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Dynamical Volume Element in Scale-Invariant and Supergravity Theories*

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Abstract. The use in the action integral of a volume element of the form $\Phi d^D x$, where Φ is a metric-independent measure density, can yield new interesting results in all types of known generally coordinate-invariant theories: (1) 4-D theories of gravity plus matter fields; (2) reparametrization invariant theories of extended objects (strings and branes); (3) supergravity theories. In case (1) we obtain interesting insights concerning the cosmological constant problem, inflation and quintessence without the fifth force problem. In case (2) the above formalism leads to dynamically induced tension and to string models of non-abelian confinement. In case (3), we show that the modified-measure supergravity generates an arbitrary dynamically induced cosmological constant, i.e., a new mechanism of dynamical supersymmetry breaking.

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1 Introduction

In Refs. [1, 2] we have studied a new class of gravity theories based on the idea that the action integral may contain a new metric-independent measure of integration. For example, in $D = 4$ space-time dimensions the new measure density can be built out of four auxiliary scalar fields φ^i $(i = 1, 2, 3, 4)$:

$$\Phi(\varphi) = \frac{1}{4!}\varepsilon^{\mu\nu\kappa\lambda}\varepsilon_{ijkl}\partial_\mu\varphi^i\partial_\nu\varphi^j\partial_\kappa\varphi^k\partial_\lambda\varphi^l \ . \tag{1}$$

$\Phi(\varphi)$ is a scalar density under general coordinate transformations. Here we will discuss three applications:

- (i) Study of $D = 4$-dimensional models of gravity and matter fields containing the new measure of integration (1), which appears to be promising candidates for resolution of the dark energy and dark matter problems, the fifth force problem, *etc*.

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- (ii) Study of a new type of string and brane models based on employing of a modified world-sheet/world-volume integration measure. It allows for the appearance of new types of objects and effects like, for example, a spontaneously induced variable string tension.
- (iii) Studying modified supergravity models. Here we will find some outstanding new features: (a) the cosmological constant arises as an arbitrary integration constant, totally unrelated to the original parameters of the action, and (b) spontaneously breaking of local supersymmetry invariance.

2 Gravity and Cosmology Two Measures Theory

. We consider action principle of the following general form:

$$S = \int L_1 \Phi d^4 x + \int L_2 \sqrt{-g} d^4 x \;, \tag{2}$$

including two Lagrangians L_1 and L_2 and two measures of the volume elements ($\Phi d^4 x$ and the standard one $\sqrt{-g} d^4 x$, respectively). In constructing field theory with the action (2) we make only two basic additional assumptions:

(A) L_1 and L_2 are independent of the measure fields φ_i. Then the action (2) is invariant under volume-preserving diffeomorphisms on the target space of the latter [1]. Besides, it is invariant (up to an integral of a total divergence) under the infinite-dimensional group of shifts of the measure fields φ^i: $\varphi^i \to \varphi^i + f^i(L_1)$, where $f^i(L_1)$ is an arbitrary differentiable function of the Lagrangian density L_1.

(B) We proceed in the first-order formalism where all fields, including the metric $g_{\mu\nu}$ (or the vierbeins e^a_μ), connection coefficients (or spin-connection $\omega_{\mu\, ab}$) and the measure fields φ^i are *a priori* independent dynamical variables. All the relations between them follow subsequently as a result of the equations of motion.

The field theory based on the listed assumptions we call “Two Measures Theory” (TMT). It turns out that the measure fields φ_i affect the theory only via the ratio of the two measure densities $\chi \equiv \Phi/\sqrt{-g}$, which is a scalar field. It is determined by a constraint in the form of an algebraic equation, which is precisely a consistency condition of the equations of motion. *This constraint determines χ in terms of the fermion density and scalar fields*.

By an appropriate change of the dynamical variables, consisting of a conformal rescaling of the metric and a multiplicative redefinitions of the fermion fields, one can formulate the theory as a model in a Riemannian (or Riemann-Cartan) space-time. The corresponding conformal frame we call “the Einstein frame”.

We have started a detailed study of gravity-matter models with a general form for L_1 and L_2 such that the action (2) possesses both non-Abelian gauge symmetry as well as scale symmetry. For brevity, in a schematic form L_1 can be represented as ($\kappa^2 = 8\pi G_N$, G_N – Newton constant):

$$L_1 = e^{\alpha\phi/M_p}\Big[\frac{1}{\kappa^2}R(\omega,e) - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + (\text{Higgs}) + (\text{gauge}) + (\text{fermions})\Big] \quad (3)$$

and similarly for L_2 (with different choice of the normalization factors in front of each of the terms). Varying w.r.t. φ^i and assuming $\Phi \neq 0$, we get:

$$L_1 = sM^4 = \text{const} \;, \quad (4)$$

where $s = \pm 1$ and M has dimension of mass. The appearance of a nonzero integration constant sM^4 *spontaneously breaks the scale invariance* [2].

When including terms quadratic in the scalar curvature $R(\omega, e)$, these types of models can be applied not only for the late time universe, but also for the early inflationary epoch. As it has been demonstrated in Ref. [3], a smooth transition between these epochs is possible in these models. Also, these type of models provide the possibility of a non-singular “emergent” type cosmology, where the existence and stability of singularity free universe imposes an upper bound on the cosmological constant today; for a review, see Ref. [4].

3 Extended objects

Extended objects’ actions can be formulated using a modified measure analogous to (1). For simplicity we review here only the string case, where on the 2-dimensional world-sheet we introduce:

$$\Phi(\varphi) = \frac{1}{2}\varepsilon^{ab}\varepsilon_{ij}\partial_a\varphi^i\partial_b\varphi^j \;. \quad (5)$$

In Ref. [5] we have proposed the following modified-measure string action:

$$S_{\text{mstring}} = -\frac{1}{2}\int d^2\sigma\,\Phi(\varphi)\Big[\gamma^{ab}\partial_a X^\mu\partial_b X^\nu g_{\mu\nu} - \frac{\varepsilon^{ab}}{\sqrt{-\gamma}}F_{ab}\Big] \;, \quad (6)$$

where $F_{ab} = \partial_a A_b - \partial_b A_a$ with $A_a(\sigma)$ being an auxiliary abelian world-sheet gauge field. Its presence is crucial for consistency of the modified-measure string dynamics. Note that adding this term to the standard Polyakov-type string action ($\Phi(\varphi) \to \sqrt{-\gamma}$ in (6)) would make it a purely topological (total divergence) term $\frac{1}{2}\int d^2\sigma\, \varepsilon^{ab}F_{ab}$.

The action (6) is Weyl-conformally invariant under conformal rescaling of the world-sheet metric $\gamma_{ab} \to \gamma'_{ab} = J\gamma_{ab}$ combined with a diffeomorphism on the φ^i-target space $\varphi^i \to \varphi'^{\,i} = \phi^i(\varphi)$ with a Jacobian $\det \|\partial\phi^i/\partial\varphi^j\| = J$.

The equation of motion obtained from variation of (6) w.r.t. A_a is $\varepsilon^{ab}\partial_a(\Phi/\sqrt{-\gamma}) = 0$, which yields a *spontaneously induced string tension* $T = \Phi/\sqrt{-\gamma} = \mathrm{const}$. The string tension appears here as an integration constant and does not have to be introduced from the beginning. Let us stress that the string theory action (6) does not have any *ad hoc* fundamental scale parameters.

Variation of (6) w.r.t. the measure fields φ^i yields a fundamental constraint of the theory:

$$g^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\varepsilon^{ab}}{\sqrt{-\gamma}} F_{ab} = M \quad , \quad M = \mathrm{const} \; , \tag{7}$$

which allows for F_{ab} to be expressed in terms of the basic string variables. Consistency of the whole set of equations of motion demands $M = 0$, so that finally we obtain the same equations as in the standard bosonic string theory, however, with a dynamically induced "floating" string tension $T = \Phi/\sqrt{-\gamma}$.

The above modified-measure formalism can be applied to the Green-Schwarz superstring. As shown in Ref. [5] the world-sheet gauge field A_a plays crucial role for ensuring supersymmetry invariance of the modified-measure superstring theory.

4 Supergravity with Dynamically Induced Cosmological Constant

The ideas and concepts of two-measure gravitational theories [1,2] may be combined with those originating from the theory of string and branes with dynamical generation of string/brane tension [5] to consistently incorporate supersymmetry in the two-measure modification of standard Einstein gravity. Here for simplicity we will present the modified-measure construction of $N = 1$ supergravity in $D = 4$. For a recent account of modern supergravity theories and notations, see Ref. [6].

The standard component-field action of $D = 4$ (minimal) $N = 1$ supergravity reads:

$$S_{\rm SG} = \frac{1}{2\kappa^2} \int d^4x \, e \Big[R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda \Big] \; , \tag{8}$$

$$e = \det \|e^a_\mu\| \quad , \quad R(\omega, e) = e^{a\mu} e^{b\nu} R_{ab\mu\nu}(\omega) \; . \tag{9}$$

$$R_{ab\mu\nu}(\omega) = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega^c_{\mu a}\omega_{\nu cb} - \omega^c_{\nu a}\omega_{\mu cb} \; . \tag{10}$$

$$D_\nu \psi_\lambda = \partial_\nu \psi_\lambda + \frac{1}{4}\omega_{\nu ab}\gamma^{ab}\psi_\lambda \quad , \quad \gamma^{\mu\nu\lambda} = e^\mu_a e^\nu_b e^\lambda_c \gamma^{abc} \tag{11}$$

where all objects belong to the first-order "vierbein" (frame-bundle) formalism, *i.e.*, the vierbeins e^a_μ (describing the graviton) and the spin-connection $\omega_{\mu ab}$ ($SO(1,3)$ gauge field acting on the gravitino ψ_μ) are *a priori* independent fields (their relation arises subsequently on-shell); $\gamma^{ab} \equiv \frac{1}{2}\left(\gamma^a\gamma^b - \gamma^b\gamma^a\right)$ *etc.* with γ^a denoting the ordinary Dirac gamma-matrices. The invariance of the action (8) under local supersymmetry transformations:

$$\delta_\epsilon e^a_\mu = \frac{1}{2}\bar{\varepsilon}\gamma^a\psi_\mu \quad , \quad \delta_\epsilon\psi_\mu = D_\mu\varepsilon \tag{12}$$

follows from the invariance of the pertinent Lagrangian density up to a total derivative:

$$\delta_\epsilon\Big(e\big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda\big]\Big) = \partial_\mu\big[e\big(\bar{\varepsilon}\zeta^\mu\big)\big] \ , \tag{13}$$

where ζ^μ functionally depends on the gravitino field ψ_μ.

We now propose a modification of (8) by replacing the standard measure density $e = \sqrt{-g}$ by the alternative measure density $\Phi(\varphi)$ (1):

$$S_{\rm mSG} = \frac{1}{2\kappa^2}\int d^4x\,\Phi(\varphi)\,\Big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\,\partial_\mu H_{\nu\kappa\lambda}\Big] \ , \tag{14}$$

where a new term containing the field-strength of a 3-index antisymmetric tensor gauge field $H_{\nu\kappa\lambda}$ has been added. Note that its inclusion in the standard supergravity action (8) would yield a purely topological (total divergence) term like in the case of modified-measure (super)string [5] (cf. Eq. (6)).

The equations of motion w.r.t. $H_{\nu\kappa\lambda}$ and the "measure" scalars φ^i read:

$$\partial_\mu\Big(\frac{\Phi(\varphi)}{e}\Big) = 0 \quad \to \quad \frac{\Phi(\varphi)}{e} \equiv \chi = \text{const} \ , \tag{15}$$

$$R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\,\partial_\mu H_{\nu\kappa\lambda} = 2M \ , \tag{16}$$

where M is an arbitrary integration constant.

Now it is straightforward to check that the modified-measure supergravity action (14) is invariant under local supersymmetry transformations (13) supplemented by the transformation laws for $H_{\mu\nu\lambda}$ and $\Phi(\varphi)$:

$$\delta_\epsilon H_{\mu\nu\lambda} = -e\,\varepsilon_{\mu\nu\lambda\kappa}\big(\bar{\varepsilon}\zeta^\kappa\big) \quad , \quad \delta_\epsilon\Phi(\varphi) = \frac{\Phi(\varphi)}{e}\,\delta_\epsilon e \ , \tag{17}$$

which algebraically close on-shell, *i.e.*, when Eq. (15) is imposed.

The role of $H_{\nu\kappa\lambda}$ in the modified-measure action (14) is to absorb, under local supersymmetry transformation, the total derivative term coming from (13), so as to insure local supersymmetry invariance of (14) – this is a generalization of the formalism used in Ref. [5] to write down a modified-measure extension of the standard Green-Schwarz world-sheet action of space-time supersymmetric strings. Similar approach has also been employed in Refs. [7, 8].

Let us particularly stress that the appearance of the integration constant M in (16) signifies a *spontaneous (dynamical) breaking of supersymmetry* and, simultaneously, it represents a *dynamically generated cosmological constant* in the pertinent gravitational equations of motion. Indeed, varying (14) w.r.t. e^a_μ:

$$e^{b\nu}R^a_{b\mu\nu} - \frac{1}{2}\bar{\psi}_\mu\gamma^{a\nu\lambda}D_\nu\psi_\lambda + \frac{1}{2}\bar{\psi}_\nu\gamma^{a\nu\lambda}D_\mu\psi_\lambda + \frac{1}{2}\bar{\psi}_\lambda\gamma^{a\nu\lambda}D_\nu\psi_\mu + \frac{e^a_\mu}{2}\,\frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\,\partial_\mu H_{\nu\kappa\lambda} = 0 \qquad (18)$$

and using Eq. (16) to replace the last H-term on the l.h.s. of (18) we obtain the vierbein analogues of the Einstein equations including a dynamically generated *floating* cosmological constant term $e^a_\mu M$:

$$e^{b\nu}R^a_{b\mu\nu} - \frac{1}{2}e^a_\mu R(\omega,e) + e^a_\mu M = \kappa^2 T^a_\mu \; ,$$
$$\kappa^2 T^a_\mu \equiv \frac{1}{2}\bar{\psi}_\mu\gamma^{a\nu\lambda}D_\nu\psi_\lambda - \frac{1}{2}e^a_\mu\bar{\psi}_\rho\gamma^{\rho\nu\lambda}D_\nu\psi_\lambda - \frac{1}{2}\bar{\psi}_\nu\gamma^{a\nu\lambda}D_\mu\psi_\lambda - \frac{1}{2}\bar{\psi}_\lambda\gamma^{a\nu\lambda}D_\nu\psi_\mu \; . \qquad (19)$$

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