluctuations δM_i in this temperature range are very weak and can be neglected [$M_i \approx M_i^{(MF)}$]. Then one may substitute $M_j(x)$ in the exponent of Eq. (3) by $M_j^{(MF)}$ and $\mathcal{D}\delta M_j(x)$ in the functional integration by $\mathcal{D}\delta[\delta M_j(x)]$; here $\delta[z]$ denotes δ -function. This procedure totally eliminates the fluctuations $\delta M_i(\mathbf{x})$ from our consideration; hereafter we shall omit the label ‘‘MF’’ [$\mathbf{M}^{(MF)} \equiv \mathbf{M}$]. For $T > T_{FS}$ the statistical averages $\langle \psi_j \rangle = \psi_j - \delta\psi_j$ are equal to zero and we have a purely fluctuation field $\psi(\mathbf{x}) = \delta\psi(\mathbf{x})$. Thus, within the present consideration, the only integration variables in functional integral (3) are the fluctuations $\delta\psi_j(\mathbf{x}) = \psi_j(\mathbf{x})$. Neglecting the Ginzburg critical region [18, 19], which is very small in all low temperature superconductors and, hence, unobservable in experiments, we may ignore the fourth order fluctuation term $|\psi_j|^4$ in Eq. (1), and apply Gaussian approximation to the fluctuation modes $\psi(\mathbf{x})$.

For convenience, we choose the vectors \mathbf{M} and \mathbf{H} along the \hat{z} -axis: $\mathbf{M} = (0, 0, M)$, and $\mathbf{H} = (0, 0, H)$. This assumption does not essentially restrict the generality of our consideration. If the superconductors are magnetically isotropic, the magnetization vector \mathbf{M} will follow the direction of external magnetic field \mathbf{H} . If magnetic anisotropy is present, for example, an easy axis of magnetization, as in UGe₂, our assumption will be satisfied by choosing an external field \mathbf{H} parallel to this easy axis of magnetization. Then the term $\mathbf{M} \cdot (\psi \times \psi^*)$ takes the simple form $M(\psi_1 \psi_2^* - c.c.)$.

Under the supposition of uniform magnetic induction $\mathbf{B} = (0, 0, B)$, we take the gauge of the vector potential \mathbf{A} as $\mathbf{A} = (-By, 0, 0)$ and expand the fields $\psi_j(\mathbf{x})$ in series [19]

$$\psi_j(\mathbf{x}) = \sum_q c_j(q) \varphi_j(q, \mathbf{x}) \quad (4)$$

in terms of the complete set of eigenfunctions

$$\varphi_j(q, \mathbf{x}) = \frac{1}{(L_x L_z)^{1/2}} e^{i(k_x x + k_z z)} \chi_n(y) \quad (5)$$

of the operator $[i\hbar\nabla + (2e/c)\mathbf{A}]^2/4m$, corresponding to the eigenvalues

$$E(q) = \left(n + \frac{1}{2}\right) \hbar\omega_c + \frac{\hbar^2}{4m} k_z^2, \quad (6)$$

specified by the magnetic frequency $\omega_B = (eB/mc)$ and vector $q = (n, k_x, k_z)$, where $n = 0, 1, \dots, \infty$, is the quantum number corresponding to the Landau

levels, and k_x and k_z are components of the wave vector $\mathbf{k} = (k_x, k_y, k_z)$. In Eq. (5), the function $\chi_n(y)$ is related to the Hermite polynomials $H_n(y)$ by

$$\chi_n(y) = A_n e^{-\frac{(y-y_0)^2}{2a_B^2}} H_n\left(\frac{y-y_0}{a_B}\right), \quad (7)$$

where $A_n^{-1} = (a_B 2^n n! \sqrt{\pi})^{1/2}$ [24], $y_0 = a_B^2 k_x$, and $a_B = (\hbar c / 2eB)^{1/2}$.

In terms of the $c_j(q)$ -functions, the ψ^2 -part of the fluctuation Hamiltonian (1) is given by

$$\mathcal{H} = \sum_{j,q} \tilde{E}(q) c_j(q) c_j^*(q) + i\gamma_0 M [c_1(q) c_2^*(q) - \text{c.c.}], \quad (8)$$

where

$$\tilde{E}(q) = E(q) + a_s + \delta_0 M^2. \quad (9)$$

Applying the unitary transformation,

$$c_1(q) = \frac{i}{\sqrt{2}} [-\phi_+(q) + \phi_-(q)] \quad (10a)$$

$$c_2(q) = \frac{1}{\sqrt{2}} [\phi_+(q) + \phi_-(q)] \quad (10b)$$

renders the fluctuation Hamiltonian (1) as a sum of squares of field components $c_3(q)$, and $\phi_{\pm}(q)$.

In the continuum limit, $L_y \rightarrow \infty$, the integral

$$I_y = \int_{-L_y/2}^{L_y/2} dy \chi_n(y) \chi_{n'}(y) \quad (11)$$

is simply equal to the Kronecker symbol $\delta_{nn'}$ and this is a key point in the further simple representation of the Hamiltonian (1). Substituting the function (7) in the integral (11), and having in mind the properties of the Hermite polynomials $H_n(z)$ [24], we see that the integral (11) will be equal to $\delta_{n,n'}$ only if the limits of integration can be expanded to $\pm\infty$. In fact, the integration in Eq. (11) can be performed with respect to the variable $\bar{y} = (y - y_0)/a_B$ and integral limits $(-y_0 \pm L_y/2)/a_B$. The limits of integration with respect to this variable can be approximately equalized to $\pm\infty$, if only $L_y/2a_B \rightarrow \infty$; for finite samples ($L_y \gg 2a_B$). Another inevitable condition, namely, $-L_y/2 < y_0 < L_y/2$, follows from the requirement that the coordinate $y_0 = a_B^2 k_x$ must belong to the sample volume. This condition implies

$$-\frac{L_y}{2a_B^2} < k_x < \frac{L_y}{2a_B^2}. \quad (12)$$

As we show below, this condition fixes the number of states $\mathcal{N} = L_x L_y / 2\pi a_B^2$ for all possible values of quantum number k_x at any given n and k_z .

Having in mind these features of theory and considering sufficiently large L_y , we can justify the solution $I_y = \delta_{nn'}$ of the integral (11) and achieve a very useful form of the Hamiltonian, namely,

$$\mathcal{H} = \sum_{n,q} [E_-(q)|\phi_+(q)|^2 + E_+(q)|\phi_-(q)|^2 + E_3(q)|c_3(q)|^2], \quad (13)$$

with

$$E_{\pm}(q) = E(q) + a_{\pm}(M), \quad (14)$$

where $E(n, k_z)$ is given by Eq. (6),

$$a_{\pm}(M) = a_0 \pm \gamma_0 M \quad (15)$$

is represented by $a_0 = a_s + \delta_0 M^2$, and $E_3 \equiv \tilde{E}$ is given by Eq. (9).

Now the free energy (3) can be written as a functional integral over all independent field amplitudes: $\pm\phi(q)$, and $c_3(q)$. Using the short notations $\varphi_{\alpha}(q)$ with $\alpha = (+, -, 3)$ of the Fourier amplitudes $\phi_+(q)$, $\phi_-(q)$ and $c_3(q)$, respectively, and adopting the same label α to denote E_+ , E_- and E_3 by E_{α} , we obtain the free energy F in the form

$$F = - \int \prod_{\alpha,q} d\Re\varphi_{\alpha}(q) d\Im\varphi_{\alpha}(q) e^{-\beta \sum_{\alpha,q} E_{\alpha}(q) |\varphi_{\alpha}(q)|^2}. \quad (16)$$

The direct calculation of the Gaussian integrals in Eq. (16) yields

$$F = -k_B T \ln \prod_{\alpha,n,k_x,k_z} \left[\frac{\pi k_B T}{E_{\alpha}(n, k_z)} \right] = -k_B T \ln \prod_{\alpha,n,k_z} \left[\frac{\pi k_B T}{E_{\alpha}(n, k_z)} \right]^{\mathcal{N}}, \quad (17)$$

where we have used the condition (12) and the continuum limit for the k_x -product,

$$\prod_{k_x} 1 \longrightarrow \exp \left[L_x \int dk_x / 2\pi \right], \quad (18)$$

namely,

$$\mathcal{N} = \sum_{k_x} 1 \approx L_x \int_{-L_y/2a_B^2}^{L_y/2a_B^2} \frac{dk_x}{2\pi} = \frac{L_y L_x}{2\pi a_B^2}. \quad (19)$$

In the second equality (17) we point out the result of the summation over k_x with the help of the rule (18). The obtained expression (17) follows from the fact that the mode energies $E_{\alpha}(n, k_z)$ do not depend on the quantum number k_x . Thus the number of states \mathcal{N} at fixed quantum numbers n and k_z and the relation $F \sim \mathcal{N}$ are naturally deduced from the calculation.

Further, we have to pay attention to the fact that the field theories of GL type are limited to length scales $k = |\mathbf{k}| \lesssim \Lambda \sim \pi/\xi_0$, where ξ_0 is the zero temperature correlation length of the field of interest [18,19]. In the present case, we must use the quantity ξ_0 corresponding to the field $\psi(q)$ which fluctuates in the vicinity of the phase transition line $T_c(P)$. The standard expression $\xi_{0s} = \hbar/(4m\alpha_s T_s)^{1/2}$, corresponding to the generic critical temperature T_s [18,19] cannot be applied to our problem. So, we define the upper cutoff $\Lambda \simeq \pi/\xi_0$ by the zero-temperature correlation (coherence) length ξ_0 but the latter will be specified at a next stage of our consideration. Here we will mention that ξ_0 is the scaling amplitude of the correlation length ξ of the superconducting fluctuations at the FM-FS critical line $T_c(P)$: $\xi(t) = \xi_0/|t|^{1/2}$, where $t = (T - T_c)/T_c$; $|t| < 1$.

As the small wave numbers k have the main contribution to the values of the integrals in the free energy and its derivatives, we shall use the finite cutoff Λ only when the respective integral has an "ultraviolet" divergency; for example, see Eq. (19). In all other cases, the relatively large values of k do not produce essential quantitative contributions to the integrals and for this reason, we may extend the cutoff Λ to infinity. Moreover, owing to the same type of limitation of the GL theory — the long wavelength approximation $\xi_0 k \lesssim \pi$, we should take in mind that only quantum numbers n corresponding to energies $E_\alpha \lesssim \hbar^2/4m\xi_0^2$ are to be taken into account. Thus the quantum number n has a cutoff as well, and the latter is given by $n_c \omega_B \simeq \hbar/4m\xi_0^2$, namely, $n_c = [\hbar/4m\xi_0^2 \omega_B]$ ($[z]$ denotes the integer part of number z). This energy cutoff could be neglected in cases when this does not produce divergencies of the respective physical quantities.

For our further aims we shall write Eq. (17) in a more convenient form

$$F = -S \frac{ek_B T B}{\pi \hbar c} \sum_{n=0}^{n_c} \sum_{k_z=-\Lambda}^{\Lambda} \ln \frac{(\pi k_B T)^3}{E_+ E_- E_3}, \quad (20)$$

The relevant part of the free energy, which contains singularities at the critical temperature T_c is given by the term containing $\ln E_-$. All other terms are quite smooth near the line $T_c(P)$ and do not produce singularities of the physical quantities. This important circumstance follows directly from the fact, that namely the parameter $a_-(M)$ is relatively small in magnitude and changes sign at T_c — the FM-FS phase transition temperature, corresponding to the phase transition from FM phase to the phase of coexistence (FS) of ferromagnetism and a homogeneous (Meissner) superconducting state at zero external magnetic field ($H = 0$). Therefore, the critical fluctuations are described by the field $\phi_-(q)$. The other fields, $\phi_+(q)$ and $c_3(q)$, do not produce critical phenomena (singularities) because the parameters a_+ and a_0 do not pass through the null at $T = T_c$. The value of magnetization M is relatively large along the most part of the line $T_c(P)$ in this type of ferromagnetic superconductors ($T_F \gg T_c$), and therefore the parameters a_+ and a_0 are quite different from a_- except for a narrow domain around the critical pressure P_c . Note that in a non-magnetic

(standard) p -wave superconductor [18], where $M \equiv 0$, $a_{\pm} = a_0 = a_s$, all three modes $\varphi_{\alpha}(q)$ are critical in a close vicinity of the critical point T_s . For such superconductor, the free energy (21) will differ with a factor 3 from the standard result for a conventional (s -wave) superconductor with a scalar order parameter [19].

In our further analysis we shall ignore the nonsingular part of the free energy and keep only the contributions from the critical mode $\phi_{-}(q)$. Thus, we have to analyze the behavior of function

$$F = -S \frac{e k_B T B}{\pi \hbar c} \sum_{n=0}^{n_c} \sum_{k_z=-\Lambda}^{\Lambda} \ln \frac{\pi k_B T}{E(q) + a_{-}(M)}, \quad (21)$$

where $E(q)$ is given by Eq. (6).

Now we have to define the parameters in Eq. (21). For $M = (|a_f|/b_f)^{1/2}$, the parameters $a_{\pm}(M)$ are given by [8, 9]

$$a_{\pm}(T) = \alpha_s(T - T_s) + \delta_0 \frac{a_f}{b_f} \pm \gamma_0 \left(\frac{a_f}{b_f} \right)^{1/2}. \quad (22)$$

We are interested mainly on the parameter $a_{-}(T)$ which is related with the equilibrium phase transition from FM to FS. Defining $T_c \equiv T_{FS}$ from the equation $a_{-}(T_c) = 0$, we obtain

$$a_{-}(T) \approx \alpha_c(T - T_c), \quad (23)$$

where

$$\alpha_c = \alpha_s - \frac{\delta_0 \alpha_f}{b_f} + \frac{\gamma_0 \alpha_f^{1/2}}{2 [b_f (T_F - T_c)]^{1/2}} \quad (24)$$

and T_c is given as a solution of the equation

$$T_c(M_c) = T_s - \frac{\delta_0}{\alpha_s} M_c^2 + \frac{\gamma_0}{\alpha_s} M_c, \quad (25)$$

where $M_c \equiv M(T_c) = [\alpha_f(T_F - T_c)/b_f]^{1/2} > 0$. In the same way one obtains that the parameters a_0 and a_+ remain positive at T_c : $a_0(T_c) = \gamma_0 M_c$, and $a_+(T_c) = 2a_0(T_c)$, which is a demonstration that the modes $\phi_+(q)$ and $c_3(q)$ are not critical and could not have essential contributions to the thermodynamics in the vicinity of phase transition line $T_c(P)$.

Note that the parameter α_c , given by Eq. (24), is positive for requirements of stability of the ordered phases [10]. The solution of Eq. (25) with respect to T_c yields the curve $T_{FS}(P) \equiv T_c(P)$, shown in Figure 1. The dependence of T_c on the pressure P comes from the P -dependence of material parameters ($\alpha_s, \alpha_f, T_F, \dots$) in Eq. (25). In Refs. [8, 9] a simple P -dependence of

these parameters has been suggested: all material parameters except T_F are P -independent, and the form of the function $T_F(P)$ is assumed of the simple form $T_F(P) \approx T_F(0)(1 - P/P_c)$. Although this is a simple approximation of the pressure effect in these systems, it gives a remarkable agreement between theory and experimental data for the $T - P$ phase diagram [8, 9]. In the framework of the same approximation, according to Eq. (25), the P -dependence is contained in the quantity M_c and, hence, we may often consider T_c as a function of $M_c \equiv M(T_c)$ – the value of the magnetization on the FM-FS phase transition line: $T_c = T_c(M_c)$.

The upper cutoff for the wave number k_z is given by $\Lambda \simeq \pi/\xi_0$, where the zero temperature correlation length $\xi_0 = (\hbar^2/4m\alpha_c T_c)^{1/2}$ is expressed by α_c and T_c , given by Eqs. (24) and (25), respectively. The upper quantum number n_c , defined by the equality $n_c \hbar \omega_c \simeq \hbar^2/4m\xi_0^2$, can be represented by $n_c = [1/2b]$, where $b = (e\hbar B/2mc\alpha_c T_c)$ is a non-negative quantity. Having in mind the supplementary condition that $E(n, k_z)$ from Eq. (6) with $n = k_z = 0$ should also obey the condition $E(0, 0) \leq \hbar^2/4m\xi_0^2$, we find that $b \in [0, 1]$. For type II superconductors, the parameter b has the useful representation $b = B/B_{c2}(0)$ by the upper critical induction $B_{c2}(T) = B_{c2}(0)|t|$ at zero temperature, $B_{c2}(0) = e\hbar/2mc\alpha_c T_c$, *i.e.*, for $|t| = 1$; henceforth we shall denote $B_{c2}(0)$ by B_0 .

Using these remarks, we can represent the free energy in the form

$$F = \rho S B_0 f(b, t) \quad (26)$$

where $\rho = (ek_B T/\pi \hbar c)$, and

$$f(t, b) = \sum_{k_z=-\Lambda}^{\Lambda} S(k_z, t, b) \quad (27)$$

is given by the sum

$$S(k_z, t, b) = -b \sum_{n=0}^{[1/2b]} \ln \frac{(\pi k_B/\alpha_c)(1+t)}{2bn + b + t + \xi_0^2 k_z^2}. \quad (28)$$

The function $f(t, b)$ describes the shape of the free energy $F(T, B)$, whereas the functions $m(t, b) = -\partial f/\partial b$ and $m'(t, b) = \partial m/\partial b$ represent the variations of the diamagnetic moment $\mathcal{M} = -\partial F(T, H)/\partial H$ and the diamagnetic susceptibility $\chi_{dia} = -\partial^2 F(T, H)/\partial H^2$, respectively. For the choice $\mathbf{H} = (0, 0, H)$, the diamagnetic vector \mathcal{M} has only one component: $(0, 0, \mathcal{M})$. As $\partial H = \partial B$, we can use the formulae $\mathcal{M} = -\partial F(T, B)/\partial B$ and $\chi_{dia} = \partial \mathcal{M}(T, B)/\partial B$.

The wave number k_z lies in the reduced Brillouin zone: $-\pi/\xi_0 < k_z = 2\pi l/L_z \leq \pi/\xi_0$; $l = 0, \pm 1, \dots \pm [L_z/\xi_0]$. For q2D systems, where $L_z \leq \xi_0$, the only possible value of k_z is zero and, hence, for q2D superconductors the function $f(t, b)$ coincides with $S(k_z, t, b) = S(0, t, b)$. For 3D systems, we shall use

the continuum limit for $f(t, b)$, given by

$$f(t, b) = L_z \int_{-\Lambda}^{\Lambda} \frac{dk_z}{2\pi} S(k_z, t, b). \quad (29)$$

In Eqs. (21) and (28), the logarithmic divergence at maximal temperature corresponds to $n = 0$, $k_z = 0$, and $\epsilon = (b + t) = 0$. The parameter t indicates the vicinity to T_c along the T -axis and the parameter $b = B/B_0$ shows the strength of the induction B and the distance to the phase transition point $(T_c, B = M)$ along the H -axis of the (T, P, H) phase diagram. These two parameters, t and b , are suitable for investigations of the system properties for $t > 0$. It is easy to show that for $t < 0$, where the upper critical induction $B_{c2}(T) = B_0(-t) \geq 0$ of type II superconductors is defined, the parameter $\epsilon = (t + b)$ can be represented in the suitable form $\epsilon = [B - B_{c2}(T)]/B_0$, or, alternatively, in the form $\epsilon = [T - T_{c2}(B)]/T_c$, where $T_{c2}(B) = T_c(1 - b)$ is the higher critical temperature of type II superconductors [19]. Thus, for $t < 0$, the parameter ϵ shows the distance from the phase transition line described by the upper critical induction $B_{c2}(T) = B_0|t|$, or, alternatively, by the higher critical temperature $T_{c2}(B)$. The parameter ϵ is appropriate for investigations at temperatures $T < T_c(P)$, *i.e.*, $t < 0$. In this paper our consideration is restricted to temperatures $T > T_c(P)$ and for this reason we shall use the original parameters variables t and b , as given in Eqs. (26) – (29).

Performing the summation in Eq. (28) and keeping only terms which depend on $b \sim B$, we obtain

$$S(k_z, t, b) = -b \ln \left(\frac{\pi k_B}{\alpha_c} \right) - b \ln(1 + t) + \frac{\ln b}{2} + b \ln [1 + b + \varepsilon(k_z)] + b \ln \frac{\Gamma \left[\frac{1 + b + \varepsilon(k_z)}{2b} \right]}{\Gamma \left[\frac{b + \varepsilon(k_z)}{2b} \right]}, \quad (30)$$

where $\varepsilon(k_z) = (t + \xi_0^2 k_z^2)$ and $\Gamma(z)$ is the gamma function. The sum $S(k_z, t, b)$, given by Eq. (30) and the shape function $f(t, b)$, given by Eq. (27), contain redundant terms. Remind that we have neglected the contributions from the factors E_+ and E_3 in Eq. (20) although most of them depend on the parameter b , namely, on the induction B and, hence, these terms may have a finite contribution to the diamagnetic moment \mathcal{M} . The mentioned terms have been however ignored for the fact that they do not produce singularities in the free energy derivatives. Thus, within the approximations already made, we cannot evaluate correctly the magnitude of finite contributions to important quantities as \mathcal{M} and χ_{dia} at the phase transition point. We may just demonstrate that such contributions exist. To be in a consistency with our preceding consideration, we should neglect such terms in Eqs. (26), (27) and (30), too.

Up to now we have kept all b -dependent terms contained in the general formula (21) for the free energy, including terms which obviously do not lead to any singularities, for example, the first and second terms on the r.h.s. of Eq. (30). At this stage we make the stipulation to keep these terms in our further consideration with the remark that contributions to \mathcal{M} and χ_{dia} which are finite at the phase transition point will be neglected, provided divergent term is present. When no divergency occurs in some of these quantities, we shall keep the finite term only to indicate the lack of divergencies and to show that the respective quantity remains finite at the phase transition point.

4 Crossover from Weak to Strong Magnetic Induction

The free energy, the diamagnetic moment \mathcal{M} and the diamagnetic susceptibility χ_{dia} can be investigated analytically in the limiting cases of strong and weak magnetic induction B . Above the critical temperature T_c , when $t > 0$, the weak- B limit is defined by the condition

$$\varepsilon(k_z) = t + \xi_0^2 k_z^2 \gg b, \quad (31a)$$

whereas the strong- B limit is given by the opposite condition

$$\varepsilon(k_z) \ll b. \quad (31b)$$

These conditions are considered in a close vicinity ($t \ll 1$) of the phase transition line $T_c(P)$. The condition (31a) is satisfied for any k_z , provided $t \gg b$. The condition (31b), however, can be satisfied only for $t \ll b$ and $(\xi_0 k_z)^2$ sufficiently small. For large $k_z \sim \pi/\xi_0$ this condition does not hold for any $b \in [0, 1]$. However, the sum (27) is practically taken for $k_z \neq 0$ only for the 3D geometry, and in this case, the main contribution to the sum is given by the relatively small wave numbers ($\xi_0 k_z \ll \pi$). Having in mind this argument, we can use the condition (31b) without any restriction to small values of $\xi_0 k_z$ because the final result for the free energy will not essentially depend on the contribution of relatively large wave numbers. In particular, this is true in the continuum limit (29) for the sum (27) and $(t + b) \ll 1$, namely, in the close vicinity of the phase transition line $T_c(P)$.

4.1 Weak- B Limit

Applying the condition (31a) to the sum (30), we obtain the result

$$S(k_z, t, b) = b \ln \frac{1 + \varepsilon(k_z)}{(\pi k_B / \alpha_c)(1 + t)} + \frac{b^2}{12} \left[\frac{11}{1 + \varepsilon(k_z)} + \frac{1}{\varepsilon(k_z)} \right]. \quad (32)$$

In Eq. (32), all b -independent terms have been omitted as irrelevant to our consideration and small terms of type $O(b^3)$ have been neglected. From Eq. (32)

is readily seen that only the term of type $b^2/\varepsilon(k_z)$ will produce a singularity of the thermodynamic functions as this term tends to infinity for $k_z \sim 0$ and $t \sim 0$. Therefore, in this case, we can neglect the other terms in Eq. (32) and write the free energy in the form

$$F = \frac{\rho S B^2}{12 B_0} \sum_{k_z=-\Lambda}^{\Lambda} \frac{1}{\varepsilon(k_z)}. \quad (33)$$

This result has been obtained in Ref. [17] in different notations.

3D superconductors

For 3D superconductors and $t \ll 1$, the sum (27) over k_z can be substituted by the integral (29) and the cutoff Λ can be extended to infinity. Then the free energy becomes

$$F_{3D} = \frac{\rho V B^2}{24 B_0 \xi_0 t^{1/2}}. \quad (34)$$

The diamagnetic moment $\mathcal{M} \equiv M_{dia} = -\partial F(T, B)/\partial B$ takes the form

$$\mathcal{M}_{3D}(T, H) = -\frac{\rho V B}{12 B_0 \xi_0 t^{1/2}}. \quad (35)$$

In contrast to the usual case of non-magnetic superconductors [16], here the diamagnetic moment does not vanish at $H = 0$ but rather remains proportional to the magnetization M_c at the FM-FS phase transition line. In fact, as we work at a close vicinity of FM-FS phase transition line $T_c(P)$, the magnetic induction in Eq. (35) should be approximated by $B \approx B_c = H + 4\pi M_c$, where $M_c = M(T_c)$. Within the weak- B limit, this result is valid for relatively small values of M_c : $M_c \ll B_0$.

The diamagnetic susceptibility $\chi_{dia}^{(3D)} = \partial \mathcal{M}_{3D}/\partial B$ is given by

$$\chi_{dia}^{(3D)}(T) = -\frac{\rho V}{12 B_0 \xi_0 t^{1/2}}. \quad (36)$$

In these general notations this is the well known result for the fluctuation diamagnetic susceptibility above the critical point T_c of conventional superconductors [16, 19]. For p -wave ferromagnetic superconductors we have to take into account that $T_c = T_{FS}$ as given by Eq. (25) and the material parameter α_c , which enters in the zero-temperature correlation length ξ_0 , is given by Eq. (24). Thus one reveals the result for $\chi_{dia}(T)$, obtained in Ref. [17].

Quasi-2D superconductors

For thin films, where $L_z < \xi_0 = \pi/\Lambda$, only the wave number $k_z = 0$ satisfies the condition $-\pi/\xi_0 < k_z = 2\pi l/L_z \leq \pi/\xi_0$; $l = 0, \pm 1, \dots, [\pi L_z/\xi_0]$. For

such q2D geometry, $f(t, b) = S(0, b, t)$. In the weak- B limit (31a), $S(0, b, t)$ is obtained by setting $k_z = 0$ in Eq. (32). Once again we may keep only the leading singular term $b^2/12t$. Thus, we obtain the free energy in the form

$$F_{2D} = \frac{\rho S B^2}{12 B_0 t}. \quad (37)$$

In Eq. (37) and below we use the label “2D” to denote quantities corresponding to q2D systems. Now one easily finds that

$$\mathcal{M}_{2D}(T, H) = -\frac{\rho S B}{6 B_0 t}. \quad (38)$$

$$\chi_{dia}^{(2D)}(T) = -\frac{\rho S}{6 B_0 t}. \quad (39)$$

Having in mind the relations (26) and (27) as well as $\partial/\partial b = B_0 \partial/\partial B$, for q2D systems, we obtain $\chi_{dia} = (\rho S/B_0)\chi(t, b)$, where $\chi(t, b)$ is the susceptibility shape function. The latter is defined by $\chi = -\partial^2 \tilde{f}(0, t, b)/\partial b^2$ with $\tilde{f}(t, b) = \tilde{S}(0, t, b)$, where $\tilde{S}(0, t, b)$ denotes $S(0, t, b)$ with $(\pi k_B/\alpha_c) = 1$ for suitable choice of units, as given by Eq. (30) for $k_z = 0$. The function $\chi(t, b)$ is depicted in Figure 2 for $t = (0.01, \dots, 0.1)$ and $b = (0.01, \dots, 0.1)$. As shown in Figure 2,

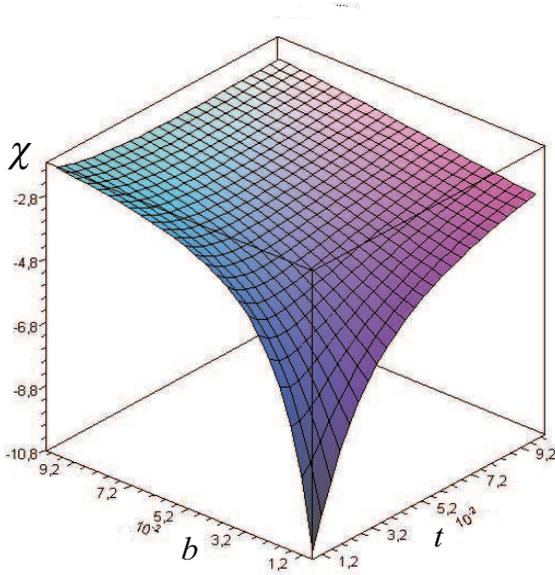


Figure 2. The susceptibility shape function $\chi(t, b)$ for q2D systems and variations of $t = (0.01, \dots, 0.1)$ and $b = (0.01, \dots, 0.1)$.

the shape function $\chi(t, b)$ exhibits a sharp decrease even at values of $(t, b) \sim (0.01, 0.05)$. The minimal value $\chi \sim -10$ for $t \sim b \sim 0.01$ is a precursor of divergency, given by Eq. (39). As we see from Figure 2, the decrease of the function $\chi(t, b)$ is more steep along the t -axis, and changes its monotonic decrease with a decrease of value of t at any fixed b changes to an increase at some finite $t_m(b) > 0$, which renders the minimal value of χ at given b . Obviously, the decrease of χ is not symmetric with respect to the axes t and b even in the pre-critical region $t \sim b \sim 0.01$. The difference in the behavior of χ with respect to t and b is better seen in the strong- B limit ($t \ll b$).

4.2 Strong- B Limit

For relatively large induction B , the leading terms in the sum (30) are

$$S(k_z, t, b) = \frac{\ln b}{2} + b \ln \frac{2\alpha_c(1+b)}{\pi k_B} + b \ln \left[\frac{\Gamma(1/b)}{\Gamma(1/2b)} \right] + O(b\varepsilon), \quad (40)$$

where b -independent terms and small term of order $O(bt)$ have been omitted. This expression of $S(k_z, t, b)$ is valid for any $0 < b \leq 1$ and does not contain $\varepsilon(k_z)$. Therefore, the result (40) can be obtained by setting $t = (\xi_0 k_z)^2 = 0$ in Eq. (30) and by applying properties of the gamma function $\Gamma(z)$ [24].

In this limiting case, F_{3D} is related with F_{2D} by

$$F_{3D} = \frac{L_z}{\xi_0} F_{2D}, \quad (41)$$

as implied by Eq. (29), and F_{2D} is given by

$$F_{2D} = \rho S B_0 S(b), \quad (42)$$

where $S(b)$ is a short notation of the expression (40) of $S(k_z, t, b) \approx S(0, 0, b)$ in the strong- B limit.

The sum (40) does not exhibit any singularity. We shall briefly discuss the case $\varepsilon \ll b \ll 1$ in order to reveal and generalize a preceding result for the diamagnetic moment [25]; see also Ref. [16]. For small b we obtain from Eq. (40) that

$$S(b) = b \ln \frac{\sqrt{2}\alpha_c}{\pi k_B} + \frac{11}{12} b^2 \quad (43)$$

(b -independent terms have been once again omitted).

Now one may obtain a simple expressions for the free energies of q2D and 3D superconductors. The q2D free energy F_{2D} will be

$$F_{2D} = \rho S B \left[\ln \frac{\sqrt{2}\alpha_c}{\pi k_B} + \frac{11B}{12B_0} \right], \quad (44)$$

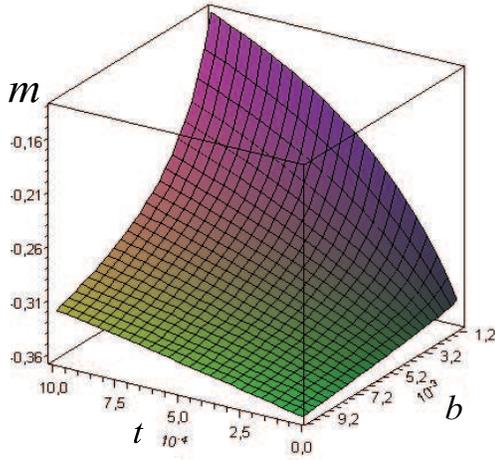


Figure 3. The magnetization shape function $m(t, b)$ for q2D systems and variations of $t = (0.00001, \dots, 0.001)$ and $b = (0.001, \dots, 0.01)$.

whereas F_{3D} is given by Eqs. (41) and (44). The diamagnetic moment \mathcal{M}_{2D} and the diamagnetic susceptibility $\chi_{dia}^{(2D)}$ will be

$$\mathcal{M}_{2D} = -\rho S \left[\ln \frac{\sqrt{2}\alpha_c}{\pi k_B} + \frac{11B}{6B_0} \right], \quad (45)$$

and

$$\chi^{(2D)} = -\frac{11\rho S}{6B_0}, \quad (46)$$

respectively. For 3D systems, in accord with Eq. (41), $\mathcal{M}_{3D} = (L_z/\xi_0)\mathcal{M}_{2D}$, and $\chi_{dia}^{(3D)} = (L_z/\xi_0)\chi_{dia}^{(2D)}$, where \mathcal{M}_{2D} and $\chi_{dia}^{(2D)}$ are given by Eqs. (45) and (46).

These results are shown in Figures 3 and 4. For q2D systems, the magnetization shape function $m = \mathcal{M}/\rho S$ is given by $m(t, b) = -\partial\tilde{S}(0, t, b)/\partial b$, where $\tilde{S}(0, t, b)$ is equal to $S(0, t, b)$ for $\pi k_B/\alpha_c = 1$; see Eq. (30). The function $m(t, b)$ is shown in Figure 3 for $t = (0.0001, \dots, 0.001) \ll b = (0.001, \dots, 0.01)$; q2D systems. When b tends to 0.01, the variations of $m(b \sim 0.01, t)$ with $t \in (10^{-5}, 10^{-3})$ are relatively small compared to those for $t \sim b \sim 10^{-3}$. At given small t , $t \sim 10^{-4}$ in Figure 3, χ slowly increases with the decrease of b following the linear law (45), and tends to $-\ln 2/2 \approx 0.345$ for $b = 10^{-3}$ in accord with Eq. (45); a result, firstly achieved in Ref. [25].

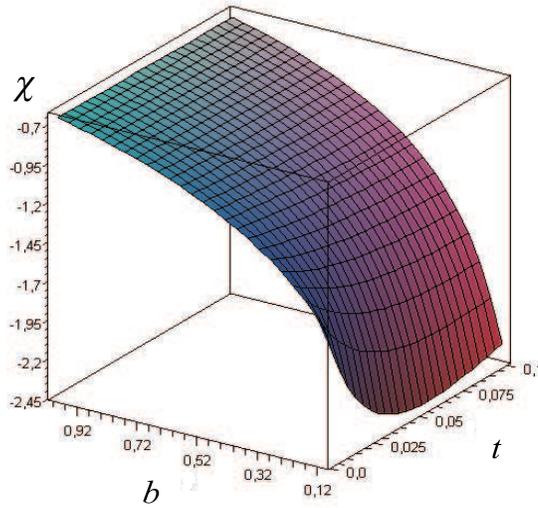


Figure 4. The susceptibility shape function $\chi(t, b)$ for q2D systems: $t = (0.001, \dots, 0.1)$ and $b = (0.1, \dots, 1)$.

The susceptibility shape function $\chi(t, b)$ of q2D systems is shown in Figure 4 for $t = (0.001, \dots, 0.1)$ and $b = (0.1, \dots, 1)$. As seen from Figure 4, the shape function $\chi(t, b)$ remains finite, provided $b \gg t$ even when b tends to zero. Besides, as we have shown analytically for $b \ll 1$, in the large- B limit this function virtually does not depend on t for any fixed $0 < b < 1$. In accord with our analytical result Eq. (46), valid for $t \ll b \ll 1$, Figure 4 shows that at fixed b , the function $\chi(t, b)$ is almost constant for variations of t under the condition $t \ll b$. At fixed $t \ll b$, however, the variations of the function $\chi(b)$ are substantial, in particular, for $b \ll 1$. When t tends to zero, the function $\chi(t \sim 0, b)$ is bounded from below at $-11/6$, as seen from both Eq. (46) and Figure 4.

4.3 Discussion of the Results: Application to p -Wave Ferromagnetic Superconductors

The results for the free energy F , the diamagnetic moment \mathcal{M} , and the diamagnetic susceptibility χ_{dia} are very similar to the respective known results for conventional non-magnetic superconductors [16]. In particular we point out the dependence of these physical quantities on the parameters t and b , describing the departure of thermodynamic states from the phase transition line $T_c(P)$. For

non-magnetic superconductors ($\mathbf{M} \equiv 0$), $\mathbf{B} = \mathbf{H}$, $T_c(M = 0) = T_{c0}$ is the usual superconducting critical temperature at zero external magnetic field, and we reveal the known results for 3D and q2D standard superconductors, summarized in the review [16].

In p -wave ferromagnetic superconductors, the magnetization \mathbf{M} in zero external magnetic field \mathbf{H} is different from zero along the whole phase transition line $T_c(P)$ and the shape of critical temperature $T_c(P)$ in zero external magnetic field \mathbf{H} strongly depends on the magnetization \mathbf{M} , as given by Eq. (25). Thus, except for a very narrow domain of the $T - P$ phase diagram above the critical pressure P_c , the magnetization above the line $T_c(P)$ is always large and, hence, for this case, we should consider large values of the induction B even when the external magnetic field \mathbf{H} is small or equal to zero. The important quantity in our consideration is the induction B , because the latter enters in the magnetic frequency ω_B and the magnetic length a_B . Now the magnetic induction B plays a role similar to that of external magnetic field \mathbf{H} in the theory of diamagnetic moment and diamagnetic susceptibility in usual superconductors [16, 19]. Therefore, for p -wave ferromagnetic superconductors with phase diagrams of the types shown in Figure 1, we should use only those of our results, which correspond to the large- B limit. Our results show that both diamagnetic moment and diamagnetic susceptibility do not exhibit any singularity and remain finite up to $T = T_c(P)$ along the most part of the FM-FS phase transition line $T_c(P)$. This means that the diamagnetic singularities are dumped by the ferromagnetic order.

The results in the weak- B limit could be valid in a close vicinity of the critical pressure P_c , where the magnetization M of the ferromagnetic phase is very small and the criterion for weak- B limit is fulfilled: $(H + 4\pi M) \ll B_0$. Then the results in this limit will be valid for enough small external field H and $M \simeq M_c = M(T_c) \ll B_0$. In this case, as mentioned in Section IV.A.1, the diamagnetic moment $\mathcal{M} < 0$ will exist even for $H = 0$ and will be proportional to the ferromagnetic moment $M \simeq M_c$; see Eq. (35). According to Eq. (35), the 3D superconductor will have a negative total magnetic momentum $M_{tot} = \mathcal{M} + VM$ at $H = 0$ provided

$$\frac{\pi\rho}{3B_0\xi_0} > t^{1/2}. \quad (47a)$$

According to Eq. (38), the total magnetic moment in 2D superconductors will be negative, $M_{tot} = \mathcal{M} + SM < 0$, provided

$$\frac{2\pi\rho}{3B_0} > t. \quad (47b)$$

When the criteria (47a) and (47b) are satisfied the diamagnetism prevails and the overall magnetization of the system is negative. However, under certain conditions, some relevant fluctuation contribution of the magnetization vector

M may occur, and this is an issue which need a study beyond the Gaussian approximation. A reliable application of our results in the weak- B limit could be performed in ferromagnetic superconductors, where a line of phase transition of type N-FS exists, *i.e.*, when the lines of the N-FM and FM-FS phase transitions of first order meet at some finite temperature critical-end point $(T, P'_c \sim P_c)$ which is connected with the zero temperature point $(0, P_c)$ by a second-order (N-FS) phase transition line (Section 1).

5 Conclusion

Introducing an advanced theoretical approach, we have been able to investigate the basic properties of the superconducting fluctuations in p -wave ferromagnetic superconductors in zero external magnetic field. For the presence of a strong magnetization due to the ferromagnetic state, the p -wave ferromagnetic superconductors with $T_F(P) \gg T_c(P)$ exhibit a diamagnetic behavior, which is typical for usual superconductors in the strong- H limit. For this type of ferromagnetic superconductors we have demonstrated a form of universality. It is known [16] that at a strong external field \mathbf{H} , the diamagnetic quantities do not exhibit singularities. Here the same quantities undergo the same dumping, *i.e.* lack of singularities in the strong- B limit, including their values at external field equal to zero.

In the weak- B limit the diamagnetic moment and the diamagnetic susceptibility exhibit scaling singularities with respect to the parameter t of type known from the theory of non-magnetic superconductors [17]. As demonstrated in Ref. [17] for 3D geometry, and here for both 2D and 3D superconductors, the scaling amplitudes for p -wave ferromagnetic superconductors are quite different from the known scaling amplitudes for nonmagnetic superconductors [16]. The difference is due to the existence of ferromagnetic moment M above the FM-FS phase transition line $T_{FS}(P)$; see Figure 1. Therefore, our new results in the weak- B limit may have application to p -wave superconductors with $T - P$ diagrams of shape shown in Figure 1, where the FM-FS phase transition line lies below the N-FM phase transition line. Besides, in order to apply the weak- B limit results, the ferromagnetic moment M should be enough small. Thus, the results for weak- B could be applied only in a close vicinity of the critical pressure P_c , where the N-FM and FM-FS phase transition lines are very close to each other and the ferromagnetic states between them possess a small magnetization M . If the N-FM phase transition in this domain of $T - P$ diagram is of first order, as indicated by the experimental data, the ferromagnetic fluctuations are suppressed and could not affect on the fluctuation diamagnetism.

The weak- B limit may be applied to p -wave superconductors containing a N-FS line of phase transition. Then the fluctuation diamagnetism in the N -phase will be described by the known formulae [16]. For the lack of ferromagnetic moment

in the N-phase ($M = 0, B = H$), in such cases we should consider weak- H limit [16]. The physics of p -wave ferromagnetic superconductors is not limited to the ferromagnetic compounds enumerated in this paper and those discovered until now. In future, new substances exhibiting p -wave ferromagnetic superconductivity with different shape of the $T - P$ phase diagram may be discovered. The theory predicts a variety of possible $T - P$ phase diagrams, including both cases with $T_F > T_s$ and $T_s > T_F$. In the last case, stable pure (non-magnetic) phases are possible [7]. This means that the results for the weak- H may have a wider application.

Except for the location of the phase transition line $T_c(P)$ at $\mathbf{H} = 0$, given by Eq. (25), all results for the diamagnetic quantities p -wave superconductors in Gaussian approximation can be obtained from the known results for usual (non-magnetic) superconductors by the substitution $\mathbf{H} \rightarrow \mathbf{B}$. This is a form of universality, deduced in the present paper.

We have used the Gaussian approximation, which is not valid in the critical region [18] of anomalous fluctuations. As the critical region of real ferromagnetic superconductors with spin-triplet electron pairing is often very narrow and, hence, virtually of no interest, the present results can be reliably used in interpretation of experimental data for real itinerant ferromagnets, which exhibit low-temperature p -wave superconductivity triggered by the ferromagnetic order.

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