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Dynamical Couplings and Charge Confinement/Deconfinement from Gravity Coupled to Nonlinear Gauge Fields*

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Abstract. We briefly outline several main results concerning various new physically relevant features found in gravity – both ordinary Einstein or $f(R) = R + R^2$ gravity in the first-order formalism, coupled to a special kind of nonlinear electrodynamics containing a square-root of the standard Maxwell Lagrangian and known to produce charge confinement in flat spacetime.

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1 Introduction

G. 't Hooft [1] has shown that in any effective quantum gauge theory, which is able to describe QCD-like charge confinement phenomena, the energy density of electrostatic field configurations should be a linear function of the electric displacement field in the infrared region (the latter appearing as a quantum "infrared counterterm"). The simplest way to realize these ideas in flat spacetime is to incorporate into the full gauge field action an additional term being a square-root of the standard Maxwell (or Yang-Mills) gauge field Lagrangian [2–4]:

$$S = \int d^4x L(F^2) \quad , \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \, , \tag{1}$$
$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \, .$$

The "square-root" Maxwell term is naturally produced as a result of spontaneous breakdown of scale symmetry in the standard gauge theory [2]. Moreover, the (dimensionfull) coupling constant f_0 measures the strength of the effective confining potential among quantized fermions produced by (1) [3].

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In a series of recent papers [5,6] we have studied in detail physically more interesting models where the above nonlinear "square-root electrodynamics" couples to gravity – either standard Einstein gravity or generalized $f(R)$-gravity ($f(R)$ being a nonlinear function of the scalar curvature of space-time), as well as coupled to scalar dilaton field. Let us recall that $f(R)$-gravity models are attracting a lot of interest in modern cosmology as possible candidates to cure problems in the standard cosmological scenarios related to dark matter and dark energy. For a recent review, see *e.g.* [7] and references therein. The first $R + R^2$-model (in the second-order formalism) which was also the first inflationary model, was proposed by Starobinsky in [8].

Here we will briefly describe some of our main results [5,6] concerning the new physically relevant features we uncovered in the coupled gravity/"square-root" nonlinear gauge field/dilaton system (defined in Eq.(2) below):

(i) Appearance of dynamical effective gauge couplings and confinement-deconfinement transition effect as functions of the dilaton vacuum expectation value, in particular due to appearance of "flat" region of the effective dilaton potential.

(ii) New mechanism for dynamical generation of cosmological constant.

(iii) Non-standard black hole solutions with constant vacuum radial electric field with Reissner-Nordström-(anti)de-Sitter or Schwarzschild-(anti)de-Sitter type geometry and with non-asymptotically flat "hedgehog"-type spacetime asymptotics. Let us stress that constant vacuum radial electric fields do not exist as solutions of ordinary Maxwell electrodynamics.

(iv) The above non-standard black holes obey the first law of black hole thermodynamics.

(v) New "tube-like universe" solutions of Levi-Civita-Bertotti-Robinson tye [9];

(vi) Coupling to *lightlike* branes produces "*charge-hiding*" and *charge-confining* "thin-shell" wormhole solutions displaying QCD-like charge confinement (see also the previous talk [10] at this conference).

2 R + R^2-Gravity Coupled to Confining Nonlinear Gauge Field

Let us consider coupling of $f(R) = R + \alpha R^2$ gravity (possibly with a bare cosmological constant Λ_0) to a "dilaton" ϕ and the nonlinear gauge field system containing $\sqrt{-F^2}$ (1) (we are using units with the Newton constant $G_N = 1$):

$$S = \int d^4x \sqrt{-g} \Big[\frac{1}{16\pi} \Big(f\big(R(g,\Gamma)\big) - 2\Lambda_0 \Big) + L(F^2(g)) + L_D(\phi, g) \Big] \; , \quad (2)$$

$$f\big(R(g,\Gamma)\big) = R(g,\Gamma) + \alpha R^2(g,\Gamma) \quad , \quad R(g,\Gamma) = R_{\mu\nu}(\Gamma) g^{\mu\nu} \; , \tag{3}$$

$$L(F^2(g)) = -\frac{1}{4e^2} F^2(g) - \frac{f_0}{2}\sqrt{-F^2(g)} \; , \tag{4}$$

$$F^2(g) \equiv F_{\kappa\lambda} F_{\mu\nu} g^{\kappa\mu} g^{\lambda\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{5}$$

$$L_D(\phi, g) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \; . \tag{6}$$

$R_{\mu\nu}(\Gamma)$ is the Ricci curvature in the first order (Palatini) formalism, *i.e.*, the space-time metric $g_{\mu\nu}$ and the affine connection $\Gamma^\mu_{\nu\lambda}$ are *a priori* independent variables. The solution to the corresponding equation of motion w.r.t. $\Gamma^\mu_{\nu\lambda}$ – $\nabla_\lambda\left(\sqrt{-g} f'_R g^{\mu\nu}\right) = 0$ – implements transition to the "physical" Einstein-frame metrics $h_{\mu\nu}$ via conformal rescaling of the original metric $g_{\mu\nu}$:

$$g_{\mu\nu} = \frac{1}{f'_R} h_{\mu\nu} \quad , \quad \Gamma^\mu_{\nu\lambda} = \frac{1}{2} h^{\mu\kappa}\left(\partial_\nu h_{\lambda\kappa} + \partial_\lambda h_{\nu\kappa} - \partial_\kappa h_{\nu\lambda}\right) \; . \tag{7}$$

Here $f'_R \equiv \dfrac{df(R)}{dR} = 1 + 2\alpha R(g,\Gamma)$.

As shown in [6], using (7) the original $R + R^2$-gravity equations of motion resulting from (2) can be rewritten in the form of *standard* Einstein equations:

$$R^\mu_\nu(h) = 8\pi \left(T_{\rm eff}{}^\mu_\nu(h) - \frac{1}{2}\delta^\mu_\nu T_{\rm eff}{}^\lambda_\lambda(h) \right) \tag{8}$$

with effective energy-momentum tensor of the following form:

$$T_{\rm eff\,\mu\nu}(h) = h_{\mu\nu} L_{\rm eff}(h) - 2\frac{\partial L_{\rm eff}}{\partial h^{\mu\nu}} \; . \tag{9}$$

The pertinent effective "Einstein-frame" matter Lagrangian reads:

$$L_{\rm eff}(h) = -\frac{1}{4e^2_{\rm eff}(\phi)} F^2(h) - \frac{1}{2} f_{\rm eff}(\phi)\sqrt{-F^2(h)} + \frac{X(\phi,h)\big(1 + 16\pi\alpha X(\phi,h)\big) - V(\phi) - \Lambda_0/8\pi}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \tag{10}$$

with the following dynamical ϕ-dependent couplings:

$$\frac{1}{e^2_{\rm eff}(\phi)} = \frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \; , \tag{11}$$

$$f_{\rm eff}(\phi) = f_0 \frac{1 + 32\pi\alpha X(\phi,h)}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \; . \tag{12}$$

The dilaton kinetic term $X(\phi,h) \equiv -\frac{1}{2} h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ will be ignored in the sequel.

3 Dynamical Couplings and Confinement/Deconfinement in R + R^2 Gravity

In what follows we consider constant "dilaton" ϕ extremizing the effective Lagrangian (10). Here we observe an interesting feature of (10) – the dynamical couplings and effective potential are extremized *simultaneously*, which is an explicit realization of the so called "least coupling principle" of Damour-Polyakov [11]:

$$\begin{aligned} &\frac{\partial f_{\rm eff}}{\partial \phi} = -64\pi\alpha f_0 \frac{\partial V_{\rm eff}}{\partial \phi} \, , \\ &\frac{\partial}{\partial \phi}\frac{1}{e^2_{\rm eff}} = -(32\pi\alpha f_0)^2 \frac{\partial V_{\rm eff}}{\partial \phi} \to \frac{\partial L_{\rm eff}}{\partial \phi} \sim \frac{\partial V_{\rm eff}}{\partial \phi} \, , \end{aligned} \tag{13}$$

where:

$$V_{\rm eff}(\phi) = \frac{V(\phi) + \frac{\Lambda_0}{8\pi}}{1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)} \; . \tag{14}$$

Thus, at a constant extremum of $L_{\rm eff}$ (10) ϕ must satisfy:

$$\frac{\partial V_{\rm eff}}{\partial \phi} = \frac{V'(\phi)}{\left[1 + 8\alpha\left(8\pi V(\phi) + \Lambda_0\right)\right]^2} = 0 \; . \tag{15}$$

There are two generic cases:

(a) *Confining phase*: Eq.(15) is satisfied for some finite-value ϕ_0 extremizing the original "bare" dilaton potential $V(\phi)$: $V'(\phi_0) = 0$.

(b) *Deconfinement phase*: For polynomial or exponentially growing "bare" potential $V(\phi)$, so that $V(\phi) \to \infty$ when $\phi \to \infty$, we have:

$$\frac{\partial V_{\rm eff}}{\partial \phi} \to 0 \quad , \quad V_{\rm eff}(\phi) \to \frac{1}{64\pi\alpha} = \text{const} \quad \text{when} \;\; \phi \to \infty \; , \tag{16}$$

i.e., for sufficiently large values of ϕ we find a "flat region" in $V_{\rm eff}$. This "flat region" triggers a *transition from confining to deconfinement dynamics*.

Namely, in the "flat-region" phase (b) we have $f_{\rm eff} \to 0$, $e^2_{\rm eff} \to e^2$, and the effective gauge field Lagrangian in (10) reduces to the ordinary *non-confining* one (the "square-root" Maxwell term $\sqrt{-F^2}$ vanishes):

$$L^{(0)}_{\rm eff} = -\frac{1}{4e^2} F^2(h) - \frac{1}{64\pi\alpha} \tag{17}$$

with an *induced* cosmological constant $\Lambda_{\rm eff} = 1/8\alpha$, which is *completely independent* of the bare cosmological constant Λ_0.

4 Non-Standard Black Holes and "Tube-Like" Universes

In the *confining phase* (phase (a) above: ϕ_0 – generic minimum of the effective dilaton potential; $e_{\rm eff}(\phi)$), $f_{\rm eff}(\phi)$ as in (11)–(12)) we obtain several physically interesting solutions of the Einstein-frame gravity Eqs.(8) due to the special form of the effective Einstein-frame matter Lagrangian (10).

First, we find *non-standard* Reissner-Nordström-(anti-)de-Sitter-type black holes with metric:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2\big(d\theta^2 + \sin^2\theta d\varphi^2\big) \;, \tag{18}$$

$$A(r) = 1 - \sqrt{8\pi}|Q|f_{\rm eff}(\phi_0)e_{\rm eff}(\phi_0) - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\rm eff}(\phi_0)}{3}r^2 \;, \tag{19}$$

where $\Lambda_{\rm eff}(\phi_0)$ is a *dynamically generated* cosmological constant:

$$\Lambda_{\rm eff}(\phi_0) = \frac{\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2}{1 + 8\alpha\left(\Lambda_0 + 8\pi V(\phi_0) + 2\pi e^2 f_0^2\right)} \;. \tag{20}$$

The black hole's static radial electric field contains apart from the Coulomb term an *additional constant "vacuum" piece*:

$$|F_{0r}| = |\vec{E}_{\rm vac}| - \frac{Q}{\sqrt{4\pi}\,r^2}\Big(\frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha\left(8\pi V(\phi_0) + \Lambda_0\right)}\Big)^{-\frac{1}{2}} \tag{21}$$

$$|\vec{E}_{\rm vac}| \equiv \Big(\frac{1}{e^2} + \frac{16\pi\alpha f_0^2}{1 + 8\alpha\left(8\pi V(\phi_0) + \Lambda_0\right)}\Big)^{-1}\frac{f_0/\sqrt{2}}{1 + 8\alpha\left(8\pi V(\phi_0) + \Lambda_0\right)} \;. \tag{22}$$

Let us emphasize again that constant vacuum radial electric fields do not exist as solutions of ordinary Maxwell electrodynamics.

In the special case $\Lambda_{\rm eff}(\phi_0) = 0$ we obtain a non-standard Reissner-Nordström-type black hole with a "hedgehog" *non-flat-spacetime* asymptotics (cf. Refs. [12]: $A(r) \to 1 - \sqrt{8\pi}|Q|f_{\rm eff}(\phi_0)e_{\rm eff}(\phi_0) \neq 1$ for $r \to \infty$.

Apart from non-standard black hole solutions we also obtain "tube-like" space-time solutions of the Einstein-frame gravity Eqs.(8), which are of Levi-Civitta-Bertotti-Robinson type (cf. Refs. [9]) with geometries $AdS_2 \times S^2$, $Rind_2 \times S^2$ and $dS_2 \times S^2$, where AdS_2, $Rind_2$ and dS_2 are two-dimensional anti-de Sitter, Rindler and de Sitter space, respectively. The corresponding metric is of the form:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r_0^2\big(d\theta^2 + \sin^2\theta d\varphi^2\big) \;\;,\;\; -\infty < \eta < \infty \;, \tag{23}$$

carrying constant vacuum "radial" electric field $|F_{0\eta}| = |\vec{E}_{\rm vac}|$. The radius of the spherical factor S^2 is given by (using short-hand $\Lambda(\phi_0) \equiv 8\pi V(\phi_0) + \Lambda_0$):

$$\frac{1}{r_0^2} = \frac{4\pi}{1+8\alpha\Lambda(\phi_0)}\Big[\Big(1+8\alpha\left(\Lambda(\phi_0)+2\pi f_0^2\right)\Big)\vec{E}^2_{\rm vac} + \frac{1}{4\pi}\Lambda(\phi_0)\Big] \; , \qquad (24)$$

and the metric coefficient in (23) reads: $A(\eta) = 4\pi K(\vec{E}_{\rm vac})\eta^2 \; , \; K(\vec{E}_{\rm vac}) > 0$ for AdS_2; $A(\eta) = \pm\eta$ for $\eta \in (0, +\infty)$ or $\eta \in (-\infty, 0)$ for $Rind_2$; $A(\eta) = 1 - 4\pi|K(\vec{E}_{\rm vac})|\,\eta^2 \; , \; K(\vec{E}_{\rm vac}) < 0$ for dS_2, using notation:

$$K(\vec{E}) \equiv \Big(1+8\alpha\left(\Lambda(\phi_0)+2\pi f_0^2\right)\Big)\vec{E}^2 - \sqrt{2}f_0|\vec{E}| - \frac{1}{4\pi}\Lambda(\phi_0).$$

To conclude, let us particularly stress that all physically relevant features described above are exclusively due to the *combined* effect of both square-root nonlinearity $\sqrt{-F^2}$ in the gauge field Lagrangian as well as the R^2 term in the gravity action.

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