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A New Mechanism of Dynamical Spontaneous Breaking of Supersymmetry *

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Abstract. We present a qualitatively new mechanism for dynamical spontaneous breakdown of supersymmetry. Specifically, we construct a modified formulation of standard minimal $N = 1$ supergravity. The modification is based on an idea worked out in detail in previous publications by some of us, where we proposed a new formulation of (non-supersymmetric) gravity theories employing an alternative volume form (volume element, or generally-covariant integration measure) in the pertinent Lagrangian action, defined in terms of auxiliary (pure-gauge) fields instead of the standard Riemannian metric volume form. Invariance under supersymmetry of the new modified $N = 1$ supergravity action is preserved due to the addition of an appropriate compensating antisymmetric tensor gauge field. This new formalism naturally triggers the appearance of a dynamically generated cosmological constant as an arbitrary integration constant which signifies a spontaneous (dynamical) breaking of supersymmetry. Furthermore, applying the same formalism to anti-de Sitter supergravity allows us to appropriately choose the above mentioned arbitrary integration constant so as to obtain simultaneously a very small positive effective observable cosmological constant as well as a large physical gravitino mass as required by modern cosmological scenarios for slowly expanding universe of today.

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1 Introduction – Gravity-Matter Theories in Terms of Non-Riemannian Volume-Forms on Space-Time Manifold

In a series of previous papers [1–3] (for recent developments, see Refs. [4, 5]) some of us have proposed a new class of generally-covariant (non-supersymmetric) field theory models including gravity – called "two-measure

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theories" (TMT), which appear to be promising candidates for resolution of the dark energy and dark matter problems, the fifth force problem, etc. The principal idea is to employ an alternative volume form (volume element or generally-covariant integration measure on the space-time manifold) in the pertinent Lagrangian action, which is defined in terms of auxiliary pure-gauge fields independent of the standard Riemannian volume form in terms of the Riemannian space-time metric.

To illustrate the formalism let us consider the following type of TMT action of the form relevant in the present context::

$$S = \int d^D x\, \Phi(B) \Big[L^{(1)} + \frac{\varepsilon^{\mu_1 \ldots \mu_D}}{(D-1)!\sqrt{-g}} \partial_{\mu_1} H_{\mu_2 \ldots \mu_D} \Big] + \int d^D x \sqrt{-g}\, L^{(2)} \quad (1)$$

with the following notations:

- The first term in (1) contains a non-Riemannian integration measure density:
$$\Phi(B) \equiv \frac{1}{D!} \varepsilon^{\mu_1 \ldots \mu_D}\, \partial_{\mu_1} B_{\mu_2 \ldots \mu_D} \ , \quad (2)$$
where $B_{\mu_1 \ldots \mu_{D-1}}$ is a rank $(D-1)$ antisymmetric tensor gauge field.
- The Lagrangians $L^{(1,2)} \equiv \frac{1}{2\kappa^2} R + L^{(1,2)}_{\rm matter}$ include both standard Einstein-Hilbert gravity action as well as matter/gauge-field parts. Here $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ is the scalar curvature within the first-order (Palatini) formalism for the Riemannian space-time metric $g_{\mu\nu}$ and $R_{\mu\nu}(\Gamma)$ is the Ricci tensor in terms of the independent affine connection $\Gamma^{\mu}_{\lambda\nu}$. In general, $L^{(2)}$ might contain also higher curvature terms like R^2 (cf. Ref. [5]).
- Note that in the first modified-measure term in the action (1) we have included an additional term containing another rank $(D-1)$ antisymmetric tensor gauge field $H_{\mu_1 \ldots \mu_{D-1}}$. Such terms would be purely topological (total divergence) ones if included in standard Riemannian integration measure action terms like the second term with $L^{(2)}$ on the r.h.s. of (1).

In (1) we have taken linear combination of modified-measure and standard Riemannian measure action terms. Recently in [6] we have proposed a more general TMT gravity-matter model defined in terms of *two different* non-Riemannian integration measures, which provides interesting cosmological implications, in particular, it allows for a unified description of both an early universe inflation and present day dark energy.

Varying (1) w.r.t. $H_{\mu_1 \ldots \mu_{D-1}}$ and the "measure" tensor gauge field $B_{\mu_1 \ldots \mu_{D-1}}$ we get:

$$\partial_\mu \Big(\frac{\Phi(B)}{\sqrt{-g}} \Big) = 0 \quad \to \quad \frac{\Phi(B)}{\sqrt{-g}} \equiv \chi = {\rm const} \ , \quad (3)$$

$$L^{(1)} + \frac{\varepsilon^{\mu_1 \ldots \mu_D}}{(D-1)!\sqrt{-g}} \partial_{\mu_1} H_{\mu_2 \ldots \mu_D} = M \ , \quad (4)$$

where χ (ratio of the two measure densities) and M are arbitrary integration constants. Performing canonical Hamiltonian analysis of (1) one can show that the above integration constants M and χ are in fact constrained a'la Dirac canonical momenta of the auxiliary tensor gauge fields B and H.

Now, varying (1) w.r.t. $g^{\mu\nu}$ and taking into account (3)–(4) we arrive at the following effective Einstein equations (in the first-order formalism):

$$R_{\mu\nu}(\Gamma) - \frac{1}{2} g_{\mu\nu} R + \Lambda_{\rm eff} g_{\mu\nu} = \kappa^2 T^{\rm eff}_{\mu\nu}, \tag{5}$$

with effective energy-momentum tensor:

$$T^{\rm eff}_{\mu\nu} = g_{\mu\nu} L^{\rm eff}_{\rm matter} - 2 \frac{\partial L^{\rm eff}_{\rm matter}}{\partial g^{\mu\nu}} \quad , \quad L^{\rm eff}_{\rm matter} \equiv \frac{1}{\chi+1}\Bigl[L^{(1)} + L^{(2)}\Bigr] \; , \tag{6}$$

and with a *dynamically generated* effective cosmological constant:

$$\Lambda_{\rm eff} = \frac{\kappa^2}{\chi+1}\, \chi M \; . \tag{7}$$

2 Modified $N = 1$ Supergravity and Dynamical Supersymmetry Breaking

The ideas and concepts of two-measure gravitational theories [1]- [5] may be combined with those applied to contsruct a theory of strings and branes with dynamical generation of (variable) string/brane tension [7] to consistently incorporate supersymmetry into the two-measure modification of standard Einstein gravity. Here for simplicity we will present the modified-measure construction of $N = 1$ supergravity in $D = 4$. For a recent account of modern supergravity theories and notations, see Ref. [8].

Let us recall the standard component-field action of $D = 4$ (minimal) $N = 1$ supergravity:

$$S_{\rm SG} = \frac{1}{2\kappa^2} \int d^4 x\, e \Bigl[R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda\Bigr] \; , \tag{8}$$

$$e = \det \|e^a_\mu\| \;\; , \;\; R(\omega, e) = e^{a\mu} e^{b\nu} R_{ab\mu\nu}(\omega) \; . \tag{9}$$

$$R_{ab\mu\nu}(\omega) = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega^c_{\mu a} \omega_{\nu cb} - \omega^c_{\nu a} \omega_{\mu cb} \; . \tag{10}$$

$$D_\nu \psi_\lambda = \partial_\nu \psi_\lambda + \frac{1}{4} \omega_{\nu ab} \gamma^{ab} \psi_\lambda \;\; , \;\; \gamma^{\mu\nu\lambda} = e^\mu_a e^\nu_b e^\lambda_c \gamma^{abc} \; , \tag{11}$$

where all objects belong to the first-order "vierbein" (frame-bundle) formalism, *i.e.*, the vierbeins e^a_μ (describing the graviton) and the spin-connection $\omega_{\mu ab}$ ($SO(1,3)$ gauge field acting on the gravitino ψ_μ) are *a priori* independent fields (their relation arises subsequently on-shell); $\gamma^{ab} \equiv \frac{1}{2}\left(\gamma^a \gamma^b - \gamma^b \gamma^a\right)$ *etc.* with

γ^a denoting the ordinary Dirac gamma-matrices. The invariance of the action (8) under local supersymmetry transformations:

$$\delta_\epsilon e^a_\mu = \frac{1}{2}\bar{\varepsilon}\gamma^a\psi_\mu \quad , \quad \delta_\epsilon\psi_\mu = D_\mu\varepsilon \tag{12}$$

follows from the invariance of the pertinent Lagrangian density up to a total derivative:

$$\delta_\epsilon\Big(e\big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda\big]\Big) = \partial_\mu\big[e\big(\bar{\varepsilon}\zeta^\mu\big)\big] \; , \tag{13}$$

where ζ^μ functionally depends on the gravitino field ψ_μ.

We now propose a modification of (8) by replacing the standard generally-covariant measure density $e = \sqrt{-g}$ by the alternative measure density $\Phi(B)$ (Eq.(2) for $D = 4$):

$$\Phi(B) \equiv \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\,\partial_\mu B_{\nu\kappa\lambda} \; , \tag{14}$$

and we will use the general framework described above in (1)–(7). The modified-measure supergravity action reads:

$$S_{\rm mSG} = \frac{1}{2\kappa^2}\int d^4x\,\Phi(B)\,\Big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\,\partial_\mu H_{\nu\kappa\lambda}\Big] \; , \tag{15}$$

where a new term containing the field-strength of a 3-index antisymmetric tensor gauge field $H_{\nu\kappa\lambda}$ has been added. As already explained above, its inclusion in the Lagrangian of the standard supergravity action (8) would yield a purely topological (total divergence) term. Similar construction has been previously used in ref. [7] to formulate a new version of Green-Schwarz superstring using an alternative non-Riemannian world-sheet volume form.

The equations of motion w.r.t. $H_{\nu\kappa\lambda}$ and $B_{\nu\kappa\lambda}$ yield as in (3)–(4):

$$\partial_\mu\Big(\frac{\Phi(B)}{e}\Big) = 0 \;\; \to \;\; \frac{\Phi(B)}{e} \equiv \chi = \text{const} \; , \tag{16}$$

$$R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\,\partial_\mu H_{\nu\kappa\lambda} = 2M \; , \tag{17}$$

where M is an arbitrary integration constant.

Now it is straightforward to check that the modified-measure supergravity action (15) is invariant under local supersymmetry transformations (12) supplemented by the transformation laws for $H_{\mu\nu\lambda}$ and $\Phi(B)$:

$$\delta_\epsilon H_{\mu\nu\lambda} = -e\,\varepsilon_{\mu\nu\lambda\kappa}\big(\bar{\varepsilon}\zeta^\kappa\big) \quad , \quad \delta_\epsilon\Phi(B) = \frac{\Phi(B)}{e}\,\delta_\epsilon e \; , \tag{18}$$

which algebraically close on-shell, *i.e.*, when Eq.(16) is imposed.

The role of $H_{\nu\kappa\lambda}$ in the modified-measure action (15) is to absorb, under local supersymmetry transformation, the total derivative term coming from (13), so as to insure local supersymmetry invariance of (15) – this is a generalization of the formalism used in Ref. [7] to write down a modified-measure extension of the standard Green-Schwarz world-sheet action of space-time supersymmetric strings. Similar approach has also been employed in Refs. [9, 10] in the context of $f(R)$ supergravity.

The appearance of the integration constant M represents a *dynamically generated cosmological constant* in the pertinent gravitational equations of motion and, thus, it signifies a *new mechanism of spontaneous (dynamical) breaking of supersymmetry*. Indeed, varying (15) w.r.t. e^a_μ:

$$e^{b\nu}R^a_{b\mu\nu} - \frac{1}{2}\bar{\psi}_\mu\gamma^{a\nu\lambda}D_\nu\psi_\lambda + \frac{1}{2}\bar{\psi}_\nu\gamma^{a\nu\lambda}D_\mu\psi_\lambda + \frac{1}{2}\bar{\psi}_\lambda\gamma^{a\nu\lambda}D_\nu\psi_\mu + \frac{e^a_\mu}{2}\frac{\varepsilon^{\rho\nu\kappa\lambda}}{3!\,e}\partial_\rho H_{\nu\kappa\lambda} = 0 \tag{19}$$

and using Eq.(17) to replace the last H-term on the l.h.s. of (19) we obtain the vierbein analogues of the Einstein equations including a dynamically generated *floating* cosmological constant term $e^a_\mu M$ (cf. Eqs.(5)-(7) above):

$$e^{b\nu}R^a_{b\mu\nu} - \frac{1}{2}e^a_\mu R(\omega, e) + e^a_\mu M = \kappa^2 T^a_\mu \;, \tag{20}$$

$$\kappa^2 T^a_\mu \equiv \frac{1}{2}\bar{\psi}_\mu\gamma^{a\nu\lambda}D_\nu\psi_\lambda - \frac{1}{2}e^a_\mu\bar{\psi}_\rho\gamma^{\rho\nu\lambda}D_\nu\psi_\lambda - \frac{1}{2}\bar{\psi}_\nu\gamma^{a\nu\lambda}D_\mu\psi_\lambda - \frac{1}{2}\bar{\psi}_\lambda\gamma^{a\nu\lambda}D_\nu\psi_\mu \;. \tag{21}$$

Let us recall that according to the classic paper [11] the sole presence of a cosmological constant in supergravity, even in the absence of manifest mass term for the gravitino, implies that the gravitino is *massive*, i.e., it absorbs the Goldstone fermion of spontaneous supersymmetry breakdown – a supersymmetric Higgs effect.

3 Dynamical Supersymmetry Breaking in Modified Anti-de Sitter Supergravity – Large Gravitino Mass versus Small Cosmological Constant

Let us now start from anti-de Sitter (AdS) supergravity (see, *e.g.* [8, 11]; using the same notations as in (8)):

$$S_{\rm AdS} = \frac{1}{2\kappa^2}\int d^4x\, e\Big[R(\omega, e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda - m\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu - 2\Lambda_0\Big] \;, \tag{22}$$

$$m \equiv \frac{1}{L} \quad , \quad \Lambda_0 \equiv -\frac{3}{L^2} \;. \tag{23}$$

The action (22) contains additional explicit mass term for the gravitino as well as a bare negative cosmological constant Λ_0 balanced in a precise way $|\Lambda_0| = 3m^2$ so as to maintain local supersymmetry invariance and, in particular, according to [11] – keeping the physical gravitino mass zero.

Then, application of the above formalism from Section 2 to the action (22) allows us to construct a modified-measure AdS supergravity in complete analogy with (15):

$$S_{\rm mod-AdS} = \frac{1}{2\kappa^2}\int d^4x\,\Phi(B)\Big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda \\ -m\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu - 2\Lambda_0 + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\,\partial_\mu H_{\nu\kappa\lambda}\Big]\,, \tag{24}$$

with $\Phi(B)$ as in (14) and m, Λ_0 as in (22).

Repeating the same steps as in Section 2 the AdS action (24) will trigger dynamical spontaneous supersymmetry breaking resulting in the appearance of the dynamically generated floating cosmological constant M which will add to the bare cosmological constant Λ_0. Thus, we can achieve via appropriate choice of $M \sim |\Lambda_0|$ a *very small positive effective observable cosmological constant*:

$$\Lambda_{\rm eff} = M + \Lambda_0 = M - 3m^2 \ll |\Lambda_0| \tag{25}$$

and, simultaneously, a *large physical gravitino mass* $m_{\rm eff}$ which in the case of very small effective cosmological constant (25) will be very close to the bare gravitino mass parameter m:

$$m_{\rm eff} \simeq m = \sqrt{\frac{1}{3}|\Lambda_0|}\,, \tag{26}$$

since now the background space-time geometry becomes almost flat. This is precisely what is required by modern cosmological scenarios for slowly expanding universe of today [12]- [14].

4 Conclusions

- Two-measure formalism in gravity/matter theories (employing alternative non-Riemannian volume form, *i.e.* reparametrization covariant integration measure, on the spacetime manifold alongside standard Riemannian volume form) naturally generates a ***dynamical cosmological constant*** as an arbitrary dimensionful integration constant.
- Within modified-measure minimal $N = 1$ supergravity the dynamically generated cosmological constant implies spontaneous supersymmetry breaking and mass generation for the gravitino (supersymmetric Brout-Englert-Higgs effect).

- Within modified-measure anti-de Sitter supergravity we can fine-tune the dynamically generated cosmological integration constant in order to achieve simultaneously a *very small physical observable positive cosmological constant* and a *very large physical observable gravitino mass* – a paradigm of modern cosmological scenarios for slowly expanding universe of today.

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