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Lightlike Membranes in Black Hole and Wormhole Physics, and Cosmology*

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Abstract. We shortly outline the principal results concerning the reparametrization-invariant world-volume Lagrangian formulation of *lightlike* brane dynamics and its impact as a source for gravity and (nonlinear) electromagnetism in black hole and wormhole physics.

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1 Introduction

Extended objects (strings and p-branes) are of primary importance for the construction of self-consistent unified modern theory of fundamental forces in Nature [1]. In a series of recent papers of ours [2–4] we have proposed for the first time in the literature a systematic world-volume Lagrangian description and studied in detail the physical properties of a new class of brane theories called *lightlike branes* (*LL-branes*), which are qualitatively distinct from the standard Nambu-Goto type brane models which describe intrinsically *massive* world-volume modes.

As it is well known, *LL-branes* (also called *null-branes*) are of substantial interest in general relativity as they describe impulsive lightlike signals arising in various violent astrophysical events, *e.g.*, final explosion in cataclysmic processes such as supernovae and collision of neutron stars. *LL-branes* also play important role in the description of various other physically important cosmological and astrophysical phenomena such as the “membrane paradigm” of black hole physics and the thin-wall approach to domain walls coupled to gravity. For a detailed account, see [5]. More recently they became significant also in the context of modern non-perturbative string theory [6].

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Here we will shortly describe some of our principal results concerning the physics of *LL-branes* and their implications in black hole and wormhole physics, and cosmology:

(a) Horizon "straddling" effect: the dynamics of *LL-branes* requires the bulk space-time geometry to possess one or more horizons, for instance, to be of black hole type, and it dictates that *LL-branes* automatically occupy (one of these) horizon(s).

(b) *LL-branes* are natural candidates for matter and charged sources of "thin-shell" traversable wormholes of various types (one- or multi-"throat" "tube-like", rotating etc.) [2].

(c) *LL-branes* naturally produce regular black holes, i.e., black holes free of "inside" (below the inner horizon) physical space-time singularities [3].

(d) *LL-branes* trigger spontaneous compactification of space-time, as well as compactification/decompactification transitions [4].

(e) *LL-branes* are consistent matter sources for lightlike braneworlds [7].

(f) *LL-branes* produce new wormhole "universes" exhibiting *charge-hiding* and *charge-confining* effects [8], physically analogous to the quark confinement mechanism in quantum chromodynamics.

2 Gravity and Nonlinear Gauge Fields Coupled to LL-Brane Sources

In Refs. [2–4, 7, 8] we proposed and extensively studied a manifestly reparametrization invariant world-volume Lagrangian action of *LL-branes*:

$$S_{\rm LL}[q] = -\frac{1}{2}\int d^{p+1}\sigma\, T b_0^{\frac{p-1}{2}} \sqrt{-\gamma}\left[\gamma^{ab}\bar{g}_{ab} - b_0(p-1)\right] , \tag{1}$$

$$\bar{g}_{ab} \equiv g_{ab} - \frac{1}{T^2}(\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b) \quad , \quad \mathcal{A}_a \equiv \partial_a X^\mu A_\mu \,. \tag{2}$$

Here and below the following notations are used:

- γ_{ab} is the *intrinsic* world-volume Riemannian metric; $g_{ab} = \partial_a X^\mu G_{\mu\nu}(X)\partial_b X^\nu$ is the *induced* metric on the world-volume, which becomes *singular* on-shell (manifestation of the lightlike nature); b_0 is world-volume "cosmological constant";
- $X^\mu(\sigma)$ are the p-brane embedding coordinates in the D-dimensional bulk space-time with Riemannian metric $G_{\mu\nu}(x)$ ($\mu,\nu = 0,1,\ldots,D-1$); $(\sigma) \equiv (\sigma^0 \equiv \tau, \sigma^i)$ with $i = 1,\ldots,p$; $\partial_a \equiv \frac{\partial}{\partial\sigma^a}$.

- u is auxiliary world-volume scalar field defining the lightlike direction of the induced metric;
- T is *dynamical (variable)* brane tension;
- q – the coupling to bulk spacetime gauge field $\mathcal{A}_\mu$ is *LL-brane* surface charge density.

The on-shell singularity, *i.e.*, the lightlike property of the induced metric g_{ab}, directly follows from the equations of motion resulting from (1):

$$g_{ab}\left(\bar{g}^{bc}(\partial_c u + q\mathcal{A}_c)\right) = 0 \; . \tag{3}$$

Now, the full action of gravity and (nonlinear) gauge fields interacting self-consistently with *LL-branes* reads (we specialize to $D = 4$ space-time dimensions and use units with the Newton constant $G_N = 1$):

$$S = \int d^4x \sqrt{-G}\Big[\frac{R(G) - 2\Lambda_0}{16\pi} + L(F^2)\Big] + \sum_{k=1}^{N} S_{\rm LL}[q^{(k)}] \; , \tag{4}$$

where the superscript (k) indicates the k-th *LL-brane.* Here $R(G) = G^{\mu\nu}R_{\mu\nu}$ and $R_{\mu\nu}$ denote the Riemannian scalar curvature and the Ricci tensor of the bulk space-time geometry. $L(F^2)$ is the Lagrangian of a remarkable non-standard nonlinear electrodynamics containing *square root* of ordinary Maxwell Lagrangian [10]:

$$L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \quad , \quad F^2 \equiv F_{\mu\kappa}F_{\nu\lambda}G^{\mu\nu}G^{\kappa\lambda} \; . \tag{5}$$

This is an explicit realization of ‘t Hooft's proposal (in flat space-time) for *infrared charge confinement* [11] (see also next talk [12] at this congress).

3 Charge-Confinement via "Tube-Like" Wormhole

The general scheme to construct "lightlike thin-shell" wormholes of static "spherically-symmetric" type (in Eddington-Finkelstein coordinates $dt = dv - \frac{d\eta}{A(\eta)}$ and "radial"-like coordinate $\eta \in (-\infty, +\infty)$):

$$ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j \quad , \quad F_{v\eta} = F_{v\eta}(\eta) \, , \tag{6}$$

$$-\infty < \eta < \infty \quad , \quad A(\eta_0^{(k)}) = 0 \;\; \text{for} \;\; \eta_0^{(1)} < \ldots < \eta_0^{(N)} \tag{7}$$

is as follows (cf. Section 5 in Ref. [8]):

(1) Take "vacuum" solutions of Einstein and (nonlinear) Maxwell equations resulting from (4) (*i.e.*, without the delta-function *LL-brane* contributions)

in each space-time region (separate "universe") given by $\left(-\infty < \eta < \eta_0^{(1)}\right), \ldots, \left(\eta_0^{(N)} < \eta < \infty\right)$ with common horizon(s) at $\eta = \eta_0^{(k)}$ ($k = 1, \ldots, N$).

(2) Each k-th *LL-brane* automatically locates itself on the horizon at $\eta = \eta_0^{(k)}$ – intrinsic property of *LL-brane* dynamics defined by the action (1).

(3) Match discontinuities of the derivatives of the metric and the gauge field strength across each horizon at $\eta = \eta_0^{(k)}$ using the explicit expressions for the *LL-brane* stress-energy tensor and charge current density systematically derived from the action (4) with (1).

Let us now consider the gravity/nonlinear-gauge-field system coupled to two oppositely charged *LL-branes*, *i.e.*, $N = 2$ and $q_1 = -q_2 \equiv q$ in (4). We obtain a particularly interesting "two-throat" wormhole-type solution exhibiting a QCD-like charge confinement effect. The total space-time manifold consists of three "universes" with different geometry glued together at their common horizons occupied by the two oppositely charged *LL-branes*:

(i) "Left-most" non-compact "universe" comprising the exterior region of a new kind of *non-standard* Schwarzschild-de-Sitter-type black hole, with additional *constant vacuum radial electric field* $\vec{E}_{\rm vac}$, beyond the Schwarzschild-type horizon r_0 for the "radial-like" η-coordinate interval $-\infty < \eta < -\eta_0 \equiv -\Big[4\pi\left(\sqrt{2}f_0|\vec{E}| - \vec{E}^2\right) + \Lambda_0\Big]^{-\frac{1}{2}}$, where (using notations as in (6)):

$$A(\eta) = 1 - \frac{2m}{r_0 - \eta_0 - \eta} - \frac{\Lambda_{\rm eff}}{3}(r_0 - \eta_0 - \eta)^2\,, \tag{8}$$

$$C(\eta) = (r_0 - \eta_0 - \eta)^2 \quad , \quad |F_{v\eta}(\eta)| \equiv |\vec{E}_{\rm vac}| = \frac{f_0}{\sqrt{2}} < |\vec{E}|\,. \tag{9}$$

Here $\vec{E}$ is the constant electric field in the "middle" "tube-like" "universe" (ii) (Eq. (12) below); $\Lambda_{\rm eff} \equiv \Lambda_0 + 2\pi f_0^2$ in (8) is *dynamically generated/shifted* cosmological constant, which is non-vanishing even in the absence of the "bare" cosmological constant Λ_0. Let us stress that *constant vacuum radial electric fields* such as in (9) *do not* exist as solutions of ordinary Maxwell electrodynamics on generic non-compact space-times – the former are due exclusively to the nonlinear "square-root" term in (5).

(ii) "Middle" "tube-like" "universe" of Levi-Civita-Bertotti-Robinson type [9] with geometry $dS_2 \times S^2$ (dS_2 denotes two-dimensional de Sitter space; S^2 – sphere with constant radius r_0), comprising the finite extent (w.r.t. η-coordinate) region between the two horizons of dS_2 at $\eta = \pm\eta_0$ occupied by the two *LL-branes* with charges $\pm q$:

$$-\eta_0 < \eta < \eta_0 \equiv \left[4\pi\left(\sqrt{2}f_0|\vec{E}| - \vec{E}^2\right) + \Lambda_0\right]^{-\frac{1}{2}} \; , \qquad (10)$$

where the metric coefficients and electric field are:

$$A(\eta) = 1 - \left[4\pi\left(\sqrt{2}f_0|\vec{E}| - \vec{E}^2\right) + \Lambda_0\right]\eta^2 \;\; , \;\; A(\pm\eta_0) = 0 \; , \quad (11)$$

$$C(\eta) = r_0^2 = \frac{1}{4\pi\vec{E}^2 + \Lambda_0} \;\; , \;\; |\vec{E}| = |q| + \frac{f_0}{\sqrt{2}} = \text{const} \; . \quad (12)$$

(iii) "Right-most" non-compact "universe" of the same type as (i) above for the "radial-like" η-coordinate interval $\eta_0 < \eta < \infty$ (η_0 as in (10)). Its metric is given by Eq. (8) upon changing $-\eta \to \eta$ and the elecric field is the same as in (9).

The equations for the electric field (second relations in (9) and (12)) have profound consequences:

- The "left-most" and "right-most" non-compact "universes" are two identical copies of the *electrically neutral* exterior region of Schwarzschild-de-Sitter black hole beyond the Schwarzschild horizon. They both carry a constant vacuum radial electric field with magnitude $|\vec{E}| = \frac{f_0}{\sqrt{2}}$ pointing inbound/outbound w.r.t. pertinent horizon. The corresponding electric

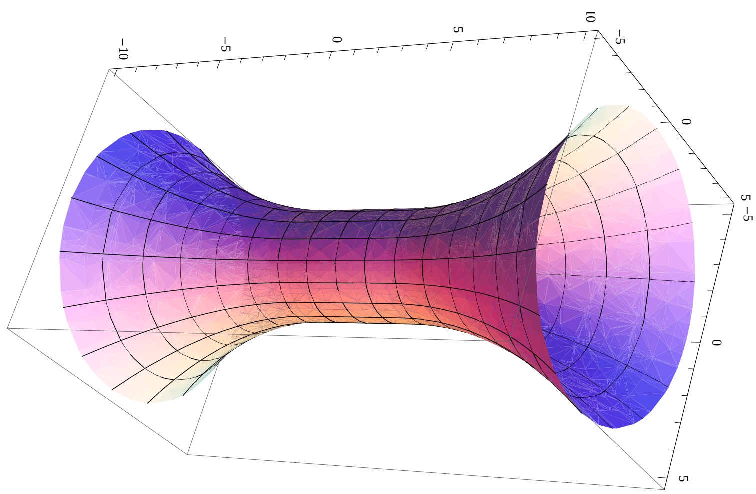

Figure 1: Shape of $t = \text{const}$ and $\theta = \pi/2$ slice of charge-confining wormhole geometry. The whole electric flux is confined within the middle cylindric tube.

displacement field $\vec{D} = \Big(1 - \frac{f_0}{\sqrt{2}|\vec{E}|}\Big)\,\vec{E} = 0$, so there is *no* electric flux there.

- The whole electric flux produced by the two charged *LL-branes* with opposite charges $\pm q$ at the boundaries of the above non-compact "universes" is *confined* within the finite-extent "tube-like" middle "universe" of Levi-Civitta-Robinson-Bertotti type with geometry $dS_2 \times S^2$, where the constant electric field is $|\vec{E}| = \frac{f_0}{\sqrt{2}} + |q|$ with associated non-zero electric displacement field $|\vec{D}| = |q|$. This is *QCD-like confinement*.

The *charge-confining* wormhole geometry is visualized in Figure "1.

To conclude let us emphasize that the existence of charge-confining "thin-shell" wormholes is entirely due to the combined effect of the exceptional properties of *LL-brane* dynamics and the "square-root" nonlinear electrodynamics.

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