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Anisotropic Dark Energy Cosmological Models from Early Deceleration to Late Time Acceleration in Lyra Geometry

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Abstract. We have studied anisotropic and homogenous Bianchi Type III, Kantowaski-Sachs, Bianchi Type V, Non-Static Plane Symmetric, Hypersurface-Homogeneous, Bianchi Type II, VIII, IX space-times under the assumption on the anisotropy of the fluid within the frame work of Lyra manifold. A special form of deceleration parameter (q) which gives an early deceleration and late time accelerating cosmological models has been utilized to solve the field equations. The physical and geometrical behaviors of the models are discussed.

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1 Introduction

Einstein formulated his field equation of General Relativity by geometrizing gravitation. Shortly after Einstein's general theory of relativity, Weyl [1] has suggested the first so called unified field which is a geometrized theory of gravitation and electromagnetism. He has shown how can one introduce a vector field in the Riemannian space-time with an intrinsic geometrical significance. But this theory has never been taken seriously because it was based on the concept of non-integrability of length transfer. Later Lyra [2] has suggested a modification of Riemannian geometry, which may also be considered as a modification of Weyl's geometry, by introducing a gauge function into the structureless manifold which removes the non-integrability condition of the length of a vector under parallel transport and a cosmological constant has naturally been introduced from the geometry. The essential difference between the cosmological theories based on Lyra's geometry and Riemannian geometry lies in the fact that displacement field β has arised from geometry where as cosmological term Λ is introduced in an adhoc fashion in the usual treatment.

Sen and Dunn [3] have introduced a scalar tensor theory based on Lyra geometry in which both the scalar and tensor fields have intrinsic geometrical significance. Subsequently, Halford [4] has developed a cosmological theory within the framework of Lyra Geometry and pointed out that the vector field ϕ_μ in Lyra geometry, plays a similar role of cosmological constant Λ in General Relativity. It has been shown by Halford [5] that the scalar-tensor treatment based on Lyra's geometry predicts the same effects within observational limits as the Einstein's theory. Jeavons et al. [9] have pointed out that the field equations proposed by Sen and Dunn are heuristically useful even though they are not derived from the variational principle. Soleng [10] has pointed out that the cosmologies based on Lyra's manifold with constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation field cosmology (Hoyle [6]; Hoyle and Narlikar [7,8]) or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. In the latter case the solutions are equal to the general relativistic cosmologies with a cosmological term. Further Soleng [11] has observed that for matter with zero spin the field equations of his scalar tensor theory reduce to those of Brans-Dicke theory.

Eminent researchers, such as Bhamra [12], Karade and Borikar [13], Kalyanshetti and Wagmode [14], Reddy and Innaiah [15], Reddy and Venkateswarlu [16], Soleng [10] have studied cosmological models based on Lyra's geometry with a constant displacement field vector. Beesham [17] investigated FLRW cosmological models in Lyra's manifold with time dependent displacement field. The models so obtained, solve the singularity, entropy and horizon problems which exist in standard models of cosmology based on Riemannian geometry. Singh and Singh [18-21], Singh and Desikan [22] have studied FRW, Bianchi-type I, III, Kantowaski-Sachs space times and obtained a new class of cosmological models with time dependent displacement field based on Lyra's geometry. Rahaman et al. [23-24] have studied inhomogeneous spherically symmetric higher dimensional model in the presence of a mass less scalar field and cosmological models with negative constant deceleration parameter in Lyra's geometry. Pradhan et al. [25-30], Agarwal et al. [31], Casama et al. [32], Bali and Chandnani [33-35], Kumar and Singh [36], Singh[37], Rao, Vinutha and Santhi [38], Pradhan [39-40] and Singh and Kale [41] have studied cosmological models based on Lyra's geometry in various contexts. Ram et al.[42] have obtained exact solutions for anisotropic Bianchi type V perfect fluid cosmological models in Lyra's geometry. Very recently, Chaubey [43] has obtained exact solutions for Kantowski-Sachs cosmological model in Lyra's geometry.

Modern experimental data including the Type Ia Supernovae (SNe Ia), the Large Scale Structure and the Cosmic Microwave Background (CMB) Radiation, indicate the existence of an anisotropic universe which has gained isotropization with the passage of time. Different data from the recent astrophysical observations, such as Super-Nova Ia (Riess et al. [44-45]; Knop et al.[46]), Cosmic Microwave Background Radiations (Spergel et al. [47-48]; Komatsu et al. [49]), Wilkinson Microwave Anisotropy Probe (Bennett et al. [50]), Sloan Digital Sky Survey (Tegmark et al. [51]; Seljak et al. [52]) and Baryon Acoustic Oscillations (Eisenstein et al. [53]; Percival et al. [54]) have indicated that the universe is flat and the expansion of universe is currently accelerating. To explain mysterious dark energy, a variety of the theoretical models has been proposed in the literature such as vacuum energy $\omega = -1$, phantom $\omega < -1$, quintessence $\omega > -1$, quintom (that is the combination of phantom and quintessence), Chaplygin gas, tachyon and etc. Some other limits obtained from observational results coming from SNe Ia data (Knop et al. [46]) and SNe Ia data collaborated with CMBR anisotropy and galaxy clustering statistics (Tegmark et al. [51]) are $-1.67 < \omega < -0.62$ and $-1.33 < \omega < -0.79$, respectively. The latest results, obtained after a combination of cosmological datasets coming from CMB anisotropies, luminosity distances of high redshift type Ia supernovae and galaxy clustering, constrain the dark energy EoS to $-1.44 < \omega < -0.92$ at 68% confidence level (Hinshaw et al. [55]; Komatsu et al. [49]). Some authors (Akarsu and Kilinc [56-57]; Yadav et al. [58]; Yadav and Yadav [59]; Kumar and Yadav [60];Yadav [61]; Adhav et al. [62] and recently Yadav and Saha [63]) have studied DE models with variable EoS parameter. Pradhan and Amirhashchi [64] have investigated a new anisotropic Bianchi type-III dark energy model, in general relativity, with equation of state (EoS) parameter without assuming constant deceleration parameter. Very recently, Naidu et al.[65-67] have presented Bianchi type-II and III dark energy models in Saez-Ballester [68] scalar-tensor theory of gravitation. Recently, Adhav K.S. [69] studied LRS Bianchi type-I universe with anisotropic dark energy in Lyra Geometry.

Motivating with this research works, in this paper, we have studied anisotropic and homogenous Bianchi Type III, Kantowaski-Sachs, Bianchi Type V, Non-Static Plane Symmetric, Hypersurface-Homogeneous, Bianchi Type II, VIII, IX space-times under the assumption on the anisotropy of the fluid within the frame work of Lyra manifold. A special form of deceleration parameter (q) which gives an early deceleration and late time accelerating cosmological models has been utilized to solve the field equations. The physical and geometrical behaviors of the models are discussed.

2 Metric and Its Field Equations

We consider a Riemannian space-time (Non-static Plane Symmetric) described by the line element

$$ds^2 = e^{2h}\left\{dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2\right\}, \quad (1)$$

where r, θ, z are the usual cylindrical polar co-ordinates and h, s are the functions of cosmic time t only.

The simplest generalization of EoS parameter of perfect fluid may be to determine the EoS parameter separately on each spatial axis by preserving the diagonal form of the energy momentum tensor in a consistence way with the considered metric. Therefore, the energy momentum tensor of fluid can be written, most generally, in anisotropic diagonal form as follows: The Energy momentum tensor of fluid is taken as

$$T_v^u = \, diag\left[T_1^1, T_2^2, T_3^3, T_4^4\right], \tag{2}$$

Allowing for anisotropy in the pressure of the fluid, and thus in its EoS parameter, gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor given in equation (3) as follows:

$$\begin{aligned} T_v^u &= \, diag\left[-p_x, -p_y, -p_z, \rho\right], \\ &= \, diag\left[-w_x, -w_y, -w_z, 1\right]\rho, \\ &= \, diag\left[-(w+\delta), -\left(w+\delta\right), -w, 1\right]\rho, \end{aligned} \tag{3}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressures and w_x, w_y, w_z are the directional EoS parameter along the x, y, z axes respectively. Now, parametrizing the deviation from isotropy by setting $w_x,\ w_y,\ w_z = w$ then introducing the skewness parameter δ which is the derivative from w on x and y axes. Here w and δ are not necessarily constants and can be functions of the cosmic time t.

The Einstein Field equations in Lyra's manifold as obtained by Sen [70] are $(8\pi G = 1, c = 1)$

$$R_{uv} - \frac{1}{2}R\, g_{uv} + \frac{3}{2}\phi_u\phi_v - \frac{3}{4}g_{uv}\phi_\alpha\phi^\alpha = -T_{uv}, \tag{4}$$

where $g_{uv}u^u v^v = 1$, $u^u = (0,0,0,1)$ is the four velocity vector, ϕ_u is the displacement vector, R_{uv} is Ricci Tenser, R is the Ricci Scalar and T_{uv} is the usual stress energy momentum tensor of the matter. Here we assume the vector displacement field ϕ_u to be time-like constant vector $\phi_u = (0,0,0,\lambda)$.

In a co-moving co-ordinate system, the field equations for the metric (1) with the help of equations (3) and (4) can be written as

$$\frac{1}{e^{2h}}\left\{2\ddot{h} + \dot{h}^2 + \frac{\ddot{s}}{s} + 2\dot{h}\frac{\dot{s}}{s}\right\} + \frac{3}{4}\lambda^2 = -(w+\delta)\rho \quad , \tag{5}$$

$$\frac{1}{e^{2h}}\left\{2\ddot{h} + \dot{h}^2\right\} + \frac{3}{4}\lambda^2 = -(w)\rho, \tag{6}$$

$$\frac{1}{e^{2h}}\left\{3\dot{h}^2 + 2\dot{h}\frac{\dot{s}}{s}\right\} - \frac{3}{4}\lambda^2 = \rho, \tag{7}$$

where the overhead dot (.) denote derivative with respect to the cosmic time t.

An important observational quantity is the deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2},$$

where a is the mean scale factor of the universe. The sign of the deceleration parameter indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating models, whereas the negative sign indicates inflation.

There are three linearly independent equations (5)-(7) with five unknowns h, s, w, ρ, δ. In order to get a model consistent with recent observations, we take special form of deceleration parameter (Banerjee and Das [71], Singh and Debnath [72], Adhav et al. [73]) as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^{\alpha}}, \tag{8}$$

where $\alpha > 0$, is a constant and a is the mean scale factor of the universe.

Thus, we have a model of universe which begins with a decelerating expansion and evolves into a late time accelerating universe which is in agreement with SNe Ia astronomical observations (Riess et al. [45]).

After solving (8) one can obtain the mean Hubble parameter H as

$$H = \frac{\dot{a}}{a} = k(1 + a^{-\alpha}), \tag{9}$$

where k is a constant of integration.

On integrating equation (9), we obtain the mean scale factor as

$$a = (e^{k\alpha t} - 1)^{\frac{1}{\alpha}}. \tag{10}$$

The physical parameters that are of cosmological importance are as follows:

- The mean anisotropy parameter:

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2. \tag{11}$$

- The shear scalar:

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}A_m H^2, \tag{12}$$

  where $H_i\ (i = 1, 2, 3)$ represent the directional Hubble parameters in the directions of x, y and z axes respectively.

- The expansion scalar:

$$\theta = 3H. \tag{13}$$

We define the spatial volume V of Riemannian space-time (Non-static Plane Symmetric) as

$$V = a^3 = rse^{4h}. \tag{14}$$

Using equations (10) and (14), we have

$$(e^{\alpha kt} - 1)^{\frac{3}{\alpha}} = rse^{4h}. \tag{15}$$

To solve the above set of highly non linear equations, we assume the relation between the metric coefficients

$$e^h = \beta s^n, \tag{16}$$

where β is constant and $n > 0$.

Using equations (15) and (16), we obtain

$$s = \left(\frac{1}{r\beta^4}\right)^{\frac{1}{4n+1}} (e^{\alpha kt} - 1)^{\frac{3}{\alpha(4n+1)}}, \tag{17}$$

$$e^h = \beta \left(\frac{1}{r\beta^4}\right)^{\frac{n}{4n+1}} (e^{\alpha kt} - 1)^{\frac{3n}{\alpha(4n+1)}}. \tag{18}$$

Hence a Riemannian space-time describe by line element (1) corresponding to equations (17) and (18) can be written in the form

$$\begin{aligned} ds^2 = {} & \beta^2 \left(\frac{1}{r\beta^4}\right)^{\frac{2n}{4n+1}} (e^{\alpha kt} - 1)^{\frac{6n}{\alpha(4n+1)}} \\ & \times \left\{ dt^2 - dr^2 - r^2 d\theta^2 - \left(\frac{1}{r\beta^4}\right)^{\frac{2}{4n+1}} (e^{\alpha kt} - 1)^{\frac{6}{\alpha(4n+1)}} dz^2 \right\}. \end{aligned} \tag{19}$$

Using equations (10) and (14), we get

$$V = (e^{k\alpha t} - 1)^{\frac{3}{\alpha}}. \tag{20}$$

Using equations (17) and (18), the directional Hubble parameters are found as

$$H_r = H_\theta = \left(\frac{3nk}{4n+1}\right) \frac{e^{\alpha kt}}{e^{\alpha kt} - 1}, \tag{21}$$

$$H_z = \left(\frac{6k(n+1)}{4n+1}\right) \frac{e^{\alpha kt}}{e^{\alpha kt} - 1}. \tag{22}$$

Using equations (7),(17) and (18), we obtain the energy density as

$$\rho = \left(\frac{1}{\beta^2}\right)\left(\frac{1}{r\beta^4}\right)^{\frac{-2n}{4n+1}}\left(e^{\alpha kt}-1\right)^{\frac{-6n}{\alpha(4n+1)}} \times \frac{9nk^2}{(4n+1)^2}\frac{e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}\{3n+2\} - \frac{3}{4}\lambda^2. \quad (23)$$

Using equations (6), (17) and (18), the EoS parameter is

$$w = \frac{-1}{\rho}\Bigg\{\left(\frac{1}{\beta^2}\right)\left(\frac{1}{r\beta^4}\right)^{\frac{-2n}{4n+1}}\left(e^{\alpha kt}-1\right)^{\frac{-6n}{\alpha(4n+1)}} \times \frac{3nk^2}{(4n+1)}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}\left\{-2\alpha + \frac{3n}{4n+1}e^{\alpha kt}\right\} + \frac{3}{4}\lambda^2\Bigg\}. \quad (24)$$

Using equations (6), (7), (17) and (18) , the skewness parameter as

$$\delta = \frac{-1}{\rho}\Bigg\{\left(\frac{1}{\beta^2}\right)\left(\frac{1}{r\beta^4}\right)^{\frac{-2n}{4n+1}}\left(e^{\alpha kt}-1\right)^{\frac{-6n}{\alpha(4n+1)}} \times \Bigg[\left(\frac{3\alpha k^2}{(4n+1)}\right)\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left(1 + \frac{3-\alpha-4\alpha n}{\alpha(4n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\right) + \left(\frac{18nk^2}{(4n+1)^2}\right)\frac{e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}\Bigg]\Bigg\} \quad (25)$$

The physical parameters such as anisotropy parameter of the expansion, shear scalar and expansion scalar as defined in (11), (12) and (13) are

$$\theta = \frac{3ke^{\alpha kt}}{e^{\alpha kt}-1}; \quad (26)$$

$$A_m = \frac{(e^{\alpha kt}-1)}{ke^{\alpha kt}}; \quad (27)$$

$$\sigma^2 = \frac{3k}{2}\frac{e^{\alpha kt}}{e^{\alpha kt}-1}. \quad (28)$$

3 Kantowaski-Sachs Model

The Kantowaski-Sachs line element can be written as

$$ds^2 = dt^2 - A^2dr^2 - B^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad (29)$$

where A, B are the scale factors and functions of the cosmic time only.

The Energy momentum tensor of fluid is taken as

$$T_v^u = diag\left[T_1^1, T_2^2, T_3^3, T_4^4\right], \tag{30}$$

allowing for anisotropy in the pressure of the fluid, and thus in its EoS parameter, gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor given in equation (30) as follows:

$$\begin{aligned} T_v^u &= diag\left[-p_x, -p_y, -p_z, \rho\right], \\ &= diag\left[-w_x, -w_y, -w_z, 1\right]\rho, \\ &= diag\left[-w, -(w+\delta), -(w+\delta), 1\right]\rho, \end{aligned} \tag{31}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressures and w_x, w_y, w_z are the directional EoS parameter along the x, y, z axes respectively. Now, parametrizing the deviation from isotropy by setting $w_x, w_y, w_z = w$ then introducing skewness parameter δ which is the derivative from w on y and z axes. Here w and δ are not necessarily constants and can be functions of the cosmic time t.

In a co-moving co-ordinate system, the field equations for the metric (29) with the help of equations (31) and (4) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} + \frac{3}{4}\lambda^2 = -w\rho, \tag{32}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{3}{4}\lambda^2 = -(w+\delta)\rho, \tag{33}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{3}{4}\lambda^2 = \rho, \tag{34}$$

where the overhead dot (.) denote derivative with respect to the cosmic time t.

There are three linearly independent equations (32)-(34) with five unknowns A, B, w, ρ, δ.

We define the spatial volume V of Kantowaski–Sachs space-time as

$$V = a^3 = AB^2. \tag{35}$$

Using equations (10) and (35), we have

$$\left(e^{\alpha k t} - 1\right)^{\frac{3}{\alpha}} = AB^2. \tag{36}$$

To solve the above set of highly non linear equations, we assume the relation between the metric coefficients

$$B = A^n, \tag{37}$$

where n is an arbitrary constant.

Using equations (36) and (37), we obtain

$$A = (e^{\alpha k t} - 1)^{\frac{3}{\alpha(2n+1)}}, \tag{38}$$

$$B = (e^{\alpha k t} - 1)^{\frac{3n}{\alpha(2n+1)}}. \tag{39}$$

Hence a Kantowaski–Sachs space-time described by the line element (29) corresponding to equations (38) and (39) can be written in the form

$$ds^2 = dt^2 - (e^{\alpha k t} - 1)^{\frac{6}{\alpha(2n+1)}} dr^2 - (e^{\alpha k t} - 1)^{\frac{6n}{\alpha(2n+1)}} \left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{40}$$

Using equations (10) and (35), we get

$$V = (e^{k\alpha t} - 1)^{\frac{3}{\alpha}}. \tag{41}$$

Using equations (38) and (39), the directional Hubble parameters are found as

$$H_r = \left(\frac{3k}{2n+1}\right) \frac{e^{\alpha k t}}{e^{\alpha k t} - 1}, \tag{42}$$

$$H_\theta = H_\phi = \left(\frac{3nk}{2n+1}\right) \frac{e^{\alpha k t}}{e^{\alpha k t} - 1}. \tag{43}$$

Using equations (34),(38) and (39), we obtain the energy density as

$$\rho = \frac{9nk^2}{(2n+1)^2} \frac{e^{2\alpha k t}}{(e^{\alpha k t} - 1)^2}(2+n) + \frac{1}{(e^{\alpha k t} - 1)^{\frac{6n}{\alpha(2n+1)}}} - \frac{3}{4}\lambda^2. \tag{44}$$

Using equations (32),(38) and (39), the EoS parameter as

$$w = \frac{-1}{\rho}\left\{\frac{6\alpha n k^2}{(2n+1)} \frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)} \left[\frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)}\left(\frac{3n}{\alpha(2n+1)} - 1\right) + 1\right] \right.$$
$$\left. + \frac{9n^2k^2}{(2n+1)^2} \frac{e^{2\alpha k t}}{(e^{\alpha k t} - 1)^2} + \frac{1}{(e^{\alpha k t} - 1)^{\frac{6n}{\alpha(2n+1)}}} + \frac{3}{4}\lambda^2\right\}. \tag{45}$$

Using equations (32), (33), (38) and (39), the skewness parameter is

$$\delta = \frac{1}{\rho}\left\{\frac{3\alpha k^2}{(2n+1)} \frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)} \left[\frac{e^{\alpha k t}}{(e^{\alpha k t} - 1)}\left(\frac{3}{\alpha(2n+1)} - 1\right) + 1\right](n-1) \right.$$
$$\left. + \frac{9n^2k^2}{(2n+1)^2} \frac{e^{2\alpha k t}}{(e^{\alpha k t} - 1)^2} - \frac{9nk^2}{(2n+1)^2} \frac{e^{2\alpha k t}}{(e^{\alpha k t} - 1)^2} + \frac{1}{(e^{\alpha k t} - 1)^{\frac{6n}{\alpha(2n+1)}}}\right\} \tag{46}$$

The physical parameters such as anisotropy parameter of the expansion, shear scalar and expansion scalar as defined in (11), (12) and (13) are

$$\theta = \frac{3ke^{\alpha kt}}{e^{\alpha kt}-1}; \tag{47}$$

$$A_m = \frac{2(n-1)^2}{(2n+1)^2}; \tag{48}$$

$$\sigma^2 = \frac{3k^2(n-1)^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}. \tag{49}$$

4 Bianchi type-III Model

The Bianchi type-III line element can be written as

$$ds^2 = dt^2 - A^2dr^2 - B^2\left(d\theta^2 + \sinh^2\theta d\phi^2\right), \tag{50}$$

where A, B are the scale factors and functions of the cosmic time only.

The Energy momentum tensor of fluid is taken as

$$T_v^u = diag\left[T_1^1, T_2^2, T_3^3, T_4^4\right], \tag{51}$$

allowing for anisotropy in the pressure of the fluid, and thus in its EoS parameter, gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor given in equation (51) as follows:

$$\begin{aligned} T_v^u &= diag\left[-p_x, -p_y, -p_z, \rho\right], \\ &= diag\left[-w_x, -w_y, -w_z, 1\right]\rho, \\ &= diag\left[-w, -(w+\delta), -\left(w+\delta\right), 1\right]\rho, \end{aligned} \tag{52}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressures and w_x, w_y, w_z are the directional EoS parameter along the x, y, z axes respectively. Now, parametrizing the deviation from isotropy by setting $w_x, w_y, w_z = w$ then introducing skewness parameter δ which is the derivative from w on y and z axes. Here w and δ are not necessarily constants and can be functions of the cosmic time t.

In a co-moving co-ordinate system, the field equations for the metric (50) with the help of equations (52) and (4) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} + \frac{3}{4}\lambda^2 = -w\rho, \tag{53}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{3}{4}\lambda^2 = -(w+\delta)\rho, \tag{54}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{1}{B^2}-\frac{3}{4}\lambda^2=\rho, \tag{55}$$

where the overhead dot (.) denote derivative with respect to the cosmic time t.

There are three linearly independent equations (53)–(55) with five unknowns A, B, w, ρ, δ.

We define the spatial volume V of Bianchi type-III space-time as

$$V=a^3=AB^2. \tag{56}$$

Using equations (10) and (56), we have

$$(e^{\alpha kt}-1)^{\frac{3}{\alpha}}=AB^2. \tag{57}$$

To solve the above set of highly non linear equations, we assume the relation between the metric coefficients

$$B=A^n, \tag{58}$$

where n is arbitrary constant.

Using equations (57) and (58), we obtain

$$A=(e^{\alpha kt}-1)^{\frac{3}{\alpha(2n+1)}}, \tag{59}$$

$$B=(e^{\alpha kt}-1)^{\frac{3n}{\alpha(2n+1)}}. \tag{60}$$

Hence a Bianchi type-III space-time described by the line element (50) corresponding to equations (59) and (60) can be written in the form

$$ds^2=dt^2-(e^{\alpha kt}-1)^{\frac{6}{\alpha(2n+1)}}dr^2-(e^{\alpha kt}-1)^{\frac{6n}{\alpha(2n+1)}}\left(d\theta^2+\sin^2\theta d\phi^2\right). \tag{61}$$

Using equations (10) and (56), we get

$$V=(e^{k\alpha t}-1)^{\frac{3}{\alpha}}. \tag{62}$$

Using equations (59) and (60), the directional Hubble parameters are found as

$$H_r=\left(\frac{3k}{2n+1}\right)\frac{e^{\alpha kt}}{e^{\alpha kt}-1}, \tag{63}$$

$$H_\theta=H_\phi=\left(\frac{3nk}{2n+1}\right)\frac{e^{\alpha kt}}{e^{\alpha kt}-1}. \tag{64}$$

Using equations (55), (59) and (60), we obtain the energy density as

$$\rho=\frac{9nk^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}(2+n)-\frac{1}{(e^{\alpha kt}-1)^{\frac{6n}{\alpha(2n+1)}}}-\frac{3}{4}\lambda^2. \tag{65}$$

Using equations (53), (59) and (60), the EoS parameter is

$$w = \frac{-1}{\rho}\left\{\frac{6\alpha nk^2}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left(\frac{3n}{\alpha(2n+1)}-1\right)+1\right]\right.$$
$$\left.+\frac{9n^2k^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}-\frac{1}{(e^{\alpha kt}-1)^{\frac{6n}{\alpha(2n+1)}}}+\frac{3}{4}\lambda^2\right\}. \quad (66)$$

Using equations (53), (54), (59) and (60), the skewness parameter is

$$\delta = \frac{1}{\rho}\left\{\frac{3\alpha k^2}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left(\frac{3}{\alpha(2n+1)}-1\right)+1\right](n-1)\right.$$
$$\left.+\frac{9n^2k^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}-\frac{9nk^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}-\frac{1}{(e^{\alpha kt}-1)^{\frac{6n}{\alpha(2n+1)}}}\right\} \quad (67)$$

The physical parameters such as anisotropy parameter of the expansion, shear scalar and expansion scalar as defined in (11), (12) and (13) are

$$\theta = \frac{3ke^{\alpha kt}}{e^{\alpha kt}-1}; \quad (68)$$

$$A_m = \frac{2(n-1)^2}{(2n+1)^2}; \quad (69)$$

$$\sigma^2 = \frac{3k^2(n-1)^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}. \quad (70)$$

5 Bianchi type-V Model

The Bianchi type-V line element can be written as

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}\left(dy^2+dz^2\right), \quad (71)$$

where A, B are the scale factors and functions of the cosmic time only.

The Energy momentum tensor of fluid is taken as

$$T^u_v = diag\left[T^1_1, T^2_2, T^3_3, T^4_4\right], \quad (72)$$

Allowing for anisotropy in the pressure of the fluid, and thus in its EoS parameter, gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor given in equation (72) as follows:

$$T_v^u = diag\left[p_x, p_y, p_z, -\rho\right], \\ = diag\left[w_x, w_y, w_z, -1\right]\rho, \\ = diag\left[w, (w+\delta), (w+\delta), -1\right]\rho, \tag{73}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressures and w_x, w_y, w_z are the directional EoS parameter along the x, y, z axes respectively. Now, parametrizing the deviation from isotropy by setting $w_x, w_y, w_z = w$ then introducing skewness parameter δ which is the derivative from w on $y$and z axes. Here w and δ are not necessarily constants and can be functions of the cosmic time t.

In a co-moving co-ordinate system, the field equations for the metric (71) with the help of equations (73) and (4) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} - \frac{3}{4}\lambda^2 = -w\rho, \tag{74}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} - \frac{3}{4}\lambda^2 = -(w+\delta)\rho, \tag{75}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} + \frac{3}{4}\lambda^2 = \rho, \tag{76}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0, \tag{77}$$

where the overhead dot (.) denotes derivative with respect to the cosmic time t.

Integrating equation (77), we obtain

$$A = \mu B, \tag{78}$$

where μ is the positive constant of integration.

With loss of generality for $\mu = 1$, we have $A = B$.

Using equations (74), (75) and (78), we obtain the skewness parameter on $z-$axis is null, i.e.

$$\delta = 0. \tag{79}$$

Thus the system of equations (74)–(77) may be reduced to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} - \frac{3}{4}\lambda^2 = -w\rho, \tag{80}$$

$$3\frac{\dot{B}^2}{B^2} - \frac{3}{B^2} + \frac{3}{4}\lambda^2 = \rho. \tag{81}$$

Now there are two linearly independent equations (80)–(81) with three unknowns B, w, ρ.

We define the spatial volume V of Bianchi type-V space-time as

$$V = a^3 = AB^2 e^{2x}. \tag{82}$$

Using equations (10), (78) and (82), we have

$$\left(e^{\alpha kt} - 1\right)^{\frac{3}{\alpha}} = B^3 e^{2x}. \tag{83}$$

Using equations (78) and (83), we obtain

$$A = B = e^{\frac{-2x}{3}} \left(e^{\alpha kt} - 1\right)^{\frac{1}{\alpha}}. \tag{84}$$

Hence a Bianchi type-V space-time described by the line element (71) corresponding to equation (84) can be written in the form

$$ds^2 = -dt^2 + e^{\frac{-4x}{3}} \left(e^{\alpha kt} - 1\right)^{\frac{2}{\alpha}} dx^2 + e^{\frac{-4x}{3}} \left(e^{\alpha kt} - 1\right)^{\frac{2}{\alpha}} e^{2x} \left(dy^2 + dz^2\right). \tag{85}$$

Using equations (10) and (82), we get

$$V = \left(e^{k\alpha t} - 1\right)^{\frac{3}{\alpha}}. \tag{86}$$

Using equation (84), the directional Hubble parameters are found as

$$H_x = H_y = H_z = \frac{k e^{\alpha kt}}{e^{\alpha kt} - 1}. \tag{87}$$

Using equations (81) and (84), we obtain the energy density as

$$\rho = \frac{3k^2 e^{2\alpha kt}}{\left(e^{\alpha kt} - 1\right)^2} - \frac{3}{e^{\frac{-4x}{3}} \left(e^{\alpha kt} - 1\right)^{\frac{2}{\alpha}}} + \frac{3}{4}\lambda^2. \tag{88}$$

Using equations (80) and (84), the EoS parameter is

$$w = \frac{-1}{\rho} \left\{ 2\alpha k^2 \frac{e^{\alpha kt}}{\left(e^{\alpha kt} - 1\right)} \left[\frac{e^{\alpha kt}}{\left(e^{\alpha kt} - 1\right)} \left(\frac{1-\alpha}{\alpha} \right) + 1 \right] + k^2 \frac{e^{2\alpha kt}}{\left(e^{\alpha kt} - 1\right)^2} - \frac{1}{e^{\frac{-4x}{3}} \left(e^{\alpha kt} - 1\right)^{\frac{2}{\alpha}}} - \frac{3}{4}\lambda^2 \right\}. \tag{89}$$

The physical parameters such as anisotropy parameter of the expansion, shear scalar and expansion scalar as defined in (11), (12) and (13) are

$$\theta = \frac{3k e^{\alpha kt}}{e^{\alpha kt} - 1}; \tag{90}$$

$$A_m = 0; \tag{91}$$

$$\sigma^2 = 0. \tag{92}$$

6 Hypersurface-Homogenous Model

The general form of a hypersurface-homogeneous space-time can be described by the metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)[dy^2 + \Sigma^2(y,K)dz^2], \tag{93}$$

where A and B are functions of time t. The function $\Sigma^2(y,K)$ is associated with the group acting on hypersurface-homogeneous space-time and $\Sigma^2(y,K) = (\sin y, y, \sinh y)$ for $K = (1, 0, -1)$ with K is the curvature.

The Energy momentum tensor of fluid is taken as

$$T_v^u = diag\left[T_1^1, T_2^2, T_3^3, T_4^4\right], \tag{94}$$

Allowing for anisotropy in the pressure of the fluid, and thus in its EoS parameter, gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor given in equation (94) as follows:

$$\begin{aligned} T_v^u &= diag\left[p_x, p_y, p_z, -\rho\right], \\ &= diag\left[w_x, w_y, w_z, -1\right]\rho, \\ &= diag\left[w, (w+\delta), (w+\delta), -1\right]\rho, \end{aligned} \tag{95}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressures and w_x, w_y, w_z are the directional EoS parameter along the x, y, z axes respectively. Now, parametrizing the deviation from isotropy by setting $w_x, w_y, w_z = w$ then introducing skewness parameter δ which is the derivative from w on y and z axes. Here w and δ are not necessarily constants and can be functions of the cosmic time t.

In a co-moving co-ordinate system, the field equations for the metric (93) with the help of equations (95) and (4) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} - \frac{3}{4}\lambda^2 = -w\rho, \tag{96}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{3}{4}\lambda^2 = -(w+\delta)\rho, \tag{97}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} + \frac{3}{4}\lambda^2 = \rho, \tag{98}$$

where the overhead dot (.) denotes derivative with respect to the cosmic time t.

There are three linearly independent equations (96)–(98) with five unknowns A, B, w, ρ, δ.

We define the spatial volume V of hypersurface-homogeneous space-time as

$$V = a^3 = AB^2. \tag{99}$$

Using equations (10) and (99), we have

$$(e^{\alpha kt} - 1)^{\frac{3}{\alpha}} = AB^2. \tag{100}$$

To solve the above set of highly non linear equations, we assume the relation between the metric coefficients

$$B = A^n, \tag{101}$$

where n is an arbitrary constant.

Using equations (100) and (101), we obtain

$$A = (e^{\alpha kt} - 1)^{\frac{3}{\alpha(2n+1)}}, \tag{102}$$

$$B = (e^{\alpha kt} - 1)^{\frac{3n}{\alpha(2n+1)}}. \tag{103}$$

Hence a hypersurface-homogeneous space-time described by the line element (93) corresponding to equations (102) and (103) can be written in the form

$$ds^2 = -dt^2 + (e^{\alpha kt} - 1)^{\frac{6}{\alpha(2n+1)}} dx^2 + (e^{\alpha kt} - 1)^{\frac{6n}{\alpha(2n+1)}} [dy^2 + \Sigma^2(y, K) dz^2]. \tag{104}$$

Using equations (10) and (99), we get

$$V = (e^{k\alpha t} - 1)^{\frac{3}{\alpha}}. \tag{105}$$

Using equations (102) and (103), the directional Hubble parameters are found as

$$H_x = \left(\frac{3k}{2n+1}\right) \frac{e^{\alpha kt}}{e^{\alpha kt} - 1}, \tag{106}$$

$$H_y = H_z = \left(\frac{3nk}{2n+1}\right) \frac{e^{\alpha kt}}{e^{\alpha kt} - 1}. \tag{107}$$

Using equations (98), (102) and (103), we obtain the energy density as

$$\rho = \frac{9nk^2}{(2n+1)^2} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} (2+n) + \frac{K}{(e^{\alpha kt} - 1)^{\frac{6n}{\alpha(2n+1)}}} + \frac{3}{4}\lambda^2. \tag{108}$$

Using equations (96), (102) and (103), the EoS parameter is

$$w = \frac{-1}{\rho} \left\{ \frac{6\alpha nk^2}{(2n+1)} \frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} \left[\frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)} \left(\frac{3n}{\alpha(2n+1)} - 1 \right) + 1 \right] + \frac{9n^2k^2}{(2n+1)^2} \frac{e^{2\alpha kt}}{(e^{\alpha kt} - 1)^2} + \frac{1}{(e^{\alpha kt} - 1)^{\frac{6n}{\alpha(2n+1)}}} - \frac{3}{4}\lambda^2 \right\}. \tag{109}$$

Using equations (96), (97), (102) and (103), the skewness parameter is

$$\delta = -\frac{1}{\rho}\left\{ \frac{3\alpha k^2}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left(\frac{3}{\alpha(2n+1)}-1\right)+1\right] \right.$$
$$- \frac{3n\alpha k^2}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left(\frac{3n}{\alpha(2n+1)}-1\right)+1\right]$$
$$+ \frac{9n^2k^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2} - \frac{9nk^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}$$
$$\left. + \frac{K}{(e^{\alpha kt}-1)^{\frac{6n}{\alpha(2n+1)}}}\right\}. \quad (110)$$

The physical parameters such as anisotropy parameter of the expansion, shear scalar and expansion scalar as defined in (11), (12) and (13) are

$$\theta = \frac{3ke^{\alpha kt}}{e^{\alpha kt}-1}; \quad (111)$$

$$A_m = \frac{2(n-1)^2}{(2n+1)^2}; \quad (112)$$

$$\sigma^2 = \frac{3k^2(n-1)^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}. \quad (113)$$

7 Bianchi type-II, VIII and IX Model

We consider a spatially homogeneous Bianchi type-II, VIII and IX metric of the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (B^2I^2(y) + A^2h^2(y))dz^2 + 2A^2h(y)dxdz, \quad (114)$$

where A and B are functions of cosmic time t only. It represents

— Bianchi type-II: if $I(y) = 1)$ and $h(y) = y$

— Bianchi type-VIII: if $I(y) = \cosh y$ and $h(y) = \sinh y$

— Bianchi type-IX: if $I(y) = \sin y$ and $h(y) = \cos y$

The energy momentum tensor of fluid is taken as

$$T^u_v = diag\left[T^1_1, T^2_2, T^3_3, T^4_4\right], \quad (115)$$

allowing for anisotropy in the pressure of the fluid, and thus in its EoS parameter, gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor given in equation (115) as follows:

$$\begin{aligned} T_v^u &= diag\left[p_x, p_y, p_z, -\rho\right], \\ &= diag\left[w_x, w_y, w_z, -1\right]\rho, \\ &= diag\left[w, (w+\delta), (w+\delta), -1\right]\rho, \end{aligned} \tag{116}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressures and w_x, w_y, w_z are the directional EoS parameter along the x, y, z axes respectively. Now, parametrizing the deviation from isotropy by setting $w_x, w_y, w_z = w$ then introducing the skewness parameter δ which is the derivative from w on $y$and z axes. Here w and δ are not necessarily constants and can be functions of the cosmic time t.

In a co-moving co-ordinate system, the field equations for the metric (114) with the help of equations (116) and (4) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\delta^{'}}{B^2} - \frac{3}{4}\frac{A^2}{B^4} - \frac{3}{4}\lambda^2 = -w\rho, \tag{117}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{1}{4}\frac{A^2}{B^4} - \frac{3}{4}\lambda^2 = -(w+\delta)\rho, \tag{118}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\delta^{'}}{B^2} - \frac{1}{4}\frac{A^2}{B^4} + \frac{3}{4}\lambda^2 = \rho, \tag{119}$$

where the overhead dot (.) denote derivative with respect to the cosmic time t.

When $\delta^{'} = 0, -1, 1$ the field equations (117)-(119) correspond to the Bianchi type II,VIII and IX universe respectively. The directional parameters in the direction of x, y and z for the Bianchi type-II, VIII and IX metric may be defined as follows

$$H_x = \frac{\dot{A}}{A}, H_y = H_z = \frac{\dot{B}}{B}\,.$$

There are three linearly independent equations (117)-(119) with five unknowns A, B, w, ρ, δ.

We define the spatial volume $V$of Bianchi type-II, VIII and IX space-time as

$$V = a^3 = AB^2. \tag{120}$$

Using equations (10) and (120), we have

$$(e^{\alpha kt} - 1)^{\frac{3}{\alpha}} = AB^2. \tag{121}$$

To solve the above set of highly non linear equations, we assume the relation between the metric coefficients

$$B = A^n, \tag{122}$$

where n is arbitrary constant.

Using equations (121) and (122), we obtain

$$A = (e^{\alpha kt} - 1)^{\frac{3}{\alpha(2n+1)}}, \tag{123}$$

$$B = (e^{\alpha kt} - 1)^{\frac{3n}{\alpha(2n+1)}}. \tag{124}$$

Using equations (10) and (121), we get

$$V = (e^{k\alpha t} - 1)^{\frac{3}{\alpha}}. \tag{125}$$

Using equations (123) and (124), the directional Hubble parameters are found as

$$H_x = \left(\frac{3k}{2n+1}\right)\frac{e^{\alpha kt}}{e^{\alpha kt} - 1}, \tag{126}$$

$$H_y = H_z = \left(\frac{3nk}{2n+1}\right)\frac{e^{\alpha kt}}{e^{\alpha kt} - 1}. \tag{127}$$

Using equations (119), (123) and (124), we obtain the energy density as

$$\rho = \frac{9nk^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{\left(e^{\alpha kt} - 1\right)^2}(2+n) + \frac{\delta'}{\left(e^{\alpha kt} - 1\right)^{\frac{6n}{\alpha(2n+1)}}} - \frac{1}{4}\left(e^{\alpha kt} - 1\right)^{\frac{6(1-2n)}{\alpha(2n+1)}} + \frac{3}{4}\lambda^2. \tag{128}$$

Using equations (117), (123) and (124), the EoS parameter is

$$w = \frac{-1}{\rho}\left\{\frac{6\alpha nk^2}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)}\left[\frac{e^{\alpha kt}}{(e^{\alpha kt} - 1)}\left(\frac{3n}{\alpha(2n+1)} - 1\right) + 1\right] + \frac{9n^2k^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{\left(e^{\alpha kt} - 1\right)^2} + \frac{\delta'}{\left(e^{\alpha kt} - 1\right)^{\frac{6n}{\alpha(2n+1)}}} - \frac{3}{4}\left(e^{\alpha kt} - 1\right)^{\frac{6(1-2n)}{\alpha(2n+1)}} - \frac{3}{4}\lambda^2\right\}. \tag{129}$$

Using equations (117), (118), (123) and (124), the skewness parameter is

$$\delta = -\frac{1}{\rho}\Bigg\{ \frac{3\alpha k^2}{(2n+1)} \frac{e^{\alpha kt}}{(e^{\alpha kt}-1)} \left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)} \left(\frac{3}{\alpha(2n+1)} - 1 \right) + 1 \right]$$
$$- \frac{3n\alpha k^2}{(2n+1)} \frac{e^{\alpha kt}}{(e^{\alpha kt}-1)} \left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)} \left(\frac{3n}{\alpha(2n+1)} - 1 \right) + 1 \right]$$
$$+ \frac{9n^2k^2}{(2n+1)^2} \frac{e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2} - \frac{9n^2k^2}{(2n+1)^2} \frac{e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}$$
$$- \frac{\delta'}{\left(e^{\alpha kt}-1\right)^{\frac{6n}{\alpha(2n+1)}}} + \left(e^{\alpha kt}-1\right)^{\frac{6(1-2n_-}{\alpha(2n+1)}} \Bigg\}. \quad (130)$$

The physical parameters such as anisotropy parameter of the expansion, shear scalar and expansion scalar as defined in (11), (12) and (13) are

$$\theta = \frac{3ke^{\alpha kt}}{e^{\alpha kt}-1}; \quad (131)$$

$$A_m = \frac{2(n-1)^2}{(2n+1)^2}; \quad (132)$$

$$\sigma^2 = \frac{3k^2(n-1)^2}{(2n+1)^2} \frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}. \quad (133)$$

8 Discussion

The physical and geometrical behaviors of all the anisotropic models are as follows:

a) The expansion scalar (θ):

The evolution of expansion scalar for different values of $\alpha(\alpha = 0.5, 1, 2, 3)$ is as shown in Figure 1. It is observed that the expansion is infinite at $t = 0$ but as cosmic time increases it decreases and halts after some finite value of t. The model starts with a big-bang from its singular state at $t = 0$ and continues to expand till $t = \infty$.

b) The deceleration parameter q

The variation of deceleration parameter q against mean scale factor a for different values of $\alpha(\alpha = 0.5, 1, 2, 3)$ is as shown in Figure 2. The sign of q indicates whether the model inflates or not. A positive sign of q corresponds to the standard decelerating model whereas the negative sign of q indicates inflation. The deceleration parameter is in the range $-1 \leq q \leq 0.5$ which matches with the observations made by (Riess et al. [44]; Permutter et al. [74]) reveal that the present universe is accelerating. The present DE models have a transition of the

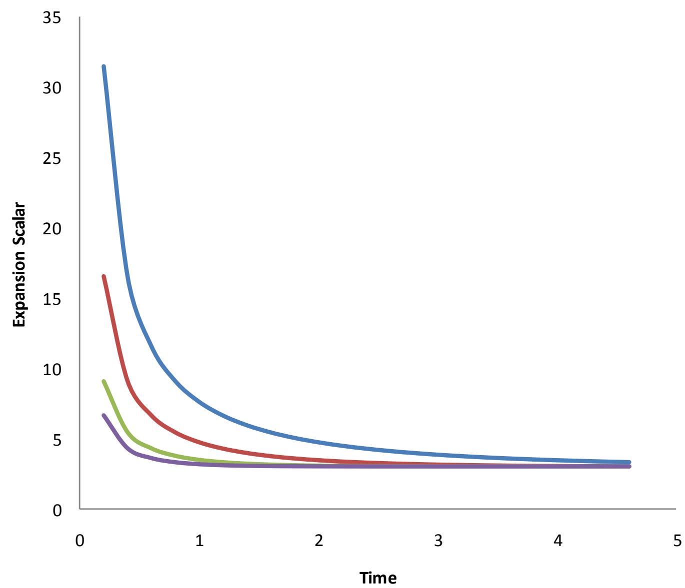

Figure 1. Evolution of expansion scalar.

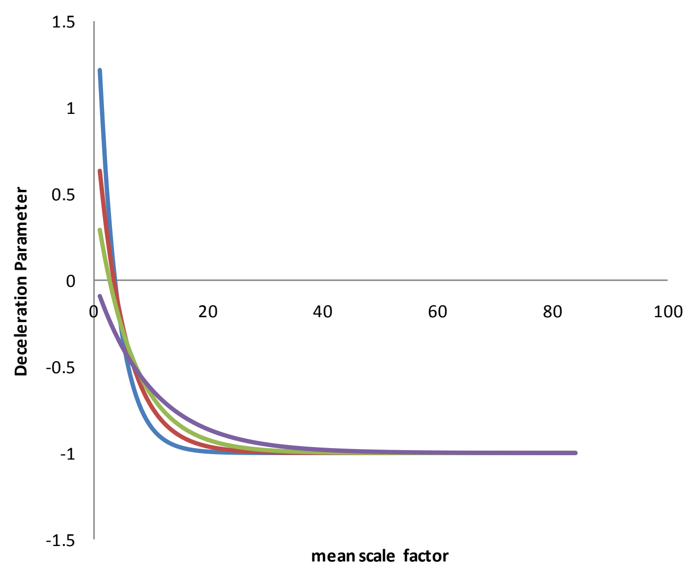

Figure 2. Deceleration parameter vs mean scale factor.

universe from the early deceleration phase to the recent acceleration phase (see, Figure 2) which agrees with recent observations (Caldwell et al. [75]).

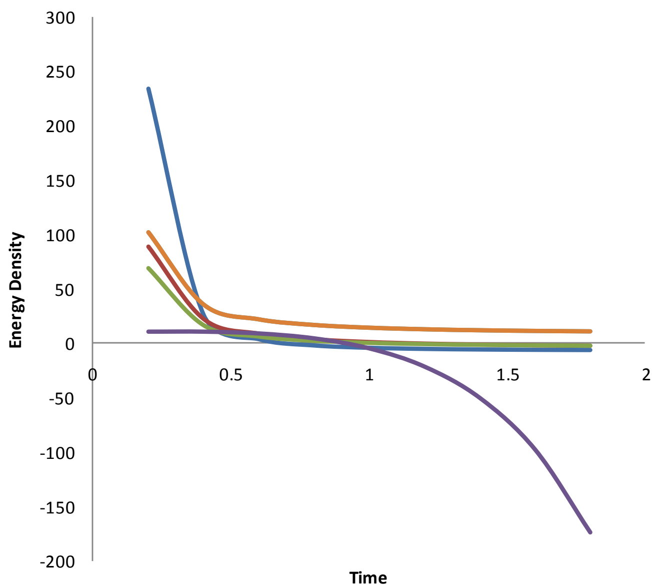

Figure 3. Evolution of density energy ρ.

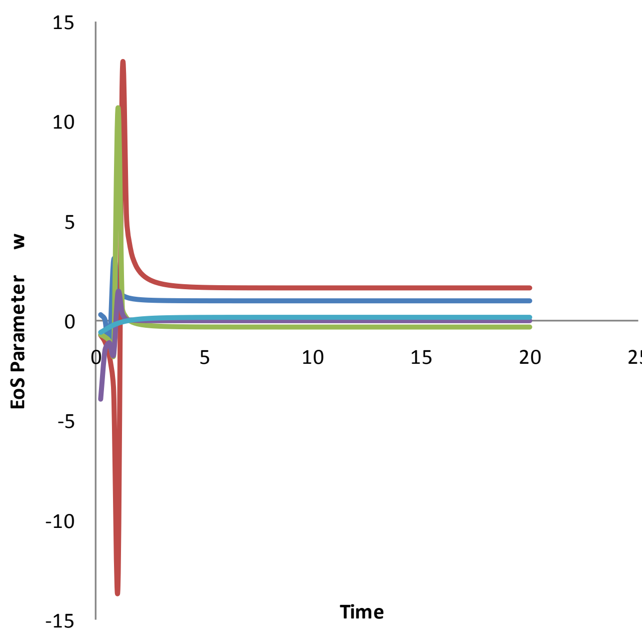

Figure 4. EoS parameter w vs. time t.

c) The anisotropic parameter of expansion A_m

It is observed that in non-static plane symmetric model anisotropy increases as time increases and then decreases to zero after some time and remains zero after some finite time. Hence, the model reaches to isotropy after some finite time which matches with the recent observation as the universe is isotropic at large scale. In case of Bianchi type-V, we observe from equations (91) and (92), the model in this theory becomes isotropic and shear free during the evolution of the universe. That is, there is a transition from decelerated phase to accelerated phase in accordance with the observations (Caldwell et al. [75]). Whereas the B-III, Kantowaski–Sachs, Hypersurface-Homogenous, B-II, VIII & IX models are anisotropic throughout the evolution of the universe and it does not depend on the cosmic time.

d) The energy density ρ

We observe that the models start evolving with a big bang at $t = 0$ with energy density ρ having infinite. With the increase in time t, i.e. $t \to \infty$, the energy density tends to finite value. The physical behavior of the energy density resembles with the investigations of J.P. Singh [76].

e) The equation of state parameter (EoS) w

The SN Ia data suggests that $-1.67 < w < -0.62$ (Knop et al. [46]) while the limit imposed on w by a combination of SN Ia data (with CMB anisotropy) and galaxy clustering statistics is $-1.33 < w < -0.79$ (Tegmark et al. [51]). Figure 4, clearly shows that w evolves within a range, which is in nice agreement with SN Ia and CMB observations. We observe that in early stage of evolution of the universe, the EoS parameter ω was positive (i.e. the universe was matter dominated) and at late time it is evolving with negative value (i.e. at the present epoch).

9 Conclusion

In this paper, we have studied anisotropic and homogenous Bianchi Type-III, Kantowaski-Sachs, Bianchi Type-V, Non-Static Plane Symmetric, Hypersurface-Homogeneous, Bianchi Type-II, VIII, IX space-times under the assumption on the anisotropy of the fluid within the frame work of Lyra manifold. A special form of deceleration parameter (q) which gives an early deceleration and late time accelerating cosmological models has been utilized to solve the field equations. The physical and geometrical behaviors of the models are also studied and analyzed in details. Thus the present Dark Energy models have transition of universe from the early deceleration phase to current acceleration phase which is in good agreement with recent observations (Caldwell et al.[75]). For the models, we observe that the spatial volume V is zero at $t = 0$ and expansion scalar θ is infinite at $t = 0$ which shows that the universe starts evolving with zero volume and infinite rate of expansion at $t = 0$. The scale factors also vanish at $t = 0$ and hence the model has a "point type" singularity at the initial epoch which resembles with R.K. Tiwari [77].

The theoretical arguments suggest and observational data show, the universe was anisotropic at the early stage. Here we are dealing not only with the present state of the universe, but drawing a picture of the universe from the remote past to present day. We use the Bianchi model as one of many models able to describe initial anisotropy that dies away as the universe evolves. Hence from the theoretical perspective, the present models can be a viable model to explain the late time acceleration of the universe. In other words, the solutions presented here can be one of the potential candidates to describe the present universe as well as the early universe. Also late time acceleration is in agreement with the observations of type Ia supernovae made by Hubble Space Telescope (HST) (Riess et al. [45]). It is interesting to note that our investigations resembles with the results of Adhav et al.[73]. Thus the resulting models are astrophysical relevant.

References

[1] H. Weyl (1918) Sber. Preuss. Akad. d. Wissenschaften.

[2] G. Lyra (1951) *Math. Z.* **54** 52.

[3] D.K. Sen, K.A. Dunn (1971) *J. Math. Phys*. **12** 578.

[4] W.D. Halford (1970) *Aust. J. Phys.* **23** 863.

[5] W.D. Halford (1972) *J. Math. Phys*. **13** 1399.

[6] F. Hoyle (1948) *Mon. Not. R. Astron. Soc*. **108** 252.

[7] F. Hoyle, J.V. Narlikar (1963) *Proc. R. Soc. Lond. Ser. A* **273** 1.

[8] F. Hoyle, J.V. Narlikar (1964) *Proc. R. Soc. Lond. Ser. A* **282** 1.

[9] J.S. Jeavons, C.B.G. McIntosh, D.K. Sen (1975) *J. Math. Phys*. **16** 320.

[10] H.H. Soleng (1987) *Gen. Relativ. Gravit*. **19** 1213.

[11] H.H. Soleng (1988) *Class. Quantum Grav*. **5** 1489.

[12] K.S. Bhamra (1974) *Aust. J. Phys.* **27** 541.

[13] T.M. Karade, S.M. Borikar (1978) *Gen. Relativ. Gravit*. **9** 431.

[14] S.B. Kalyanshetti, S.M. Wagmode (1982) *Gen. Relativ. Gravit*. **14** 823.

[15] D.R.K. Reddy, R. Innaiah (1986) *Astrophys. Space Sci*. **123** 49.

[16] D.R.K. Reddy, R. Wenkateswar (1987) *Astrophys. Space Sci*. **136** 183.

[17] A. Beesham (1988) *Aust. J. Phys*. **41** 833.

[18] T. Singh, G.P. Singh (1991) *J. Math. Phys*. **32** 2456.

[19] T. Singh, G.P. Singh (1991) *Nuovo Cimento B* **106** 617.

[20] T. Singh, C.P. Singh (1992) *Int. J. Theor. Phys*. **31** 1433.

[21] T. Singh, G.P. Singh (1993) *Fortschr. Phys*. **41** 737.

[22] G.P. Singh, K. Desikan (1997) *Pramana J. Phys*. **49** 205.

[23] F. Rahaman, S. Chakraborty, S. Das, R. Mukherjee, M. Hossain, N. Begum (2003) *Astrophys. Space Sci*.**288** 483.

[24] F. Rahaman, N. Begum, G. Bag, B.C. Bhui (2005) *Astrophys. Space Sci*. **299** 211.

[25] A. Pradhan, I. Aotemshi, G.P. Singh (2003) *Astrophys. Space Sci*. **288** 315.
[26] A. Pradhan, A.K. Vishwakarma (2004) *J. Geom. Phys*. **49** 332.
[27] A. Pradhan, L. Yadav, A.K. Yadav (2005) *Astrophys. Space Sci*. **299** 31.
[28] A. Pradhan, V. Rai, S. Otarod *Fizika B* **15** 23.
[29] A. Pradhan, K.K. Rai, A.K. Yadav (2007) *Braz. J. Phys*. **37** 1084.
[30] A. Pradhan, H. Amirhashchi, H. Zainuddin (2011) *Int. J. Theor. Phys*. **50** 56.
[31] S. Agarwal, R.K. Pandey, A. Pradhan (2011) *Int. J. Theor. Phys*. **50** 296.
[32] R. Casama, C. Melo, B. Pimentel (2006) *Astrophys. Space Sci*. **305** 125.
[33] R. Bali, N.K. Chandnani (2008) *J. Math. Phys*. **49** 032502.
[34] R. Bali, N.K. Chandnani (2009) *Int. J. Theor. Phys*. **48** 1523.
[35] R. Bali, N.K. Chandnani (2009) *Int. J. Theor. Phys*. **48** 3101.
[36] S. Kumar, C.P. Singh (2008) *Int. J. Mod. Phys. A* **23** 813.
[37] J.K. Singh (2008) *Astrophys. Space Sci*. **314** 361.
[38] V.U.M. Rao, T. Vinutha, M.V. Santhi (2008) *Astrophys. Space Sci*. **314** 213.
[39] A. Pradhan (2009) *J. Math. Phys*. **50** 022501.
[40] A. Pradhan (2007) *Fizika B* **16** 205.
[41] G.P. Singh, A.Y. Kale (2009) *Int. J. Theor. Phys*. **48** 3049.
[42] S. Ram, M. Zeyauddin, C.P. Singh (2010) *J. Geom. Phys*. **60** 3049.
[43] R. Chaubey (2012) *Int. J. Theor. Phys*.; doi:10.1007/s10773-012-1285-5.
[44] A.G. Riess, et al. (1998) *Astron. J.* **116** 1009.
[45] A.G. Riess, et al. (2004) *Astrophys. J*. **607** 665.
[46] R.A. Knop, et al. (2003) *Astrophys. J.* **598** 102.
[47] D.N. Spergel, et al. (2003) *Astrophys. J. Suppl. Ser*. **148** 175.
[48] D.N. Spergel, et al. (2007) *Astrophys. J. Suppl. Ser*. **170** 377.
[49] E. Komatsu, et al. (2009) *Astrophys. J. Suppl. Ser*. **180** 330.
[50] C.L. Bennett, et al. (2003) *Astrophys. J. Suppl. Ser*. **148** 1.
[51] M. Tegmark, et al. (2004) *Astrophys. J*. **606** 702.
[52] U. Seljak, et al. (2005) *Phys. Rev. D* **71** 103515.
[53] D.J. Eisenstein, et al. (2005) *Astrophys. J*. **633** 560.
[54] W.J. Percival, et al. (2007) *Mon. Not. R. Astron. Soc*. **381** 1053.
[55] G. Hinshaw, et al. (WMAP Collaboration) (2009) *Astrophys. J. Suppl*. **180** 225.
[56] O. Akarsu, C.B. Kilinc (2010) *Gen. Relativ. Gravit*. **42** 119.
[57] O. Akarsu, C.B. Kilinc (2010) *Gen. Relativ. Gravit*. **42** 763.
[58] A.K. Yadav, F. Rahaman, S. Ray (2011) *Int. J. Theor. Phys*. **50** 871.
[59] A.K. Yadav, L. Yadav (2011) *Int. J. Theor. Phys*. **50** 218.
[60] S. Kumar, A.K. Yadav (2011) *Mod. Phys. Lett. A* **26** 647.
[61] A.K. Yadav (2011) *Astrophys. Space Sci*. **335** 565.
[62] K.S. Adhav, et al. (2011) *Astrophys. Space Sci*. **331** 689.
[63] A.K. Yadav, B. Saha (2012) *Astrophys. Space Sci.* **337** 759.
[64] A. Pradhan, H. Amirhashchi (2011) *Astrophys. Space Sci*. **332** 441; arXiv:1010.2362 [physics.gen-ph].
[65] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy (2012) *Astrophys. Space Sci*. **338** 351-354.

[66] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy (2012) *Int. J. Theor. Phys*. **51** 2857; doi:10.1007/s10773-012-1161-3.

[67] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy (2012) *Int. J. Theor. Phys*. **51** 1997-2002.

[68] D. Saez, V.J. Ballester (1986) *Phys. Lett. A* **113** 467.

[69] K.S. Adhav (2011) *Int. J. Astron. Astrophys.* **1** 204-209.

[70] D.K. Sen (1957) *Z. Phys.* **149** 311.

[71] N. Banerjee, S. Das (2005) *Gen. Relativ. Gravit*. **37** 1695.

[72] A.K. Singh, U. Debnath (2009) *Int. J. Theor. Phys*. **48** 351.

[73] K.S. Adhav, A.S. Bansod, H.G. Ajmire (2013) *Astrophys. Space Sci.* **345** 405-413.

[74] S. Perlmutter, et al. (1999) *Astrophys. J.* **517** 565.

[75] R.R. Caldwell, et al. (2006) *Phys. Rev. D* **73** 023513.

[76] J.P. Singh (2008) *Astrophys. Space Sci.* **318** 103-107.

[77] R.K. Tiwari (2008) *Astrophys. Space Sci.* **318** 243-247.