Bulg. J. Phys. **41** (2014) 258–273

Barotropic Bulk Viscous FRW Cosmological Model in Teleparallel Gravity

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Received *20 September 2014*

Abstract. We study the spatially homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmological model with barotropic viscous fluid in the framework of $f(T)$ theory of gravity, where T denotes the torsion scalar. The behavior of accelerating universe is discussed for some well-known $f(T)$ models with the help of a power law solution. Also we have discussed the basic thermodynamic aspects of the model. For a different option of barotropic fluid, we have generated a class of coefficient of bulk viscous, thermodynamic temperatures and Entropy densities of the models. The physical and geometric properties of cosmological models are also discussed.

PACS codes: 04.40.Nr, 04.50.Kd

1 Introduction

The results coming from the analysis of type-Ia supernovae (SNeIa) surveys, large scale structure (LSS), and cosmic microwave background (CMB) anisotropy spectrum strongly indicate that our Universe is spatially flat and has a phase transition from decelerating to accelerating [1-4]. The proposals that have been put forward to explain this observed phenomenon can basically be classified into two categories. One is to assume that in the framework of Einstein's general relativity (GR), an exotic component with negative pressure called mysterious energy or dark energy (DE) is necessary to explain this observed phenomenon, for a good review of the dynamics of different DE models, see [5-8]. Another alternative to account for the current accelerating cosmic expansion is to modify GR theory.

In order to explain the current accelerating expansion without introducing dark energy, Einstein [9,10] has presented another form of the theory of gravity called teleparallel gravity, namely $f(T)$ theory. Although teleparallel gravity is not an alternative to general relativity, but its different formulation allows one to say:

gravity is not due to curvature, but due to torsion. In the teleparallel gravity (TG) theory the dynamical object is not the metric, but a set of tetrad fields. In other word, using curvature-less Weitzenbck connection instead of torsion-less Levi-Civita connection in standard general relativity leads to subsequently replacing curvature by torsion. The advantage of this theory is that the dynamics is governed by second-order field equations.

Many authors have discussed several features of $f(T)$ gravity, such as energy in an expanding universe investigated by Sousa et al. [11]; dynamical behavior discussed by P. Wu and H. Yu [12]; T. Wang [13] has studied a spherically symmetric solutions in $f(T)$ gravity, the existence of relativistic stars investigated by C.G. Bohmer [14]. Recently, Jamil et al. [15] have studied the interacting DE model in the framework of $f(T)$ modified gravity theory for a particular choice of $f(T)$. M. Sharif et al. [16] have investigated the evolution of dark energy models in some exponential, logarithmic, and combination of both exponential and logarithmic $f(T)$ gravity. Han Dong et al. [17] showed that the distinction between ΛCDM and $f(T)$ gravity according to Nother symmetry. M.E. Rodrigues et al. [18] have inspected from early to dark energy dominated universe with LRS Bianchi type-I model in $f(T)$ gravity. In their work they have disclosed that: i) at the beginning the universe presents unequal scalar factor in the directions of x and y (anisotropy) and as the late-time is reached, they become equal (isotropy) reflecting the isotropization process; and ii) the non-linear model should be favored by observational data. K. Bamba et al. [19] have deliberated the generalized second law of thermodynamics in the framework of $f(T)$ modified gravity. In $f(T)$ gravity M.R. Setare et al. [20] have investigated power-law solutions and obtained the real valued solutions. M.R. Setare et al. [21] have examined finite time future singularities in $f(T)$ gravity with and without viscosity. Effective dark energy models with a bounce in frame of $f(T)$ gravity studied by A.V. Astashenok [22]. Jung-Tsung Liet et al. [23] have considered two $f(T)$ models in order to explain the pre-inflation universe and be reduced back to the teleparallel gravity equivalent to RG in the universe.

The bulk viscosity plays an important role in the early phase evolution of the universe. This is supported by the fact that when neutrino decoupling occurred, the matter behaved like viscous fluid in early stages of the universe. Moreover, a combination of cosmic fluid with bulk deceptive pressure can generate the accelerated expansion [24]. Bulk viscosity leading to an accelerated phase of the universe today has been studied by Fabris et al. [25]. Considering the bulk viscous coefficient as a power function of mass density Santos et al. [26] have derived exact solution with bulk viscosity. Wang [27,28] have discussed LRS Bianchi type-I and Bianchi type-III model for a cloud string with bulk viscosity. Yadav et al. [29] have investigated the integrability of the cosmic string in Bianchi type-III space-time in the presence of bulk viscous fluid. The effect of bulk viscosity with a time varying bulk viscous coefficient, on the evolution of isotropic FRW models in the context of open thermodynamic system has been discussed by Desikan [30]. A.K. Yadav et al. [31] have studied the thermodynamic properties of plane-symmetric inhomogeneous universe. C. Chawala et al. [32] have inspected Bianchi type-I space-time in the presence of a dissipative fluid with thermodynamic aspects by considering time dependent deceleration parameter (DP). R.L. Naidu et al. have investigated FRW viscous fluid cosmological model in $f(R,T)$ gravity [33].

Motivated by the situations discussed above in this paper, we investigate the spatially homogeneous and isotropic FRW cosmological model in $f(T)$ theory of gravitation in the presence of viscous barotropic fluid. This paper is organized as follows: Section 2 contains the brief review of $f(T)$ formalism. Section 3 contains FRW metric and the field equations. Section 4 deals with the exact matter dominated power law solution of the field equation. Secion 5 contains the thermodynamic equation and its aspects of the models. In Sections 6 and 7 we discussed the behavior of the accelerating universe by considering depiction and exponential $f(T)$ models. In Section 8 we discussed the kinematical properties of the model. Finally, Section 9 deals with concluding remarks.

2 A Brief Review of *f(T)* Cosmology

The line element of the Riemannian manifold is given by

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{2.1}$$

This line element can be converted to the Minkowskian description of the transformation called tetrad, as follows

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij}\theta^i\theta^j, \tag{2.2}$$

$$dx^\mu = e_i^\mu \theta^i, \quad \theta^i = e_\mu^i dx^\mu, \tag{2.3}$$

where $\eta_{ij} = diag[1,-1,-1,-1]$ and $e_i^\mu e_\mu^i = \delta_\nu^\mu$ or $e_i^\mu e_\mu^j = \delta_i^j$.

The root of metric determinant is given by $\sqrt{-g} = \det[e_\mu^i] = e$. For a manifold in which the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the non-zero torsion terms exist, the Weitzenbocks connection components are defined as

$$\Gamma^\alpha_{\mu\nu} = e_i^\alpha \partial_\nu e_\mu^i = -e_\mu^i \partial_\nu e_i^\alpha. \tag{2.4}$$

Through the connection, we can define the components of the torsion and contorsion tensors as

$$T^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\mu\nu} = e_i^\alpha \left(\partial_\mu e_\nu^i - \partial_\mu e_\mu^i\right), \tag{2.5}$$

$$K^{\mu\nu}_\alpha = \left(-\frac{1}{2}\right)\left(T^{\mu\nu}_\alpha + T^{\nu\mu}_\alpha - T^{\mu\nu}_\alpha\right). \tag{2.6}$$

For facilitating the description of the Lagrangian and the equations of motion, we can define another tensor $S_\alpha^{\mu\nu}$ from the components of the torsion and contorsion tensors, as

$$S_\alpha^{\mu\nu} = \left(\frac{1}{2}\right)\left(K_\alpha^{\mu\nu} + \delta_\alpha^\mu T_\beta^{\beta\nu} - \delta_\alpha^\nu T_\beta^{\beta\mu}\right). \tag{2.7}$$

The torsion scalar T is

$$T = T_{\mu\nu}^\alpha S_\alpha^{\mu\nu}. \tag{2.8}$$

Now, we define the action by generalizing the teleparallel theory, i.e. $f(T)$ theory as [18]

$$S = \int [T + f(T) + L_{\text{matter}}] \; e \; d^4x. \tag{2.9}$$

Here $f(T)$ denotes an algebraic function of the torsion scalar T. Making the functional variation of the action (2.9) with respect to the tetrads, we get the following equations of motion

$$S_\mu^{\nu\rho}\partial_\rho T f_{TT} + [e^{-1}e_\mu^i \partial_\rho(e e_i^\alpha S_\alpha^{\nu\rho}) + T_{\lambda\mu}^\alpha S_\alpha^{\nu\lambda}](1+f_T) + \frac{1}{4}\delta_\mu^\nu(T+f) = 4\pi T_\mu^\nu, \tag{2.10}$$

where T_μ^ν is the energy, momentum tensor, $f_T = df(T)/dT$, and by setting $f(T) = a_0 =$ const. this is dynamically equivalent to the general relativity.

3 Metric and Field Equations

We consider the spatially homogeneous and isotropic Friedman-Robertson-Walker (FRW) line element in the form

$$ds^2 = -dt^2 + a_1^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \tag{3.1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$. The angle θ and φ is the usual azimuthal and polar angles of spherical coordinates, with $0 \le \theta \le \pi$ and $0 \le \varphi \le 2\pi$. The coordinates (t, r, θ, φ) are called commoving coordinates. This means that the coordinate system follows the expansion of space, so that the space coordinates of objects which do not move with respect to the background remain the same. The homogeneity of the universe fixes a special frame of reference, the cosmic rest frame given by the above coordinate system. Also k is a constant representing the curvature of the space. The case of $k = 1$ corresponds to closed universe, flat universe obtained from $k = 0$ and the case of $k = -1$ corresponds to open universe. In view of the above universe M.R. Setare et al. [20] have investigated finite time future singularities models in $f(T)$ theory. Also the same author has analyzed two possible mechanisms of avoiding final singularity in $f(T)$ theory: the first is that the rate of universe expansion in these gravity can grow with energy density more slowly than in GR; and the second is bounce in the future similar to that which may occur at the beginning of the universe [21]. In this work we deliberate on the flat universe taken after $k = 0$ with infinite radius.

The energy momentum tensor T^i_j for bulk viscous fluid distribution is taken as

$$T^i_j = (\bar{p} + \rho)\, u^i u_j + \bar{p} g^i_j, \tag{3.2}$$

together with commoving co-ordinates $u^i = (0, 0, 0, 1)$ and (3.3)

$$u^i u_i = -1, \;\; \bar{p} = p - \xi \;\, u^i_{;i}, \tag{3.3}$$

where u^i is the 4-velocity vector of the cosmic fluid, $\bar{p}$, p and ρ are the effective pressure, isotropic pressure and energy density of the matter respectively, ξ is the coefficient of bulk viscosity which is a function of time t.

The components of the energy momentum tensor are $T_{4i} = 0$. Therefore the total effect of bulk viscosity is to reduce the pressure p of the perfect fluid by an amount $\xi\theta$, so that the effective pressure of the viscous fluid turns out to be $\bar{p} = (p - \xi\theta)$. Bulk viscosity is very important in cosmology, since it has a greater role in getting accelerated expansion of the universe popularly known as inflationary phase. There are many circumstances in the evolution of the universe in which bulk viscosity could arise [34]. When neutrinos decouple from the cosmic fluid [35] at the time of formation of galaxies and during particle creation in the early universe [36] viscosity arises. C. Chawala et al. [32] have investigated the characteristics of bulk viscosity that it is a positive decreasing function with time (in an anisotropic universe).

From the equation of motion (2.10), the Friedman equation for the viscous fluid of the stress energy tensor (3.2) can be written as

$$\frac{\dot{a}}{a}\;\dot{T}\;\, f_{TT} + \left\{\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2}\right\}(1 + f_T) + \frac{1}{4}\,(T + f) = (4\pi)\,\bar{p}, \tag{3.4}$$

$$3\,(1 + f_T)\,\frac{\dot{a}^2}{a^2} + \frac{1}{4}\,(T + f) = (4\pi)\,(-\rho)\,. \tag{3.5}$$

The overhead dot represents the differentiation with respect to time t.

The universe filled with barotropic perfect fluid, leads to [32]

$$p = \gamma\rho, \tag{3.6}$$

here γ is a constant and lies in the interval $\gamma \in [0, 1]$.

4 Exact Matter Dominated Power Law Solution of the Field Equation

The universe is characterized by power law scale factors with constant exponent. Due to the complexity of the field equation, it is very difficult to evaluate explicit analytical forms of the scale factor, so we have chosen the scale factor of the form Setare et al. [20]

$$a\ \ (t) = t^n, \tag{4.1}$$

n is any positive real number.

The motivation to choose such scale factor is that the universe has accelerated expansion at present and decelerated expansion in the past. The expansion of the universe depends on n: (i) $n > 0$ leads to an expanding universe; (ii) $n < 0$ describes a contracting universe; and (iii) $n = 0$ describes a static universe. Therefore, the power law solution exists for the $f(T)$ subject to the first and second situations [20].

5 Thermodynamical Behavior and Entropy of Universe

Thermodynamic analysis has become a powerful tool to inspect a gravitational theory. As pivotal events, black hole thermodynamics [37,38] and recent AdS/CFT (anti-de Sitter Space / conformal field theory) correspondence [39] show explicit significance and strongly suggest the deep connection between gravity and thermodynamics. A recent landmark of the identification of gravity theories and thermodynamics is the seminal work of Jacobson where the inverse problem of reproducing gravity theories from thermodynamical systems has been seriously dealt with and successfully realized [40].

From the thermodynamics, we apply the combination of first and second law of thermodynamics to the system with volume V [41].

$$\tau\, ds = d\,(\rho V) + pdV, \tag{5.1}$$

where τ and s represent the temperature and entropy, respectively.

The above equation may be written as

$$\tau\, ds = d\left[(p+\rho)\, V\right] - V dp. \tag{5.2}$$

The integrability condition is necessary to define a perfect fluid as a thermodynamical system; it is given by [42,43].

$$dp = \left(\frac{p+\rho}{\tau}\right) d\tau. \tag{5.3}$$

Using equations (5.2) and (5.3) we have the differential equation

$$ds = \frac{1}{\tau} d\left[(p+\rho)\, V\right] - (p+\rho)\, V \frac{d\tau}{\tau^2}. \tag{5.4}$$

Rewriting the above equation

$$ds = d\left[\frac{(p+\rho)\, V}{\tau}\right]. \tag{5.5}$$

Therefore the entropy is defined as

$$s = \left[\frac{(p+\rho)\,V}{\tau}\right]. \tag{5.6}$$

Let the entropy density be s', so that

$$s' = \frac{s}{V} = \left(\frac{p+\rho}{\tau}\right) = \frac{(1+\gamma)\,\rho}{\tau}. \tag{5.7}$$

If we define the entropy density in terms of temperature, the first law of thermodynamics may be written as

$$d\,(\rho V) + \gamma\rho dV = (1+\gamma)\,\tau\;\; d\left(\frac{\rho V}{\tau}\right), \tag{5.8}$$

which on integration yields

$$\tau = \rho^{\frac{\gamma}{1+\gamma}}. \tag{5.9}$$

From equation (5.7), we obtain

$$s' = (1+\gamma)\,\rho^{\frac{1}{1+\gamma}}. \tag{5.10}$$

Equation (5.6) representing the thermodynamics of the universe (entropy) does not depend on any individual fluids, it depends on the total matter density and the isotropic pressure of the fluid.

A.K. Yadav et al. [29] and C. Chawala et al. [32] investigated the actions of thermodynamic parameters, these parameters are directly related to the energy density of the universe, hence our outcomes in equations (5.9) and (5.10) show the same features with the work prepared by the above authors.

In the following section we inspect some well recognized $f(T)$ models occupied with barotropic viscous fluid with thermodynamic aspects**.**

6 Case – I: Depiction $f(T)$ Model

Here we assume that the simplest and commonly considered depiction $f(T)$ model, which is already considered by Han Dong et al. [17]

$$f(T) = T^{\eta}. \tag{6.1}$$

In the depiction model (6.1) and using equations (3.5) and (4.1), we obtain the expression for energy density of the universe (as energy density is independent of γ).

Energy density:

$$\rho = \left(\frac{1}{4\pi}\right)\left\{\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right\}. \tag{6.2}$$

Depending on its numerical value γ, we describe the following types of universe with the interesting physical parameters of the universe.

6.1 Dust universe $(\gamma = 0)$

For $\gamma = 0$, the dust model corresponds with equation of state $p = 0$. The physical and thermodynamic parameters in terms of cosmic time t_0 have the following expressions:

Coefficient of bulk viscous:

$$\xi = \frac{(3n-2)}{6t} + \left\{\frac{1}{4} - \frac{3n^2\eta - n\eta}{6n^2} + \frac{\eta(\eta-1)\,t}{3n}\right\}\left(\frac{t}{3n}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}. \quad (6.3)$$

Temperature:

$$\tau = \left\{\left(\frac{1}{4\pi}\right)\left[\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right]\right\}^{0}, \quad \text{i.e.,} \quad \tau = 1. \qquad (6.4)$$

Entropy density:

$$s' = \left(\frac{1}{4\pi}\right)\left\{\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right\}. \qquad (6.5)$$

In the dust universe pressure is found to be zero. The coefficient of bulk viscosity is a positive decreasing function of time and its approaches to a constant quantity which is near to zero. This is in good agreement with the physical behavior of ξ, the demeanor shown in Figure 1 corresponds to $\gamma = 0$. It is observed that the thermodynamic parameters such as temperature come out to be constant. The entropy density is influenced by t, in expanding universe $n > 0$ entropy density decrease, while entropy of the universe increases. In 1934, Tolman studied

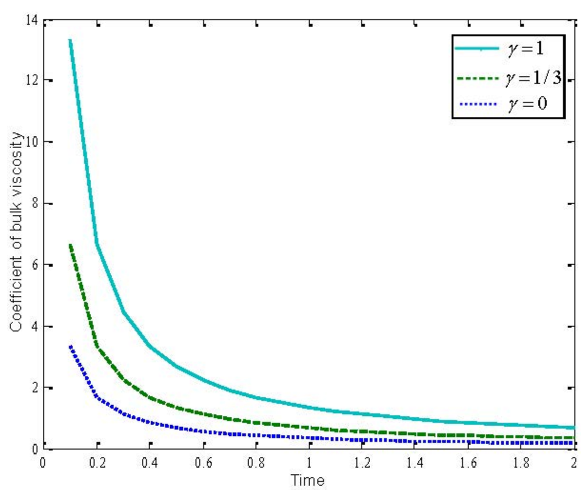

Figure 1. Coefficient of bulk viscosity versus time t.

the principle of entropy increase on a periodic sequence of closed Friedmann Robertson Walker universes. In our result, we observe the relation $(\ddot{s}/s) > 0$. This implies that the total entropy increases with time, hence our results (for flat universe) derived in the present paper are in good agreement with the second law of thermodynamics as well as with the observation of [29,32].

6.2 Radiation dominated universe $(\gamma = 1/3)$

For $\gamma = 1/3$, the disordered radiation corresponds with equation of state $\rho = 3p$. The physical and thermodynamic parameters in terms of cosmic time t have the following expressions:

Isotropic pressure

$$p = \frac{1}{3}\left(\frac{1}{4\pi}\right)\left\{\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right\}. \quad (6.6)$$

Coefficient of bulk viscous

$$\xi = \frac{(2n-1)}{3t} + \left\{\frac{1}{3} - \frac{4n^2\eta - n\eta}{6n^2} + \frac{\eta\,(\eta-1)\,t}{3n}\right\}\left(\frac{t}{3n}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}. \quad (6.7)$$

Temperature

$$\tau = \left(\frac{1}{4\pi}\right)^{\frac{1}{4}}\left(\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right)^{\frac{1}{4}}. \quad (6.8)$$

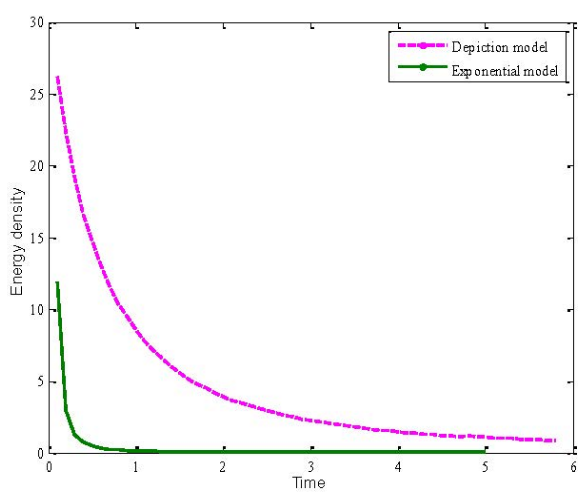

Figure 2. Energy density versus time t.

Entropy density

$$s' = \frac{4}{3}\left(\frac{1}{4\pi}\right)^{\frac{3}{4}}\left(\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right)^{\frac{3}{4}}. \tag{6.9}$$

In the radiation dominated universe pressure and the coefficient of bulk viscosity comes out to be a positive decreasing function of time and its approaches to a constant quantity which is near to zero. The behavior shown in Figure 2 corresponds to $\gamma = 1/3$. In an expanding universe density of the universe, entropy density and temperature are high but at infinite expansion these parameters are null [32].

6.3 Stiff fluid universe $(\gamma = 1)$

For, the fluid distribution corresponds to the equation of state $\rho = p$ which is known as Zeldovich fluid or stiff fluid. The physical and thermodynamic parameters in terms of cosmic time t have the following expressions:

Isotropic pressure:

$$p = \left(\frac{1}{4\pi}\right)\left\{\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right\}. \tag{6.10}$$

Coefficient of bulk viscous:

$$\xi = \frac{(3n-1)}{3t} + \left\{\frac{1}{2} - \frac{6n^2\eta - n\eta}{6n^2} + \frac{\eta(\eta-1)t}{3n}\right\}\left(\frac{t}{3n}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}. \tag{6.11}$$

Temperature:

$$\tau = \left(\frac{1}{4\pi}\right)^{\frac{1}{2}}\left(\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right)^{\frac{1}{2}}. \tag{6.12}$$

Entropy density:

$$s' = 2\left(\frac{1}{4\pi}\right)^{\frac{1}{2}}\left(\frac{3n^2}{2t^2} + \left(\frac{1-2\eta}{4}\right)\left(\frac{-6n^2}{t^2}\right)^{\eta}\right)^{\frac{1}{2}}. \tag{6.13}$$

In the stiff (Zeldovich) universe the physical parameters pressure, the coefficient of bulk viscosity and thermodynamic parameters show the same behavior as that of radiating dominated universe, positive decreasing function and approaches to a constant quantity. The behavior shown in Figure 2 corresponds to $\gamma = 1$. In a stiff universe, pressure and coefficient of bulk viscosity are resembling with results in Ref. [32], and the thermodynamic parameters with those in Refs. [29,32]

In depiction model, it is observed that the energy density distribution is positive decreasing function of time t. At the initial stage where the model starts from to expand the energy density $\rho \to \infty$ at $t \to 0$ whereas $\rho \to 0$ at $t \to \infty$ thus the model is asymptotically empty and this behavior is shown in Figure 2.

Figure 1 validates the behavior of the coefficient of bulk viscosity verses time in the evolution of universe with appropriate choice of constants of integration and other parameters for depiction model.

7 Case – II: Exponential $f(T)$ Model

M. Sharif et al. [16] have investigated the well-known phenomenon of the expansion of the universe, in the context of one of the exponential $f(T)$ gravity. For now, we select an exponential model.

In this section, we will concentrate on the following types of the exponential $f(T)$ models:

$$f(T) = \alpha T \left\{1 - e^{\beta T_0/T}\right\}. \tag{7.1}$$

Proceeding just as in Section 6 case – I we again obtain the expression for the energy density as

Energy density:

$$\rho = \left(\frac{1}{4\pi}\right)\left(\frac{3n^2}{2t^2}\right)\left\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} - \left(\frac{2\alpha\beta T_0 t^2}{6n^2}\right)e^{-\beta T_0 t^2/6n^2}\right\}. \tag{7.2}$$

7.1 Dust universe

Coefficient of bulk viscous:

$$\begin{aligned}\xi = \left(\frac{-n}{8t}\right)&\left\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2}\right\} \\ &+ \left(\frac{3n-1}{3t}\right)\left\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} - \left(\frac{2\alpha\beta T_0 t^2}{6n^2}\right)e^{-\beta T_0 t^2/6n^2}\right\} \\ &- \left(\frac{2n(\beta T_0 t)^2}{3}\right)e^{-\beta T_0 t^2/6n^2}.\end{aligned} \tag{7.3}$$

Temperature:

$$\begin{aligned}\tau = \Bigg\{\left(\frac{1}{4\pi}\right)\left(\frac{3n^2}{2t^2}\right)&\left\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} \right. \\ &\left. - \left(\frac{2\alpha\beta T_0 t^2}{6n^2}\right)e^{-\beta T_0 t^2/6n^2}\right\}\Bigg\}^0, \qquad \text{i.e. } \tau = 1.\end{aligned} \tag{7.4}$$

Entropy density:

$$s' = \Big(\frac{1}{4\pi}\Big)\Big(\frac{3n^2}{2t^2}\Big)\Big\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} - \Big(\frac{2\alpha\beta T_0 t^2}{6n^2}\Big)e^{-\beta T_0 t^2/6n^2}\Big\}. \quad (7.5)$$

In the Dust universe the physical and thermodynamical parameters of dust universe in an exponential model showing the same results as those of the dusty universe in the depiction model as the isotropic pressure is zero. The coefficient of bulk viscosity is a positive decreasing function of time. The appearance is shown as in Figure 1 and corresponds to $\gamma = 0$. In the exponential model, the dusty universe having constant temperature. At the initial stage of the universe the entropy density is infinite and at the infinite expansion of the universe the entropy density comes out to be nearly equal to zero.

7.2 Radiation dominated universe

Isotropic pressure:

$$p = \Big(\frac{1}{4\pi}\Big)\Big(\frac{n^2}{2t^2}\Big)\left\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} - \Big(\frac{2\alpha\beta T_0 t^2}{6n^2}\Big)e^{-\beta T_0 t^2/6n^2}\right\}, \quad (7.6)$$

Coefficient of bulk viscous

$$\begin{aligned}\xi = {} & \Big(\frac{-n}{6t}\Big)\Big\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2}\Big\} \\ & + \Big(\frac{4n-1}{3t}\Big)\Big\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} - \Big(\frac{2\alpha\beta T_0 t^2}{6n^2}\Big)e^{-\beta T_0 t^2/6n^2}\Big\} \\ & - \Big(\frac{2n(\beta T_0 t)^2}{3}\Big)e^{-\beta T_0 t^2/6n^2}. \quad (7.7)\end{aligned}$$

Temperature:

$$\begin{aligned}\tau = {} & \left\{\Big(\frac{1}{4\pi}\Big)\Big(\frac{3n^2}{2t^2}\Big)\right. \\ & \left. \times \Big(1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} - \Big(\frac{2\alpha\beta T_0 t^2}{6n^2}\Big)e^{-\beta T_0 t^2/6n^2}\Big)\right\}^{\frac{1}{4}}. \quad (7.8)\end{aligned}$$

Entropy density:

$$\begin{aligned}s' = {} & \frac{4}{3}\left\{\Big(\frac{1}{4\pi}\Big)\Big(\frac{3n^2}{2t^2}\Big)\right. \\ & \left. \times \Big(1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} - \Big(\frac{2\alpha\beta T_0 t^2}{6n^2}\Big)e^{-\beta T_0 t^2/6n^2}\Big)\right\}^{\frac{3}{4}}. \quad (7.9)\end{aligned}$$

In the radiation dominated universe the pressure is found to be a positive decreasing function of time t. The coefficient of bulk viscosity shows the same result as the result obtained in [29,32], i.e. the positive decreasing function of time t. The behavior shown in Figure 1 corresponds to $\gamma = 1/3$. As time increases the thermodynamic parameters decrease. This is in good agreement with the results in [29,32].

7.3 Stiff fluid universe

Isotropic pressure:

$$p = \left(\frac{1}{4\pi}\right)\left(\frac{3n^2}{2t^2}\right)\left\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2}-\left(\frac{2\alpha\beta T_0 t^2}{6n^2}\right)e^{-\beta T_0 t^2/6n^2}\right\}. \quad (7.10)$$

Coefficient of bulk viscous:

$$\xi = \left(\frac{-n}{4t}\right)\left\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2}\right\} + \left(\frac{6n-1}{3t}\right)\left\{1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2}-\left(\frac{2\alpha\beta T_0 t^2}{6n^2}\right)e^{-\beta T_0 t^2/6n^2}\right\} - \left(\frac{2n(\beta T_0 t)^2}{3}\right)e^{-\beta T_0 t^2/6n^2}. \quad (7.11)$$

Temperature:

$$\tau = \left\{\left(\frac{1}{4\pi}\right)\left(\frac{3n^2}{2t^2}\right)\left(1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2} - \left(\frac{2\alpha\beta T_0 t^2}{6n^2}\right)e^{-\beta T_0 t^2/6n^2}\right)\right\}^{\frac{1}{2}}. \quad (7.12)$$

Entropy density:

$$s' = 2\left\{\left(\frac{1}{4\pi}\right)\left(\frac{3n^2}{2t^2}\right) \times \left(1+\alpha-\alpha e^{-\beta T_0 t^2/6n^2}-\left(\frac{2\alpha\beta T_0 t^2}{6n^2}\right)e^{-\beta T_0 t^2/6n^2}\right)\right\}^{\frac{1}{2}}. \quad (7.13)$$

For the exponential model, the physical and thermodynamic parameters behave in the same way as for stiff fluid and radiating dominated universes. The behavior shown in Figure 1 corresponds to $\gamma = 1$.

In the exponential model, the appearance of the energy density is the same as that of depiction model; the behavior of the energy density versus time is shown in Figure 1.

Figure 2 shows the behavior of the energy density and the coefficient of bulk viscosity versus time in the evolution of universe with appropriate choice of constants of integration and other parameters for the exponential model.

8 Kinematical Properties of the Universe

The kinematical properties of the universe that are important in cosmology are spatial volume V, expansion scalar θ, anisotropic parameter A_m, shear scalar σ^2 and Hubble parameter H, which have the following expressions:

Spatial volume	$V = t^{3n}.$	(8.1)
Hubble parameter	$H = \frac{n}{t}.$	(8.2)
Expansion scalar	$\theta = \frac{3n}{t}.$	(8.3)
Anisotropic parameter	$A_m = 0.$	(8.4)
Shear scalar	$\sigma^2 = 0.$	(8.5)
Deceleration parameter	$q = n - 1.$	(8.6)

We observe that the spatial volume is zero at $t \to 0$. Thus, the singularity exists at $t \to 0$ in the model. The model starts evolving with a big-bang at $t \to 0$. The behavior is shown in Figure 3. The expansion scalar decreases as time increases. Also the mean Hubble parameter is initially large at $t \to 0$, and null at $t \to \infty$. The expansion scalar $\theta \to 0$ as $t \to \infty$ indicates that the universe is expanding with increase of time and the rate of expansion decreases with the increase of time. The anisotropic parameter and the shear scalar found to be zero, hence the universe does not approach anisotropy and the universe is shearing free [32] the behavior is shown in Figure 3.

Also it is observed that the deceleration parameter q comes out to be constant, the sign of q indicates whether the model inflates or not. For $n > 1$ the sign of

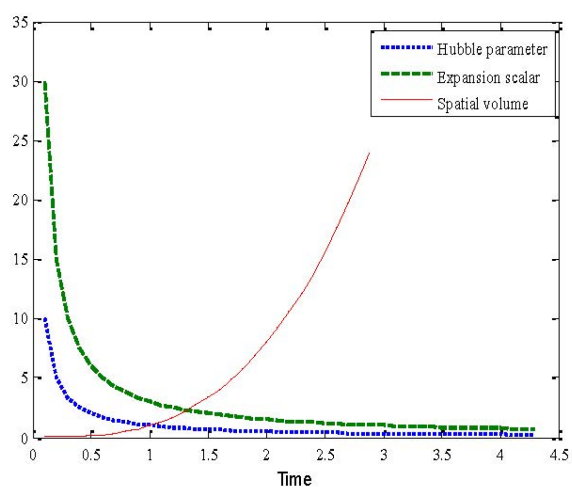

Figure 3. Hubble parameter and expansion scalar versus time t.

q becomes positive which corresponds to the standard decelerating behavior of the model. As we ensure about decelerating, the model is also consistent with the recent CMB observation model by WNAP, as well as with the high redshift supernovae Ia data [44], whereas for $n < 1$ the sign of q becomes negative, which corresponds to the standard accelerating behavior of the model. This scenario is consistent with recent observations [1-2,45-46]. This value is very close to the observed value of deceleration parameter, i.e. $-1 \leq q \leq 0$.

9 Conclusion

In this paper, we have deliberated a spatially homogeneous and isotropic Friedman-Robertson-Walker cosmological model with barotropic viscous fluid in the context of $f(T)$ theory of gravity. All the values of physical parameters have been derived. The derived models represent an expanding model, which approaches isotropy. It has also been observed that the bulk viscous coefficient plays more important role in the isotropization process of the universe. Therefore, it may be possible to explain the isotropy observed in the present universe. It is a consequence of the viscous effects in the fluid right from the beginning of the evolution of the universe.

Hence our results derived in the present paper are in good agreement with the second law of thermodynamics. In depiction model, where $\eta = 0$ for this particular choice of constant, the teleparallel gravity is equivalent to GR. Hence our result resembles with the result of [28]. Also, it resembles with the modified theory of gravity [33].

Finally, the exact solutions presented in this paper may be useful for better understanding of the evolution of the universe in FRW space-time with viscous effects as well as for perfect fluid distribution in the teleparallel theory of gravitation.

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