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Magnetized Dark Energy Cosmological Models with Time Dependent Cosmological Term in Lyra Geometry

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Abstract. In the present paper we have investigated a magnetized dark energy Bianchi type VI_0 space time with time dependent cosmological term within the framework of Lyra's manifold with uniform and time varying displacement field. In order to obtain a determinate solution of the field equations we have assumed a new special law for the deceleration parameter proposed by Akarsu and Dereli and very recently used by Abdussattar and Prajapati. Without loss of generality, by taking into consideration the constant of integration to zero one particular model with some physical parameters are discussed more in detail. We have examined the isotropy and expansion of the universe.

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1 Introduction

Recently researchers are taking a lot of interest in cosmological models with dark energy in general theory of relativity as well as in modified theories of gravity because the most of remarkable observational discoveries have shown that our universe is currently undergoing an accelerated expansion. It has been confirmed by high precession data from type Ia supernovae and cosmic microwave background radiation (CMBR) data [1-5] and large scale structure which seems to hint (support) that the present universe is dominated by an unknown form of now a day called as dark energy [6,7]. On the basis of these astronomical observations cosmologists have accepted the existence of dark energy which is a fluid with negative pressure which account more than 70% of the total energetic content of the universe to be mostly responsible for the accelerated expansion of the universe due to repulsive gravitations [8,9]. There are many proposals to explain the dark energy as well as the unknown nature of the dark energy but cosmologists have proposed many candidates for dark energy to fit the current observations such as quintessence [10,11], phantom [12], tachyon [13], EoS parameter [14,15], chaplygin gas [16,17]. In order to extract the properties of the dark energy components of the universe from the observational data, it is focused on the determination of it's equation of state $\omega = p/\rho$. Dark energy has been conventionally characterized by the equation of the state parameter which is not necessarily constant. For the determination of this parameter as a function of cosmic time an analysis of the experimental data has been carried out [18]. The vacuum energy ($\omega = -1$) is the simplest dark energy candidate which is mathematically equivalent to cosmological constant (Λ). The other conventional alternative candidates are quintessence ($\omega > -1$), phantom energy ($\omega < -1$) have the time dependent EoS parameter. Due to some limitations in observational evidence in distinguishing between constant and variable ω [19] it is considered as a constant but in general it is a function of time or red shift. In the present paper ω is a function of cosmic time.

There are many theories, which have been proposed as alternatives to Einstein's theory of general relativity. Einstein's theory of general relativity is based on geometrical description of gravitation. Many researchers have used their efforts to generalize the idea of geometrizing the gravitation to include a geometrical description of electromagnetism. Weyl [20] was the first worker who has proposed a more general theory by formulating a new kind of gauge theory which involves metric tensor to geometrize gravitation and electromagnetism. But because of non-integrability of length of vector under parallel displacement this theory was criticized. In order to remove the non-integrability condition in Wey's theory Lyra [21] has suggested a new geometry called Lyra's geometry which is a modification of Riemannian geometry introducing a gauge function into the structureless manifold, as a result of which the cosmological constant arises naturally from the geometry. But later on in further investigation Sen and Sen, et al. [22,23] formulated new scalar tensor theory of gravitation based on Lyra's geometry constructed an analogue of the Einstein field equations. In 1972 Halford [24] has put his opinion that scalar tensor treatment based on Lyra's geometry predicts the same effects as in general relativity. Some of the authors studied cosmological models based on Lyra's manifold with a constant displacement vector Pawar et al. [24,25]. As a result of the fact that the displacement field used in the model to be constant is only for the convenience. Many authors [26-32] have used uniform as well as time dependent displacement field in the cosmological models.

In the present paper we have studied Bianchi type VI_0 cosmological model with magnetized anisotropic dark energy and cosmological term in the frame work of Lyra's geometry for uniform and time dependent displacement field. Some physical parameters of the models are discussed.

2 Metric and Field Equations

We consider spatially homogeneous and anisotropic Bianchi type VI_0 metric as

$$ds^2 = -dt^2 + a^2(t)\,dx^2 + b^2(t)\,e^{-2mx}dy^2 + c^2(t)\,e^{2mx}dz^2, \tag{2.1}$$

where the metric potentials a, b and c are the functions of cosmic time t, and m is a nonzero constant.

The energy momentum tensor magnetized anisotropic dark energy fluid is given by

$$\begin{aligned} T_i^j &= diag\left[T_4^4, T_1^1, T_2^2, T_3^3\right] \\ &= diag\left[\rho + \rho_B, -p_x - \rho_B, -p_y + \rho_B, -p_z + \rho_B\right], \end{aligned} \tag{2.2}$$

where ρ is the energy density of the dark energy fluid, ρ_B is the energy density of magnetic field and p_x, p_y, p_z are the pressures on x, y and z axes, respectively.

The anisotropic fluid is characterized by the equation of state $p = \omega\rho$, where ω is EoS parameter not necessarily constant.

Thus Eq. (2.2) can be written as

$$T_i^j = diag\left[\rho + \rho_B, -\omega\rho - \rho_B, -(\omega+\gamma)\rho + \rho_B, -(\omega+\delta)\rho + \rho_B\right], \tag{2.3}$$

where $\omega_x = \omega, \omega_y = \omega + \gamma, \omega_z = \omega + \delta$ are the directional EoS parameters on the x, y and z axes, respectively, and γ, δ are the skewness parameter along y and z axes, respectively. The field equations in Lyra's manifold as obtained by Sen (1957) with cosmological term used by M.K. Verma *et al.*(2012) not necessarily constant [22,40] is given by

$$R_{ij} - \frac{1}{2}(R + 2\wedge)\,g_{ij} + \frac{3}{2}\varphi_i\varphi_j - \frac{3}{4}g_{ij}\varphi_\alpha\varphi^\alpha = -T_{ij}, \tag{2.4}$$

where $g_{ij}u^iu^j = 1$ and $u^i = (1, 0, 0, 0)$ are the four velocity vectors, $\varphi_\alpha = (0, 0, 0, \beta(t))$ is the displacement vector. R_{ij} is the Ricci tensor, R is the Ricci scalar, T_{ij} is the energy momentum tensor, and $\wedge$ is the cosmological term not necessarily constant.

Now assuming the co-moving co-ordinate system field Eqs. (2.4) of the metric (2.1) with the help of equation (2.3) takes the form

$$\frac{b_{44}}{b} + \frac{c_{44}}{c} + \frac{b_4c_4}{bc} + \left(\frac{m}{a}\right)^2 + \frac{3}{4}\beta^2 - \wedge = \omega\rho + \rho_B \tag{2.5}$$

$$\frac{a_{44}}{a} + \frac{c_{44}}{c} + \frac{a_4c_4}{ac} - \left(\frac{m}{a}\right)^2 + \frac{3}{4}\beta^2 - \wedge = (\omega+\gamma)\rho - \rho_B \tag{2.6}$$

$$\frac{a_{44}}{a} + \frac{b_{44}}{b} + \frac{a_4b_4}{ab} - \left(\frac{m}{a}\right)^2 + \frac{3}{4}\beta^2 - \wedge = (\omega+\delta)\rho - \rho_B \tag{2.7}$$

$$\frac{a_4 b_4}{ab} + \frac{b_4 c_4}{bc} + \frac{a_4 c_4}{ac} - \left(\frac{m}{a}\right)^2 - \frac{3}{4}\beta^2 - \wedge = -(\rho + \rho_B) \tag{2.8}$$

$$m\left(\frac{c_4}{c} - \frac{b_4}{b}\right) = 0 \tag{2.9}$$

3 Solution of the Field Equations

Integrating Eq. (2.9) we get

$$c = kb, \tag{3.1}$$

where k is the constant of integration. Without loss of generality if we choose $k = 1$, we have

$$b = c \tag{3.2}$$

Using equations (3.2) in Eqs. (2.5) to (2.8) we get $\gamma = \delta$ and with this relation overall system reduces and takes the form

$$\frac{2b_{44}}{b} + \left(\frac{b_4}{b}\right)^2 + \left(\frac{m}{a}\right)^2 + \frac{3}{4}\beta^2 - \wedge = \omega\rho + \rho_B \quad , \tag{3.3}$$

$$\frac{a_{44}}{a} + \frac{b_{44}}{b} + \frac{a_4 b_4}{ab} - \left(\frac{m}{a}\right)^2 + \frac{3}{4}\beta^2 - \wedge = (\omega + \delta)\,\rho - \rho_B, \tag{3.4}$$

$$\frac{2a_4 b_4}{ab} + \left(\frac{b_4}{b}\right)^2 - \left(\frac{m}{a}\right)^2 - \frac{3}{4}\beta^2 - \wedge = -(\rho + \rho_B)\,. \tag{3.5}$$

By the law of energy conservation $\left(T^{ij}_{;j} = 0\right)$ we have the Bianchi identity as

$$-\rho_4 + \left[(\omega - 1)\frac{a_4}{a} + 2(\omega + \delta - 1)\frac{b_4}{b}\right]\rho - \left[(\rho_B)_4 + 4\frac{b_4}{b}\rho_B\right] = 0. \tag{3.6}$$

Here we assume that the magnetized dark energy is minimally interacting, therefore by King and Coles [33] the Bianchi identity given by Eq. (3.6) can be split up into two components namely energy momentum tensor for the anisotropic dark energy and the energy momentum tensor for the magnetic field separately

$$-\rho_4 + \left[(\omega - 1)\frac{a_4}{a} + 2(\omega + \delta - 1)\frac{b_4}{b}\right]\rho = 0 \tag{3.7}$$

and

$$(\rho_B)_4 + 4\frac{b_4}{b}\rho_B = 0. \tag{3.8}$$

The mean generalized Hubble parameter H for the metric (2.1) is defined by

$$H = \frac{1}{3}(H_x + H_y + H_z) = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(\frac{a_4}{a} + 2\frac{b_4}{b}\right), \tag{3.9}$$

where

$$H_x = \frac{a_4}{a}, \qquad H_y = H_z = \frac{b_4}{b} \qquad (\because b = c)\,. \tag{3.10}$$

The average scale factor R and the spatial volume V of the Bianchi type VI_0 are defined by the relation

$$V = R^3 = abc = ab^2\,. \tag{3.11}$$

The expansion scalar θ and shear scalar σ^2 are respectively given by

$$\theta = \left(\frac{a_4}{a} + 2\frac{b_4}{b}\right) \tag{3.12}$$

$$\sigma^2 = \frac{2}{3}\left(\frac{a_4}{a} - \frac{b_4}{b}\right)^2. \tag{3.13}$$

The mean generalized anisotropy parameter Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right). \tag{3.14}$$

The field equations (3.3) to (3.6) are the four independent equations in seven unknowns a, b, $\wedge$, ω, δ, ρ, and ρ_B. Therefore, to get the determinate solution we have to use three more additional conditions. Generally a special law of Hubble's parameter with constant deceleration parameter proposed by Berman [31] is used in order to obtain the exact cosmological dark energy model. But according to Akarsu and Dereli [33] this law is limited only for specially closed and accelerating flat universe with cosmological fluid having quintom behavior. They have proposed a new special law for the deceleration parameter which is a function of cosmic time t, having a negative slope. This new law provides us an opportunity to generalize the different dark energy models better results with with the cosmological observations. Abdussattar and Prajapati, Chaubey and Shukla, Shriram and Priyanka [34-36] are some of the authors who have used this new law of variation deceleration parameter in their cosmological models. In the present paper we have used the same.

According to this new special law the two prominent candidates q and H are related by the relation

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \tag{3.15}$$

Integrating Eq. (3.15) to get the value of average scale factor $R\,(t)$ as

$$R\,(t) = e^{\eta} \exp\left\{\int \frac{dt}{\left[\int (1+q)\,dt + l_1\right]}\right\}, \tag{3.16}$$

where η and l_1 are the constants of integration.

For an explicit determination of $R\,(t)$ we have to integrate the Eq. (3.15). There are two different ways to integrate depending on the choice for the values of deceleration parameter q.

(i) According to Berman q is taken to be a constant either positive or negative which provides an explicit function of $R\,(t)$ and

(ii) according to new law q is taken to vary with cosmic time for an explicit determination of $R\,(t)$ which leads to a possible choice of q as

$$q = -\frac{l_2}{t^2} + (l_3 - 1)\,, \tag{3.17}$$

where $l_2 > 0$ is a parameter having the dimension of square of time and $l_3 > 1$ is a dimensionless constant. Here it is to be noted that for different values of l_2 and l_3 we are getting the different models.

Thus from Eqs. (3.16) and (3.17) we get the time variation scale factor as

$$R\,(t) = e^{\eta} \exp \int \frac{t}{l_3 t^2 + l_1 t + l_2} dt. \tag{3.18}$$

In order to obtain value to $R\,(t)$ integral appearing in Eq. (3.18) can be integrated by using the following three different cases:

(i) $\eta = 0, \quad l_1 = 0$;

(ii) $\eta = 0, \quad l_1 = 2\sqrt{l_2 l_3}$; and

(iii) $\eta = 0, \quad l_1 \neq 2\sqrt{l_2 l_3}$
(because η is the constant of integration, without loss of generality we can choose $\eta = 0$)

In the present paper we have obtained the model by using the first case.

Thus when $\eta = 0$, $l_1 = 0$, Eq. (3.18) gives

$$R\,(t) = \left(l_3 t^2 + l_2\right)^{1/(2l_3)}\,. \tag{3.19}$$

The second condition we have to assume that the expansion scalar (θ) in the model is proportional to shear scalar $(\sigma)^2$ which leads to

$$a = b^n. \tag{3.20}$$

Thus Eqs. (3.2), (3.11), (3.19) and (3.20) give value of the scale factors

$$a = \left(l_3 t^2 + l_2\right)^{\lambda n/l_3} \tag{3.21}$$

and

$$b = c = \left(l_3 t^2 + l_2\right)^{\lambda/l_3}, \tag{3.22}$$

where

$$\lambda = \frac{3}{2\,(n+2)}. \tag{3.23}$$

Thus the metric Bianchi type VI_0 given by Eq. (2.1) wih the help of equations (3.21) and (3.22) takes the form

$$\begin{aligned} ds^2 = -dt^2 + (l_3 t^2 + l_2)^{\frac{2\lambda n}{l_3}} dx^2 \\ + (l_3 t^2 + l_2)^{\frac{2\lambda}{l_3}} [e^{-2mx} dy^2 + e^{2mx} dz^2]. \end{aligned} \tag{3.24}$$

The directional Hubble parameters defined by Eq. (3.10) are

$$H_x = \frac{2n\lambda t}{(l_3 t^2 + l_2)}, \qquad H_y = H_z = \frac{2\lambda t}{(l_3 t^2 + l_2)}. \tag{3.25}$$

The mean generalized Hubble parameter defined by Eq. (3.9) is

$$H = \frac{2\,(n+2)\,\lambda t}{3\,(l_3 t^2 + l_2)}. \tag{3.26}$$

The spatial volume of the model defined by Eq. (3.11) is

$$V = \left(l_3 t^2 + l_2\right)^{\frac{3}{2l_3}}. \tag{3.27}$$

The scalar expansion (θ) and shear scalar $\left(\sigma^2\right)$ defined by Eqs. (3.12) and (3.13) are respectively given by

$$\theta = \frac{2\,(n+2)\,\lambda t}{(l_3 t^2 + l_2)}, \tag{3.28}$$

$$\sigma^2 = \frac{8\,(n-1)^2\,\lambda^2 t^2}{3\,(l_3 t^2 + l_2)^2}. \tag{3.29}$$

The mean generalized anisotropy parameter Δ defined by Eq. (3.14) is zero.

After integrating Eq. (3.8) with little manipulation and using Eq. (3.21), the energy density of the magnetic field is given by

$$\rho_B = \frac{k_1}{(l_3 t^2 + l_2)^{\frac{4\lambda}{l_3}}}, \tag{3.30}$$

where k_1 is the constant of integration.

In order to determine the remaining parameters in the field equations we have used a third condition that EoS parameter ω is proportional to the skewness parameter δ (mathematical condition) used by D.R.K. Reddy et al. [37] such that

$$\omega + \delta = 0. \tag{3.31}$$

Using Eq. (3.4) with Eqs. (3.21), (3.22), (3.30) and (3.31) the cosmological term $\wedge$ in the field equations is given by

$$\wedge = \frac{2\lambda t^2 \left[2\lambda\left(n^2+n+1\right) - l_3\left(n+1\right)\right] + 2\lambda l_2\left(n+1\right)}{\left(l_3 t^2 + l_2\right)^2} - \frac{m^2}{\left(l_3 t^2 + l_2\right)^{\frac{2\lambda n}{l_3}}} + \frac{k_1}{\left(l_3 t^2 + l_2\right)^{\frac{4\lambda}{l_3}}} + \frac{3}{4}\frac{\beta_0^2}{t^{2\alpha}} \quad . \tag{3.32}$$

Using Eq. (3.5) with Eqs. (3.21), (3.22), (3.30) and (3.32), the energy density of the dark energy is given by

$$\rho = \frac{2\lambda l_2\left(n+1\right) - 2\lambda\left[2n\left(1-n\right)\lambda + l_3\left(n+1\right)\right]t^2}{\left(l_3 t^2 + l_2\right)^2} + 2\frac{3}{4}\frac{\beta_0^2}{t^{2\alpha}} \quad . \tag{3.33}$$

Similarly using Eq. (3.3) with Eqs. (3.21), (3.22), (3.30) and (3.31), the EoS parameter ω and the skewness parameter δ are given by

$$\omega = -\delta = \frac{1}{\rho}\left\{\frac{2\lambda t^2\left[4\lambda - 2\lambda\left(n+1\right) + l_3 n\right] + 2\lambda l_2 n}{\left(l_3 t^2 + l_2\right)^2} + \frac{2m^2}{\left(l_3 t^2 + l_2\right)^{\frac{2\lambda n}{l_3}}} - \frac{2k_1}{\left(l_3 t^2 + l_2\right)^{\frac{4\lambda}{l_3}}}\right\}, \tag{3.34}$$

where ρ is given by Eq. (3.33).

In Eqs. (3.32)–(3.34) we have used vector displacement field

$$\beta = \beta_0 / t^{\alpha}\,. \tag{3.35}$$

In the above equations (3.32)–(3.34) if we put $\alpha = 0$ we are getting the model with uniform displacement field and if we put $\alpha = 1$ we are getting a model with time varying displacement field.

4 Discussion and Conclusion

In the present paper we have investigated Bianchi Type VI_0 space-time with time dependent cosmological term within the frame work of Lyra's manifold. In order to determine the exact solution of the required space-time we have used a new special law for the deceleration parameter proposed by Akarsu and Dereli and that has been very recently used by some other authors that EoS parameter ω is proportional to the skewness parameter δ. The observed value of the mean generalized anisotropic parameter of the derived model is zero hence the derived model is isotropic throughout the evolution of the universe. The directional Hubble's parameters as well as mean generalized Hubble's parameters are the function of the cosmic time t and these parameters vanishe for infinitely large value of time t where as these parameters have the finite value when cosmic time is zero. The same behaviours happen in case of the parameters scalar expansion θ and shear scalar σ with respect to cosmic time t. The spatial volume of the model is finite at the initial epoch and increases with increase in cosmic time. The cosmological term $\wedge$, the energy density ρ of the dark energy the EoS parameter ω as well as the skewness parameter δ are functions of the cosmic time t.

References

[1] A. Reiss, *et al.*(1998) *Astron. J.* **116** 1009.

[2] S. Perlmutter, *et al.*(1999) *Astrophys. J.* **517** 565.

[3] C.L. Bennett, *et al.*(2003) *Astrophys. J. Suppl. Ser.* **148** 1.

[4] M. Tegmark, *et al.*(2004) *Astrophys. J.* **606** 702.

[5] A. Reiss, *et al.*(2004) *Aston. J.* **607** 665.

[6] C. Deffayet, *et al.*(2002) *Phys. Rev. D* **65** 044023-044032.

[7] J.L. Tonry, *et al.*(2003) *Astrophys. J.* **594** 1-24.

[8] D.N. Spergel (2007) *Astrophys. J. Suppl. Ser.* **170** 337.

[9] R. Bali, S. Dave (2001) *Pramana* **56** 513-518.

[10] C. Wetterich (1988) *Nucl. Phys. B* **302** 668.

[11] B. Ratra, P.J.E. Peebles (1988) *Phys. Rev. D* **37** 321.

[12] R.R. Caldwell (2002) *Phys. Lett. B.* **545** 23-29.

[13] T. Padmanabhan (2002) *Phys. Rev. D* **66** 021301.

[14] O. Akarsu, C.B. Kilinc (2010) *Gen. Relativ. Gravitation* **42** 119.

[15] O. Akarsu, C.B. Kilinc (2010) *Gen. Relativ. Gravitation* **42** 763.

[16] S.K. Srivastav (2005) *Phys. Lett. B* **619** 1.

[17] M.C. Bento, O. Bertolami, A.A. Sen (2002) *Phys. Rev. D* **66** 043507.

[18] V. Sahni, A. Shafielooa, A. Starobinsky (2008) *Phys. Rev. D* **78** 103502.

[19] R.A. Knop, *et al.*(2003) *Astrophys. J.* **598** 102.

[20] H. Weyl (1918) *Math. Z.* **2** 384.

[21] G. Lyra (1951) *Math. Z.* **54** 52.

[22] D.K. Sen (1957) *Z. Phys. Hadrons Nucl.* **149** 311.

[23] D.K. Sen, K.A. Dunn (1971) *J. Math. Phys*. **12** 578.

[24] V.R. Patil, D.D. Pawar, G.U. Khapekar (2012) *Int. J. Theor. Phys.* **51** 2101-2108; DOI 10.1007/s10773-012-1088-8.

[25] D.D. Pawar, V.R. Patil (2013) *Prespacetime Journal* **4** 312-320.

[26] W.D. Halford (1972) *J. Math. Phys.* **13** 1399.

[27] K.S. Bhamra (1974) *Aust. J. Phys.* **27** 541-547.

[28] T.M. Karade, S.M. Borikar (1978) *Gen. Relativ. Gravitation* **19** 431-436.

[29] H. Soleng (1987) *Gen. Relativ. Gravitation* **19** 1213.

[30] G. Sing, K. Desikan (1997) *Pramana: Physics and Astronomy* **49** 205-212.

[31] A. Pradhan, V.K. Yadav, I. Chakraborty (2001) *Int. J. Mod. Phys.* **10** 339349.

[32] F. Rahaman, N. Begum, B.C. Bhui (2005) *Astrophys. Space Sci.* **299** 211-218.

[33] M.S. Berman (1983)*Nuovo Cimento B* **74** 182.

[34] E.J. King, P. Coles (2007) *Class. Quantum Gravity* **24** 2061.

[35] O. Akarsu, T. Dereli (2012) *Int. J. Theor. Phys*. **51** 612.

[36] AbdusSattar, S.R. Prajapati (20110 *Astrophys. Space Sci*. **331** 657.

[37] R. Chaubey, A.K. Shukla (2013) *Astrophys. Space Sci*. **343** 415.

[38] Shriram, Priyanka, Manishkumar Singh (2013) *Pramana J. Phys.* **81** 67-74.

[39] D.R.K. Reddy, R. Santhi Kumar, T.V. Pradip Kumar (2013) *Int. J. Theor. Phys* **52** 239-245.

[40] M.K. Verma, M.K. Singh, Shri Ram (2012) *Int. J. Theor. Phys.* **51** 1729-1736.