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Description of Alternating Parity Bands in Deformed Even-Even Nuclei in the Symplectic Extension of the Interacting Vector Boson Model

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Abstract. We revise the classification of even and odd parity states in the symplectic multiplets of the $Sp(12, R)$ dynamical symmetry of the Interacting Boson Model /IVBM/. This new classification is tested in an application of the model for the description of different parity low laying bands. We analyze the behavior of the first excited 0^- band in respect to the yrast and some of the other excited 0^+ bands. Their energies are well reproduced by means of a simple generalization of the rigid rotor expression for them. The relations between the phenomenological parameters of the model Hamiltonian and the generalized rotor are obtained. The dependence of the model parameters on the collective structure of the band head configurations is explored and analyzed.

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1 Introduction

In this work we aim to review again the application of one of the dynamical symmetries [1] of the symplectic extension of the Interacting Vector Boson Model (IVBM) [2], which has been used for the description of the behavior of the ground and octupole bands in the heavy even-even nuclei. We revise the assignment of sequences of states classified in the boson representation of the $Sp(12, R)$ algebra [3] to the experimentally observed low laying even and odd parity states clarifying in particular the definition of their parity from the physical point of view. This new approach does not simplify the description of these bands, but increases its accuracy up to rather high spins. It includes the consideration of some additional excited 0^+ bands, which as is empirically observed

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strongly influence the behavior of the ground and octupole bands [4]. They cross at some point the ground state band and so form the higher laying parts of the yrast band. In many cases the negative parity 0^- band is parallel to these bands, which simplifies to a large extend their description, which is analogous to the one of the positive parity bands [4]. We show that the energies of the considered collective bands are strongly dependent on the number of bosons that build their band head configurations [5]. This once again outlines the advantage of the symplectic extension of the IVBM, which is based on the construction of the basis states from different number of boson excitations.

2 Algebraic Structure of the U(6) Limit of the Interacting Vector Boson Model

The algebraic structure of the IVBM is realized in terms of creation (annihilation) operators $u_m^\dagger(\alpha)(u_m(\alpha))$, in a 3-dimensional oscillator potential $m = 0, \pm 1$ of two types of bosons, differing by the value of the projection $\alpha = 1/2(p)$ or $\alpha = -1/2(n)$ of an additional quantum number, called "pseudo-spin " T. The pseudo-spin has the properties of the F-spin in IBM-2 [6]. These operators satisfy the commutation relations

$$\left[u^m(\alpha), u_n^\dagger(\beta)\right] = \delta(\alpha, \beta)\, \delta_{m,\,n}$$

and Hermitian conjugation rules

$$\left[u_m^\dagger(\alpha)\right]^\dagger = u^m(\alpha), \qquad \left[u^m(\alpha)\right]^\dagger = u_m^\dagger(\alpha).$$

The operators $u_m^\dagger(\alpha)(u_m(\alpha))$ are by definition three-dimensional vectors ($l = 1, m = m = 0, \pm 1$) with respect to the group $O(3)$) belonging to two independent representations $(1, 0)$ of the group $SU(3)$ (the annihilation operators belong to the representation $(0, 1)$ [7]).

At this point it is important to note, that the transformation properties of the vector bosons used as building blocks of the algebraic structure of the model, define as well their **parity** as $\pi = (-1)^{l=1}$ in analogy with the Elliott's oscillator quarks [8] and the excitations of the p-shell of the microscopic shell model. In contrast to our previous assumptions [9] in this paper we will further use this definition for the construction of the algebraic structure and basis states.

In general the bilinear products of the creation and annihilation operators of the two vector bosons generate the boson representations of the non-compact symplectic group $Sp(12, R)$ [2]

$$\begin{aligned} F^{LM}(\alpha, \beta) &= \sum\nolimits_{k,m} C_{1k1m}^{LM} u_k^\dagger(\alpha) u_m^\dagger(\beta), \\ G^{LM}(\alpha, \beta) &= \sum\nolimits_{k,m} C_{1k1m}^{LM} u_k(\alpha) u_m(\beta), \end{aligned} \qquad (1)$$

$$A^{LM}(\alpha,\beta) = \sum\nolimits_{k,m} C^{LM}_{1k1m} u^{\dagger}_k(\alpha) u_m(\beta), \qquad (2)$$

where C^{LM}_{1k1m}, the usual Clebsch-Gordon coefficients for $L = 0,1,2$ and $M = -L, -L+1, ...L$, define the transformation properties of (1) and (2) under rotations. The commutation relations between the pair creation and annihilation operators (1) and the number preserving operators (2) are calculated in [7]. The set of operators (2) close under commutation the algebra of the maximal compact subgroup of $U(6) \subset Sp(12,R)$ [3]. The generators of the symplectic group $Sp(12,R)$ can be further defined as irreducible tensor operators according to the chain of subgroups [10]

$$U(6) \supset SU(3) \otimes U(2) \supset SO(3) \otimes U(1)$$

and expressed in terms of the operators (1) and (2).

$$F^{[\chi]_6 \;\; LM}_{[\lambda]_3[2T]_2\, TT_0} = \sum_{\alpha,\beta} C^{[1]_6 \;\; [1]_6 \;\; [\chi]_6}_{[1]_3[1]_2 [1]_3[1]_2 [\lambda]_3[2T]_2} C^{[1]_3[1]_3[\lambda]_3}_{(1)_3(1)_3(L)_3} C^{TT_0}_{\frac{1}{2}\alpha\frac{1}{2}\beta} F^{LM}(\alpha,\beta), \qquad (3)$$

$$G^{[\chi]_6 \;\; LM}_{[\lambda]_3[2T]_2\, TT_0} = \sum_{\alpha,\beta} C^{[1]^*_6 \;\; [1]^*_6 \;\; [\chi]_6}_{[1]^*_3[1]^*_2 [1]^*_3[1]^*_2 [\lambda]_3[2T]_2} C^{[1]^*_3[1]^*_3[\lambda]_3}_{(1)_3(1)_3(L)_3} C^{TT_0}_{\frac{1}{2}\alpha\frac{1}{2}\beta} G^{LM}(\alpha,\beta), \qquad (4)$$

$$A^{[\chi]_6 \;\; LM}_{[\lambda]_3[2T]_2\, TT_0} = \sum_{\alpha,\beta} C^{[1]_6 \;\; [1]^*_6 \;\; [\chi]_6}_{[1]_3[1]_2 [1]^*_3[1]^*_2 [\lambda]_3[2T]_2} C^{[1]_3[1]^*_3[\lambda]_3}_{(1)_3(1)_3(L)_3} C^{TT_0}_{\frac{1}{2}\alpha\frac{1}{2}\beta} A^{LM}(\alpha,\beta), \qquad (5)$$

where, according to the lemma of Racah [11], the Clebsch-Gordan coefficients along the chain are factorized by means of isoscalar factors (IF), defined for each step of decomposition of $U(6)$. It should be pointed out [12] that the $U(6)-$ $C^{[\chi_1]_6 \;\; [\chi_2]_6 \;\; [\chi]_6}_{[\lambda_1]_3[2T_1]_2 \; [\lambda_2]_3[2T_2]_2 \; [\lambda]_3[2T]_2}$ and $U(3) - C^{[\lambda_1]_3 \; [\lambda_2]_3 \; [\lambda]_3}_{(l_1)_3 \; (l_2)_3 \; (L)_3}$ IF's, entering in (3), (4) and (5), are equal to ± 1 and their values are taken into account in what follows. The linear invariant of $U(6)$ is the number operator

$$N = \sqrt{3}(A^0(p,p) + A^0(n,n)) = N_p + N_n, \qquad (6)$$

that counts the total number of bosons. Being the first order invariant of $U(6)$, the operator (6) splits the boson representations of $Sp(12,R)$ into a countless number of symmetric unitary irreducible representations /UIR/ of the type $[N,0,0,0,0,0] = [N]_6$, where $N = 0,2,4,...$ for the even UIR and

$N = 1, 3, 5, ...$ for the odd ones. The rest of the operators of the physical observables introduced in this limit, which define the algebra of $SU(3)$, are the truncated ("Elliott's") [8] quadrupole operator

$$Q_M = \sqrt{6} \sum_{M,\alpha} A^2_M(\alpha, \alpha), \quad M = 0, \pm 1, \pm 2$$

and the angular momentum operator with components

$$L_M = -\sqrt{2} \sum_{M,\alpha} A^1_M(\alpha, \alpha), M = 0, \pm 1$$

that generate its $SO(3)$ subalgebra. These operators are obtained exactly from the quantization of the classical momenta in the coordinate representation and further motivate the use of the vector bosons as building blocks of the model. In addition to the raising $T_+ = \sqrt{\frac{3}{2}} A^0(p, n)$ and lowering $T_- = \sqrt{\frac{3}{2}} A^0(n, p)$ components of the pseudospin T, the Cartan operators N (6) and the third projection

$$T_0 = -\sqrt{\frac{3}{2}} [A^0(p, p) - A^0(n, n)] \tag{7}$$

of the pseudospin operator T, close the pseudospin algebra $su(2)$. These operators play an important role in the consideration of the nuclear system as composed by two interacting subsystems.

Obviously the $su(2)$ generators commute with the $su(3)$ ones Q_M and L_M, so that the two algebras are mutually complementary. Since the reduction from $U(6)$ to $SO(3)$ is carried out by the direct product of the groups $SU(3)$ and $U(2)$, their quantum numbers are related in the following way:

$$T = \frac{\lambda}{2}, \qquad N = \lambda + 2\mu. \tag{8}$$

As a result of the above considerations, the rotational limit [13] of the number preserving version of the model defined by the chain

$$Sp(12, R) \supset U(6) \supset SU(3) \otimes U(2) \supset SO(3) \otimes U(1) \tag{9}$$

$$[N] \quad (\lambda, \mu) \quad (N, T) \; K \quad L \qquad T_0 \tag{10}$$

is embedded into the $Sp(12, R)$- group of dynamical symmetry of the IVBM. The labels below the subgroups are the quantum numbers (10) corresponding to their irreducible representations. Hence, all the possible irreducible representations: $[N]$ of $U(6)$, (λ, μ) of $SU(3)$ and KLM, which define the multiplicity of the the angular momentum L and its projection M in the final reduction to the $SO(3)$ representations are determined uniquely through all possible sets of the eigenvalues of the Hermitian operators $\mathbf{N}$, $\mathbf{T^2}$, $\mathbf{T_0}$ and $\mathbf{L^2}$ reducing the symplectic extension $sp(12,R) \supset u(6)$ [1] to "the unitary" limit of the model [2]. Making use of the latter we can write the basis as

$$|\ [N]_6; (\lambda,\mu); K, L, M; T_0\rangle = |\ (N,T); K, L, M; T_0\rangle \qquad (11)$$

The ground state of the system is the vacuum state $|0\rangle$ with $N = 0, T = 0, K = 0, L = 0, M = 0, T_0 = 0$.

The decomposition rules used to obtain the representation labels of the basis states constructed by $N = 0, 2, 4, 6...-$even or odd $N = 1, 3, 5, 7...-$ vector bosons are

$$T = \frac{N}{2}, \frac{N}{2} - 1, \frac{N}{2} - 2, ..., 0 \vee 1$$

and

$$T_0 = -T,\ -T + 1, ..., T - 1,\ T.$$

The values of the $SU(3)$ quantum numbers are obtained from their relations with the eigenvalues of the operators N and T (8) or from the decomposition rules for the $U(6) \supset U(3) \otimes U(2)$ reduction [14]

$$[N]_6 = \bigoplus [N - i, i, 0]_3.[N - 2i]_2; \qquad i = 0, 1 \ldots < N/2 >, \qquad (12)$$

where $\lambda = N - 2i, \mu = i$ $(T = (N - 2i)/2)$ and $< N/2 >$ denotes the smaller integer part of the ratio. The obtained values (λ, μ) facilitate the well known final reduction of $SU(3)$ to the $SO(3)$ algebra of the angular momentum.

$$K = \min(\lambda,\mu),\ \ \min(\lambda,\mu)\ - 2, \ldots, 0 \text{ or } 1$$

$$\begin{array}{lll} K = 0 \rightarrow L = \max(\lambda,\mu), & L = \max(\lambda,\mu)\ - 2, ..., & 0, \text{or}\ \ 1 \\ K \neq 0 \rightarrow L = \max(\lambda,\mu) + K, & L = \max(\lambda,\mu) + K - 1, ..., & 0, \text{or}\ \ 1. \end{array}$$

The index K appearing in this reduction is related to the projection of L in the body fixed frame and is used with the parity to label the different bands in the energy spectra of the nuclei. The parity of the states in this application of IVBM is defined as $\pi = (-1)^N$, since N gives the number of the creation operators of the vector bosons with negative parity $\pi = (-1)^{l=1}$ that build the basis states. This allows us to describe both positive and negative parity bands.

The basis states associated with the even irreducible representation of the $Sp(12,R)$ with $N-$ even and positive parity can be constructed by the application on it of powers of the raising generators $F^L_M(\alpha,\beta)$ (3) of the same group. Each raising operator will increase the number of bosons N by two. Acting on each of these states with the creation operators $u^\dagger_m(\alpha)$ we obtain the respective states of the odd representation of $Sp(12,R)$ with negative parity. Obviously by definition, the even parity states will belong to the even representations of $Sp(12,R)$ and the negative parity states to the odd ones. The classification schemes for the even ($N-$ even) and odd ($N-$ odd) $SU(3)$ boson representations are shown in Tables 1 and 2, respectively. To each fixed $U(6)$ representation N in the tables correspond all the possible values $T = \frac{N}{2}, \frac{N}{2} - 1, \ldots 0$ for the pseudospin T of the $U(2)$ algebra, given in the column next to the respective value of N, and labeling the rows of the tables. Thus when N and T are fixed, $2T + 1$ equivalent representations of the group $SU(3)$ arise. Each of them is distinguished by the eigenvalues of the operator $T_0 : -T, -T+1, ..., T$, defining the columns of Tables 1 and 2. The same $SU(3)$ representations (λ, μ) arise for the positive and negative eigenvalues of T_0.

Table 1: Classification of the basis states in the even H_+ space of $Sp(12, R)$ in the $U(6)$ limit of the IVBM.

N T	$T_0 \cdots \pm 4$	± 3	± 2	± 1	0
0 0				$\swarrow F^{[2]_6}_{[2]_3[2]_2}$	$(0,0)$
2 1			$F^{[2]_6}_{[1,1]_3[0]_2} \downarrow$	$\Longrightarrow$ $(2,0)$	$(2,0)$
0				$A^{[1-1]_6}_{[2,1]_3[0]_2}$ $-$	$(0,1)$
4 2			$(4,0)$	$(4,0)$	$(4,0)$
1			$-$	$A^{[1-1]_6}_{[2,1]_3[2]_2} \Downarrow$ $(2,1)$	$(2,1)$
0			$-$	$-$	$(0,2)$
6 3		$(6,0)$	$(6,0)$	$(6,0)$	$(6,0)$
2	$A^{[1-1]_6}_{[0]_3[2]_2}$	$-$	$(4,1)$	$(4,1)$	$(4,1)$
1	$\rightarrow$	$-$	$-$	$(2,2)$	$(2,2)$
0		$-$	$-$	$-$	$(0,3)$
8 4	$(8,0)$	$(8,0)$	$(8,0)$	$(8,0)$	$(8,0)$
3	$-$	$(6,1)$	$(6,1)$	$(6,1)$	$(6,1)$
2	$-$	$-$	$(4,2)$	$(4,2)$	$(4,2)$
1	$-$	$-$	$-$	$(2,3)$	$(2,3)$
0	$-$	$-$	$-$	$-$	$(0,4)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Now it is clear which of the tensor operators act as transition operators between the basis states ordered in the classification scheme presented in Tables 1 and 2. The action of the tensor operators on the $SU(3)$ vectors inside a given cell or between the cells of the tables is also schematically presented on it with corresponding arrows. The operators $F^{[2]_6}_{[\lambda]_3[2T]_2}{}^{LM}_{TT_0}$ (3) with $T_0 = 0$ give the transitions between two neighboring cells ($\downarrow$) from one column, while the ones with $T_0 = \pm 1$ change the column as well ($\swarrow$). The tensors $A^{[1-1]_6}_{[2,1]_3[0]_2}$ (5), which correspond to the $SU(3)$ generators do not change the $SU(3)$ representations (λ, μ), but can change the angular momentum L inside it ($\Longrightarrow$). The $SU(2)$ generating tensors $A^{[1-1]_6}_{[0]_3[2]_2}$ change the projection $T_0 (\rightarrow)$ of the pseudospin T and in this way distinguish the equivalent $SU(3)$ irreps belonging to the different columns of the same row of Tables 1 and 2. Inside a given cell the transition

Table 2: Same as in Table 1 for the H_- space of $Sp(12,R)$.

N T	$T_0 \cdots \pm {}^9\!/_2$	$\pm {}^7\!/_2$	$\pm {}^5\!/_2$	$\pm {}^3\!/_2$	$\pm {}^1\!/_2$
0 0				$\swarrow u^{\dagger\,[1]_6}_{[1]_3[1]_2}$	$(1,0)$
3 ${}^3\!/_2$			$F^{[2]_6}_{[1,1]_3[0]_2}$ $\downarrow$	$\Longrightarrow$ $(3,0)$	$(3,0)$
${}^1\!/_2$				$A^{[1-1]_6}_{[2,1]_3[0]_2}$ $-$	$(1,1)$
5 ${}^5\!/_2$			$(5,0)$	$(5,0)$	$(5,0)$
${}^3\!/_2$			$-$	$A^{[1-1]_6}_{[2,1]_3[2]_2}$ $\Downarrow$ $(3,1)$	$(3,1)$
${}^1\!/_2$			$-$	$-$	$(1,2)$
7 ${}^7\!/_2$		$(7,0)$	$(7,0)$	$(7,0)$	$(7,0)$
${}^5\!/_2$	$A^{[1-1]_6}_{[0]_3[2]_2}$ $\rightarrow$	$-$	$(5,1)$	$(5,1)$	$(5,1)$
${}^3\!/_2$		$-$	$-$	$(3,2)$	$(3,2)$
${}^1\!/_2$		$-$	$-$	$-$	$(1,3)$
9 ${}^9\!/_2$	$(9,0)$	$(9,0)$	$(9,0)$	$(9,0)$	$(9,0)$
${}^7\!/_2$	$-$	$(7,1)$	$(7,1)$	$(7,1)$	$(7,1)$
${}^5\!/_2$	$-$	$-$	$(5,2)$	$(5,2)$	$(5,2)$
${}^3\!/_2$	$-$	$-$	$-$	$(3,3)$	$(3,3)$
${}^1\!/_2$	$-$	$-$	$-$	$-$	$(1,4)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

between the different $SU(3)$ irreps ($\Downarrow$) is realized by the operators $A^{[1-1]_6}_{[2,1]_3[2]_2}$ that represent the $U(6)$ generators. In the physical applications sequences of $SU(3)$ vectors are attributed to sequences of collective states belonging to different bands in the nuclear spectra. By means of the above analysis [10], the appropriate transition operators can be defined as appropriate combinations of the tensor operators given in (3), (4) and (5).

The Hamiltonian, corresponding to this limit [1] of the IVBM, is expressed in terms of the first and second order invariant operators of the different subgroups in the chain (9). As a result of the complementarity of the $SU(3)$ and $U(2)$ groups, the Casimir operator of $SU(3)$ with eigenvalue $(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu)$ is expressed in terms of the operators N and T, so we use the expression

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 \pi_3 + \alpha_1 T_0^2, \tag{13}$$

where π_3 is the $SO(3)$ second order Casimir operator. H (13) is obviously diagonal in the basis (11) labelled by the quantum numbers of the subgroups of the chain (9). Its eigenvalues are the energies of the basis states of the boson representations of $Sp(12,R)$:

$$E((N,T),L,T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2. \tag{14}$$

The subspaces $[N,0,0,0,0,0] = [N]_6$, where $N = 0,2,4,\ldots$ and $N = 1,3,5,\ldots$ given in Tables 1 and 2 are finite dimensional, which simplifies the problem of diagonalization. Therefore the complete spectrum of the system can be calculated through the diagonalization of the Hamiltonian in the subspaces of all the UIR of $U(6)$, belonging to a given UIR of $Sp(12, R)$, which further clarifies its role of a group of dynamical symmetry.

3 Application to the Description of Positive and Negative Collective Bands Energies

The rotational spectra of some even-even nuclei in the rare earth and light actinide region exhibit, next to the ground band, a negative parity band with $K^{\pi} = 0^{-}$, which consists of the states with $L^{\pi} = 1^{-}, 3^{-}, 5^{-}, \ldots$. Such nuclei are supposed to have an asymmetric shape under reflection, associated with a static octupole deformation, which determines this new collective feature of the nuclear system. These two bands are displaced from each other, which means that fluctuations back to space symmetric shapes must also be significant. Experimentally the presence of "octupole" bands for some isotopes from the light actinide and rare earth region [15] is firmly established.

The most important application of the $U(6) \subset Sp(12, R)$ limit of the theory is the possibility it affords for describing both even and odd parity bands up to very high angular momentum. In order to do this we first have to identify the experimentally observed bands with the sequences of basis states from the even and odd representation of $Sp(12, R)$ given in Tables 1 and 2. Due to the new definition of the parity of the vector bosons we redefine in what follows the assignment of the experimentally observed states to the corresponding even and odd eigenvalues of the number of vector bosons N with the corresponding set of pseudospins T, which uniquely define the $SU(3)$ irreps, from the boson representations of the number preserving $U(6)$ symmetry, given in the tables.

The ground $(K^{\pi} = 0^{+})$ and octupole $(K^{\pi} = 0^{-})$ bands up to rather high spins in several nuclei from the rare-earth and actinide region were described rather successfully in [1], where an application of the theory was realized for both bands mapped onto $SU(3)$ basis states only from the even representation $N-$ even of $U(6)$. Now we will employ the correct definition of the parity of the vector bosons and respectively of the parity of the basis states, by considering both the even H_{+} and odd H_{-} representations of $Sp(12, R)$.

We consider first the ground band, starting with the vacuum state

$$\mid (N, T); K, L, M; T_0\rangle = \mid (0, 0); 0, 0, 0; 0\rangle,$$

and extending along the $SU(3)$ multiplets $(\lambda, 0)$ with $L = 0, 2, 4, \ldots$. In this case from the reduction rules follows that $N = L$ and the pseudospin eigenvalues are $T = N/2 = L/2$ in the column labelled by $T_0 = 0$ of Table 1. This assignment is rather similar to the one of the rotational $SU(3)-$ limit of the Interacting Boson Model [6] and correctly reflects the increase of the quadrupole deformation through the increase of $\lambda \gg \mu$ [16] with the increase of the angular momentum L.

Thus, taking into account the reduction rules, relating N, T and L with $T_0 = 0$, the energy (14) can be rewritten only in terms of the angular momentum L in the following way:

$$E_g(L) = \beta_g L(L + \Omega_g), \tag{15}$$

where

$$\beta_g = b + \frac{\alpha_3}{4} + \beta_3, \tag{16}$$

which gives the inertia parameter of the band, and

$$\Omega_g = \frac{a + \alpha_3/2 + \beta_3}{\beta_g} \tag{17}$$

is a geometrical parameter, describing the deviation of the bands'energies from the rigid rotor behavior. In this way the 4 parameter expression for the gsb energies is reduced to a simplified 2 - parameter expression (15) for the energies of the gsb, which is analogous to the one obtained empirically in [17] on the basis of the classification scheme, introduced in [18]. The geometrical parameter (17) is related there to the experimental ratio:

$$R_2(\Omega_g) = \frac{E_g(4)}{E_g(2)} = 2 + \frac{4}{2 + \Omega_g} \tag{18}$$

and is proven to be a good indicator for the nuclear collectivity [17] incorporating the limiting cases of the rigid rotor with $\Omega_g = 1$ $(a = b - \frac{\alpha_3}{4})$ and the harmonic oscillator limit where $\Omega_g \rightarrow \infty$. Here the model parameters of the IVBM are related to the generalized rotor parameters (16), (17) and in this way reduced to only two for the gsb. It is important to note that in this assignment of basis states to the states of the ground band the values of the pseudo-spin T is maximal for the given $N = L$ and changes for each of the band states. In contrast to our previous considerations [4], [19], this leads to the dependence of the parameters β_g (16) and Ω_g (17) on it.

Further we generalize the above identification of the ground state band to the other excited 0^+ bands, with $\mu_0 = 2, 4, 6...-$even fixed and $\lambda_0 = \mu_0$. We introduce the value of $\lambda = \lambda_0 + 2i$, with $i = 0, 1, 2, ...$ and $L = 2i$ for the $K = 0$ value in the $SU(3)$ multiplets labelled by $(\mu_0 + 2i, \mu_0)$. Hence $N = \lambda_0 + 2\mu_0 + 2i = N_0 + L$ or $L = N - N_0$. In this case for each L, T is changing as well, with $T = \dfrac{\lambda}{2} = \dfrac{(\lambda_0 + L)}{2}$. For $K \neq 0 = 2, 4, 6 \ldots$ the connection is $L = \mu_0 + i$ and $N = \lambda_0 + 2\mu_0 + 2i = \dfrac{2N_0}{3} + i + L$, taking into account that $\lambda_0 = \mu_0$. This cases correspond to the assignment of $K^\pi = 2^+(\gamma-\text{band}), 4^+, 6^+, \ldots$ excited positive parity bands.

If we substitute the relations for the 0^+ excited bands in (14) we obtain for their energies the expression:

$$E_{\beta(L)} = \beta_g L(L+\Omega_g) + C_\beta N_0(N_0+\omega_\beta) + D_\beta N_0 L, \qquad (19)$$

where $C_\beta = b + \dfrac{\alpha_3}{36}$ and $\omega_\beta = \dfrac{(a+\alpha_3/6)}{C_\beta}$ and $D_\beta = 2b + \dfrac{\alpha_3}{6}$.

Obviously these energies are shifted in respect to the ground state band with a constant energy, which depends on the fixed for the band value of N_0 and the parameters a, b and α_3 fitted for the ground state band of the considered even-even nucleus. The lowest such band can cross the ground state band, because the last term with the strength $(2b+\alpha_3/6)$ in (19) introduces an interaction between the two bands. After the crossing point, this excited 0^+ band becomes the second higher part of the yrast band. At the crossing point of the two bands a backbending is observed or a sudden change in ω_g, depending on N_0 [17].

The new assignment of the octupole band places it in the $N-$ odd $\boldsymbol{H}_-$ space of $Sp(12,R)$ given in Table 2. This, as explained above, ensures the negative parity of its states. Since the gsb and the lowest 0^- band are usually considered as one and the same band with alternating parity of its states, we choose a band from H_- space that is very similar to the ground state band, studied above. Hence for the octupole band we take the sequence of $SU(3)$ irreps $(\lambda,0)$ with $\mu=0-$fixed $(K=0)$ and $\lambda=\lambda_0+2i, i=0,1,2\ldots$, with $L=2i+1$ changing with the connection $N=\lambda=\lambda_0+2i=N_0+L-1$. In this case $T=\lambda/2$ is half-integer and also changing. Finally, for the energies of the negative parity bands we get:

$$E_{\text{oct}}(L) = \beta_{\text{oct}}\, L\,(L+\Omega_{\text{oct}}) + C_{\text{oct}} N_0(N_0+\omega_{\text{oct}}) + E_{\text{oct}}(0) + 2C_{\text{oct}} N_0 L. \qquad (20)$$

where if we introduce the constants $B=a-2b+3\alpha_3/4$ and $C=b+\alpha_3/4$ than $\beta_{\text{oct}}=C+\beta_3$ and $\Omega_{\text{oct}}=(B+\beta_3)/\beta_{\text{oct}}$, $C_{\text{oct}}=C$, $\omega_{\text{oct}}=B/C_{\text{oct}}$, $D_{\text{oct}}=2C$ and $E_{\text{oct}}(0)=-a+b+\alpha_3/2$.

From the expressions (15),(19) and (20) it is easy to see that as assigned the ground, excited 0^+ or $\beta-$ band and the 0^- octupole band have similar behavior, but differing strengths, expressed by different combination of the parameters of the Hamiltonian (14). Since we fit the parameters only for the energies of the yrast band Eg (15) in order to get an accurate description of the other bands we have to adjust their energies to the specific conditions of their assignment described above. Hence we suggest here to rescale the parameters in the energies of the excited bands, by introducing a specific dependence of the parameters on the number of bosons N_0 that build their band head configurations. The effectiveness of this proposition is illustrated in the next section, by the results of the given there applications of the theory. From the expressions for their energies (19) and (20) it can be seen that by means of the the last terms in (19) and (20), an interaction depending on L of these excited bands with the ground state bands is introduced. Its strength depends on the number of bosons N_0. Also the constant terms $C_\beta N_0(N_0 + \omega_\beta)$ in (19) and $C_{\rm oct} N_0(N_0 + \omega_{\rm oct})$ in (20) depend on N_0. The dependence of the parameters on the number of bosons that build the band head configurations of the considered excited bands is motivated microscopically by the relations of the nuclear surface oscillation model [20] giving the density distribution of the nucleus, in n-boson excited state [21] as a function of the number of monopole bosons n, that build it. In the framework of the IVBM, there is also a dynamical symmetry chain $Sp(12, R) \supset Sp(4, R) \otimes SO(3)$ [5], which gives the energy distribution of states with fixed angular momentum L as function of the number of vector bosons that build them. This chain is strongly related to the explored here dynamical symmetry (9) and further proves the importance of the boson structure of the band head configurations.

Further we investigate the fine structure effects in the collective rotational spectra of deformed even-even nuclei. The odd-even staggering patterns between ground and octupole bands have been investigated previously in [1], by means of the staggering function [22]

$$Stg(L) = 6(E_L - E_{L-1}) - 4(E_{L-1} - E_{L-2}) - 4(E_{L+1} - E_L) + E_{L+2} - E_{L+1} + E_{L-2} - E_{L-3}, \quad (21)$$

This function is a finite difference of fifth order in respect to the energy $E(L)$ and is characteristic for the deviation of the rotational behavior from that of the rigid rotor. The function (22) is sensitive to the interaction between the two bands and at the point where they cross the so called “beat” patterns are observed. A much simpler test illustrating the behavior of the positive and negative parity bands in respect to each other is the second order staggering function [23]:

$$S(L) = \frac{|E_{L+1} - E_L| - |E_L - E_{L-1}|}{E_2}, \quad (22)$$

also called a signature splitting index for an alternating parity band [24]. Such a band by definition should provide an equal spacing of the levels, so from (22) is clear that in the case when the positive and negative parity bands are parallel and close to each other the “amplitudes” of the function are almost constant and close to zero. Hence in the present investigation we use (22) as a measure of the strong interaction between the positive and negative parity bands forming the collective alternating parity band.

4 Results and Discussion

The aim of our present investigation is to explore the high spin behavior of the alternating parity bands and the interaction between the positive and negative

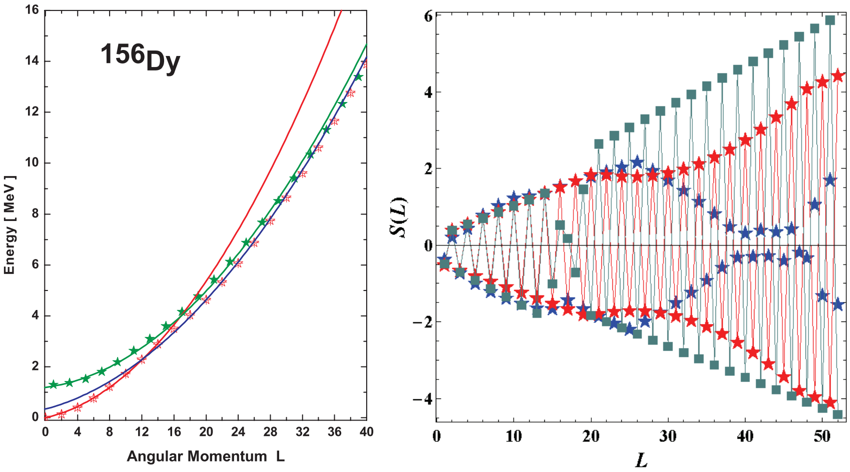

Figure 1: (Color online) Left: Experimental (symbols) and theoretical (lines) excitation energies of the ground state band $E_{\rm gr} = 0.00987259\, L\,(7.1318 + L)$ (15), first excited 0^+ band $E_\beta = 0.341952 + 0.00658172\, L\,(12.5642 + L)$ (19) and the octupole band $E_{oct} = 1.59342 + 0.00740355\, L\,(5.53277 + L)$ (20); Right: The experimental (red stars) values of the staggering functions $S(L)$ (22) between the yrast and octupole bands and theoretical values for the ground $N_0 = 0$ (gray squares) and excited 0^+ $N_0 = 6$ (blue stars) and the octupole bands $N_0 = 3$ in ^{156}Dy.

parity bands that form them. Such an investigation should be performed systematically as in [25] for the nuclei from the rare-earth and actinides but here we choose only to illustrate our interpretation of these bands on the examples of one nuclei of each region: ^{156}Dy and ^{238}U. In Figures 1 and 2 respectively we present the comparison between the experimental data and the new theoretical calculations within the $U(6)$ limit of the IVBM [1], with the redefined parity of the states and respectively assignments of the basis states to the experimentally observed one, reviewed in the previous sections. On left side of the figures are plotted the energies (14) and on the right side of the figures – the signature splitting functions of the lowest negative parity band (22) with the gsb and one of the excited 0^+ bands. A rather good agreement between the theory and experiment is obtained in the presented examples. The parameters of IVBM obtained in the overall fitting of the energies of the assigned levels to the experiment [26] are given in the figure captions.

Important characteristic introduced in this work is the dependence of the model parameters a, b, α_3, β_3 and α_1 in (14) on the number of bosons N_0 that build the band head of the excited bands. The interaction between the excited and the ground state band according to (19) and (20) also depends on N_0. In the

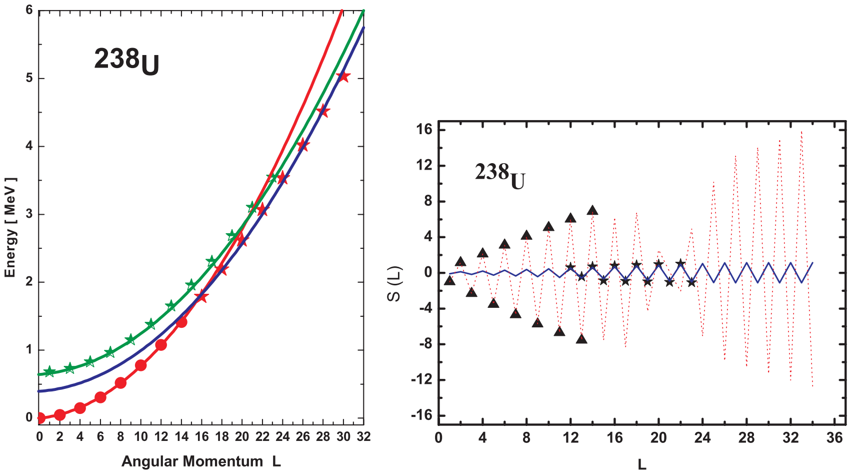

Figure 2: (Color online) Left: Experimental (symbols) and theoretical (lines) excitation energies of the ground state band $E_{gr} = 0.0062482\,L\,(2.27883 + L)$, first excited 0^+ band $E_\beta = 0.290286 + 0.00416547\,L\,(7.98107 + L)$ and the octupole band $E_{oct} = 0.633208 + 0.0047004\,L\,(3.80713 + L)$;
Right: The experimental (red dotted lines) values of the staggering functions $S(L)$ (22) between the yrast and octupole bands and theoretical values for the ground $N_0 = 0$ (black triangles) and excited 0^+ $N_0 = 6$ (black stars) and the octupole bands $N_0 = 3$ in ^{238}U.

considered nuclei these characteristics are not the same and correctly reproduce the difference in the behavior of the considered bands. These parameters for each band of the considered nuclei are given in Table 3. The investigation of the negative parity bands in the rare earths needs a more thorough and related to the microscopic structure [24] investigation, exploring staggering functions of higher order, e.g. (21).

A good characteristic of the behavior of the considered bands is the expression of their energies in terms of the generalized rotor $E(L) = \beta L(L+\Omega)+C$. The parameter Ω reveals the collectivity of the bands and clearly shows the deviation from the quadrupole rotational character of the bands. In the considered cases $\Omega > 1$ in all the collective bands, and its values reveals the influence of the vibrational collective mode on all the bands, caused by the octupole deformation of the nuclear shapes (see the captions of Figures 1 and 2). The gsb in ^{238}U is the most rotational one. The octupole bands in both cases are closer in behavior to the gsb and on the excited $\beta-$ bands the vibrations are rather prominent.

For the excited 0^+ bands in both nuclei ^{156}Dy and ^{238}U in Figures 1 and 2, $N_0 = 6$, which is actually the first band of this kind that appears in the classification of the nuclear states. For these bands, we use as a band head configuration – the

Table 3: Values of the parameters a, b, α_3, β_3 and α_1 in (14) for each of the considered bands of the nuclei ^{156}Dy and ^{238}U

Nucl.	K^π	N_0	a	b	α_3	β_3	α_1
^{156}Dy	0^+g	0	0.0608	0.0034	0.0127	0.0033	0
	$0^+\beta$	6	$\frac{a(2N_0+3)}{N_0}$	$\frac{b(2N_0+3)}{N_0}$	$\frac{\alpha_3(2N_0+3)}{N_0}$	$\frac{\beta_3(2N_0+3)}{N_0}$	0
	0^-oct.	3	$\frac{a}{N_0+1}$	$\frac{b}{N_0+1}$	$\frac{\alpha_3}{N_0+1}$	$\frac{\beta_3}{N_0+1}$	0
^{238}U	0^+g	0	0.0085	0.0024	0.0073	0.0021	0
	$0^+\beta$	6	$\frac{a(2N_0+3)}{N_0}$	$\frac{b(2N_0+3)}{N_0}$	$\frac{\alpha_3(2N_0+3)}{N_0}$	$\frac{\beta_3(2N_0+3)}{N_0}$	0.2780
	0^-oct.	3	$\frac{a(2N_0+1)}{N_0}$	$\frac{b}{N_0+1}$	$\frac{\alpha_3}{N_0+1}$	$\frac{\beta_3}{N_0+1}$	0.2780

$SU(3)$-multiplet $(2, 2)$ with $N_0 = 6$. From the behavior of the energies and the staggering functions given in the figures it could be seen that with this choice the octupole band runs in parallel with the considered excited 0^+ bands, but not with the ground state band. These two bands actually form the alternating parity band. The later band crosses at some point the ground state band and from there the respective β-band becomes the second part of the yrast band. At the crossing point a beat pattern in the staggering (21) is observed. The crossing point of the excited bands with the ground state bands appears at much higher spins - around $L = 12 - 16$ for the actinides and much lower for the rare earth nuclei. Its occurrence obviously depends on the boson structure of the band head's configuration, as the less collective states, build by smaller number of bosons shift the critical point, where the excited alternating parity bands are crossing the gsb to higher angular momenta. The behavior of the considered bands at hight spins is dominated by the behavior of the excited bands and their band head structure.

In conclusion we could summarize that in the framework of the IVBM with the present more physical assignment of the parity of the basis states and the presented new correspondence of them to the experimentally observed low laying collective 0^+ and 0^- bands, their properties for the heavy even-even nuclei from the regions of the rare-earth and the actinides are very well reproduced and interpreted in a rather simple, but physically meaningful way. It is important to proceed systematically in this way, in order to relate the observed phenomena to the microscopic and geometrical structure of the nuclei. The employed group-theoretical approach is a rather convenient tool to achieve these goals.

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