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Evolution of Optical Pulses with Broadband Spectrum in a Nonlinear Dispersive Media*

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Abstract. In the present work the dynamics of femtosecond pulses (FS) in a nonlinear dispersive media has been reviewed. The nonlinear Schrödinger equation (NSE) very well describes the propagation of the slowly varying amplitude function of envelope of nano- and picosecond pulses. However, when we examine the evolution of short optical pulses, of the order of 100 fs or less, it is necessary to use the more general nonlinear amplitude equation. In this equation effects of linear dispersion up to third order and dispersion of nonlinearity are included. In our work we ignore losses and Raman scattering of the medium.

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1 Introduction

In recent years, the nonlinear propagation of optical pulses with broadband spectrum in a dispersive media attracts considerable attention.

One of the most used equations to describe the evolution of optical pulses in a nonlinear dispersive media is NSE [1]. Based on this equation, inserting extra terms for the third-order dispersion and nonlinear addition to the group velocity, in [2] it is numerically studied the behavior of short optical pulses. It is shown that under certain conditions it may be formed a soliton.

In this work we propose a theoretical model of the propagation of soliton-like pulses with a broadband spectrum in optical fibers. We consider the nonlinear amplitude equation [3,4], in which are included the effects up to third order of linear dispersion and dispersion of nonlinearity. Losses and Raman scattering of the medium are neglected. To solve this equation we use the method described in [5].

2 Basic Equation

We work with the one dimensional scalar amplitude equation that describes the nonlinear evolution of FS optical pulses in a single mode fibers. This is one re-

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duction of the 3D scalar amplitude equation that governs the propagation of ultrashort optical pulses in Kerr-type nonlinear dispersive isotropic medium [3,4]. In this equation we make a change of variables

$$z = z_0 z', \qquad z_0 = v t_0, \qquad t = t_0 t', \qquad A = A_0 A', \tag{1}$$

where z_0 and t_0 are the initial longitudinal length and duration of the optical pulse and A_0 is the magnitude of the initial amplitude. By this substitution the amplitude equation can be presented in dimensionless form. For simplicity, in further consideration, the prims are omitted ($'$).

The dimensionless constants are introduced

$$\begin{gathered} \alpha = k_0 z_0, \qquad \beta_2 = k_0 k'' v^2, \qquad \beta_3 = \frac{2}{3} k_0^2 k''' v^3, \\ \gamma = n_2 \alpha^2 |A_0|^2, \qquad \gamma_1 = v \left(\frac{n_2 k_0}{\omega_0} + \frac{k_0}{2} \frac{\partial n_2}{P} \partial \omega \right) |A_0|^2, \end{gathered} \tag{2}$$

where

$$k_0 = k(\omega),\ k' = \frac{\partial k}{\partial \omega},\ k'' = \frac{\partial^2 k}{\partial \omega^2},\ k''' = \frac{\partial^3 k}{\partial \omega^3},\ v = \frac{1}{k'},\ n = n(\omega_0). \tag{3}$$

In the expressions above t is time; ω_0, k, v, n, n_2 are respectively the carrier frequency, the wave number, the group velocity, the linear and nonlinear refractive index of the medium. The constant $\alpha (\alpha > 1)$ determines the number of harmonic oscillations at level $1/e$ from the maximum of the pulse amplitude. The coefficients β_2 and β_3 characterize the second and third order of the linear dispersion. We examine the propagation of the pulses in an anomalous dispersion medium, i.e. $k'' < 0$. The parameters γ and $\gamma_1 (\gamma \gg \gamma_1)$ depend on the nonlinear refractive index n_2. Considering the assumptions and the substitutions that we have made above, the equation (1) can be written in the following scalar form:

$$\begin{aligned} \frac{\partial^2 A}{\partial z^2} + 2i\alpha \left[\frac{\partial A}{\partial z} + \frac{\partial A}{\partial t} + \gamma_1 \frac{\partial}{\partial t} \left(|A|^2 A \right) \right] + \frac{i}{\alpha} |B| \frac{\partial^3}{A} \partial t^3 \\ - (1 - |\beta_2|) \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0, \end{aligned} \tag{4}$$

where A is the scalar amplitude function, $\beta_2 = -|\beta_2|$, $\beta_3 - |\beta_2| = -|B|$, $|\beta_2| < |B| \ll 1$.

3 A Soliton Solution of the Scalar Amplitude Equation

Let us now consider the scalar amplitude equation (4). We search for a solution of the form

$$A(z,t) = \Phi(\xi) \exp(iat + ibz),\ \xi = z - t, \tag{5}$$

where a and b are constants and $\Phi(\xi)$ is a real function.

After substituting the expression (5) in equation (4) it is obtained a complex nonlinear ordinary differential equation of the third order and third degree of the unknown function $\Phi(\xi)$. Since $\Phi(\xi)$ is a real function and we need to define the constants a and b, the real and imaginary parts on both sides of our ordinary differential equation are equalized. After a short transformations the following two differential equations are obtained:

$$\Phi''\left(\frac{-3a|B|}{\alpha}+|\beta_2|\right)-\Phi\left[b^2+2\alpha(a+b)-\frac{|B|}{\alpha}a^3-(1-|\beta_2|)a^2\right] + \Phi^3[\gamma-2\alpha a\gamma_1]=0 \quad (6)$$

$$\frac{|B|}{\alpha}\Phi''-\Phi\left[2b+\frac{3a^2|B|}{\alpha}+2a(1-|\beta_2|)\right]+2\alpha\gamma_1\Phi^3=C=\text{const.} \quad (7)$$

The equations (6) and (7) are referred to the same unknown function. For this reason, they should match. This means that the coefficients in front of the corresponding derivatives and degrees of $\Phi(\xi)$ have to be the same and the integration constant C is then zero $(C=0)$ [5]. Since we need to define the two constants a and b, let first divide the two equations respectively by the coefficients in front of Φ'' and then to equalize the coefficients of Φ and Φ^3. Thus, we obtain two algebraic equations. From these equations we can easily define the constant a and b

$$a=\frac{\alpha|\beta_2|}{2|B|}-\frac{1}{4\alpha}\frac{\gamma}{\gamma_1} \quad (8)$$

$$b_{1,2}=-\left(\alpha+3a-\frac{\alpha|\beta_2|}{|B|}\right)\pm F\,, \quad (9)$$

where

$$F=\left[\alpha^2-\frac{\gamma}{\gamma_1}+\left(\frac{\gamma}{2\alpha\gamma_1}\right)^2\left(1-|\beta_2|+\frac{|B|}{2\alpha^2}\frac{\gamma}{\gamma_1}\right)\right]^{1/2}. \quad (10)$$

Once the two constants a and b are determined in the way that the two equations (6) and (7) match, for the unknown real function $\Phi=\Phi(\xi)$ is obtained the following nonlinear ordinary differential equation of second order:

$$\Phi''-\eta\Phi+\Gamma\Phi^3=0\,, \quad (11)$$

where

$$\eta=\frac{-2\alpha}{|B|}(\alpha-F)+\frac{1}{|B|}\frac{\gamma}{\gamma_1}-\left(\frac{1}{4\alpha}\frac{\gamma}{\gamma_1}+\frac{\alpha|\beta_2|}{2|B|}\right)^2+\frac{1}{4\alpha^2}\left(\frac{\gamma}{\gamma_1}\right)^2>0 \quad (12)$$

$$\Gamma=2\alpha^2\frac{\gamma_1}{|B|}\,. \quad (13)$$

Equation (11) has a well known soliton solution [1]

$$\Phi(\xi) = \sqrt{\frac{2\eta}{\Gamma}}\operatorname{sech}(\xi\sqrt{\eta})\,. \tag{14}$$

The obtained expression for a soliton is different from the classical one, obtained by solving the NSE. For the observation of a single soliton with a carrying wavelength of $k_0 = 4.488 \times 10^6$ m^{-1}, $k'' = -1.2 \times 10^{-26}$ s^2/m, $k''' = -1 \times 10^{-40}\, s^3$/m in a silica single-mode fiber with a nonlinear refractive index $n_2 = 4.5 \times 10^{-20}$ m^2/W and assuming that $\dfrac{\partial n_2}{\partial \omega} \approx 0$, the initial intensity of the Schrödinger soliton is $|A_0|^2_{\text{shr}} = 0.34 \times 10^{14}$ W/m^2 and for the optical pulse obtained in this work: $|A_0|^2 = 5.8 \times 10^{17}$ W/m^2. The critical intensity for ionization of the medium is $|A_0|^2_{\text{cr}} = 2.2 \times 10^{19}$ W/m^2. Thus, to observe the soliton that we have obtained, we need an initial intensity lower than the critical for ionization and significantly higher than that of the Schrödinger's soliton.

4 Conclusions

In the present work the propagation of light pulses with a broadband spectrum in a nonlinear single mode optical fiber is investigated. It is found a soliton solution of the nonlinear amplitude equation in which the effects up to third order of the linear dispersion and nonlinear addition to the group velocity are taken into account. Obviously, it differs significantly from the soliton solution of NSE.

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