Bulg. J. Phys. **42** (2015) 42–52

LRS Bianchi Type I Cosmological Model with Bulk Viscosity in Lyra Geometry

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Received *4 January 2015*

Abstract. In this paper we investigate LRS Bianchi type I bulk viscous cosmological model in the framework of Lyra geometry with time-dependent displacement vector. It is found that the bulk viscosity coefficient (ξ) is a decreasing function of time. The expression for proper distance, luminosity distance, angular diameter distance, look back time and distance modulus curve are analyzed and also the distance modulus curve of derived model nearly matches with Supernova Ia (SN Ia) observations.

PACS codes: 04.20.-q, 98.80.Jk

1 Introduction

Investigation of relativistic cosmological models usually involves the energy momentum tensor of matter generated by a perfect fluid. One must take into account viscosity mechanism to consider more realistic model, since viscosity mechanism has attracted the attention of many researchers. Misner [1,2] has suggested the strong dissipation due to neutrino viscosity may considerably reduce the anisotropy of the black body radiation. Weinberg [3,4] has suggested that a viscosity mechanism in cosmology can explain the unusual high entropy per baryon in present events. Waga et al. [5], Pacher et al. [6], Guth [7], and Murphy [8] have shown that bulk viscosity associated with the grand unified theory phase transition.

In the early evolution of the universe the bulk viscosity is very important. There are many circumstances in the evolution of the universe in which bulk viscosity could arise. The bulk viscosity coefficient determines the magnitude of the viscous stress relative to the expansion. The cosmological models with bulk viscous in general relativity have been studied by good many authors [9-14].

In order to geometrize the whole of gravitation and electromagnetism Weyl [15] has proposed a modification of Riemannian manifold. Later on Lyra [16] has proposed a modification of Riemannian geometry by introducing a gauge function into structure that bears a close resemblance to Weyl's geometry. In consecutive investigation Sen [17] and Sen and Dunn [18] have constructed a new scalar tensor theory of gravitation and form an analog of model with finite density in Lyra Einstein field equation based on Lyra geometry. Halford [19,20] has shown that the vector field φ in Lyra geometry plays a similar role of cosmological constant Λ in the general theory of relativity; also showing that the scalar-tensor theory of gravitation in Lyra geometry predicts the same effect, within observational limits, as in Einstein's theory. Several authors [21-26] have constructed different cosmological models and studied various aspects of Lyra geometry.

To describe a theory of viscosity, Eckart [27] has made the first attempt. But Eckart theory has several drawbacks including violation of causality and instabilities of equilibrium states. Readers interested in the general theory of causal thermodynamics are urged to consult the excellent survey report of Maartens [28] and Zimdahl [29] and references cited therein. Isreal and Stewart [30] and Pavon [31] have developed a fully relativistic formulation of the theory taking into account second order deviation terms. The advantages of the causal theory are as follows: (1) For stable fluid configuration, the dissipative signals propagate causally; (2) Unlike Eckart-type's theory, there is no generic short-wavelength secular instability in causal theory; (3) Even for rotating fluids, the perturbations have a well-posed initial value problem. Therefore, the best currently available theory for analyzing dissipative processes in the universe is the full Isreal and Stewart causal thermodynamics.

Motivated by above discussion, in this paper we have obtained exact solutions of the Einstein field equations of LRS Bianchi type I bulk viscous cosmological model within the framework of Lyra geometry. We have also obtained bulk viscosity coefficient in Eckart, Truncated and FIS causal theory. The expression for proper distance, luminosity distance, angular diameter distance, look back time and distance modulus curve have been analyzed.

2 The Metric and Field Equations

We consider the LRS Bianchi type I metric of the form

$$ds^2 = -dt^2 + R^2 dx^2 + S^2(dy^2 + dz^2)\,, \tag{1}$$

where R and S are functions of cosmic time t.

The energy momentum tensor for bulk viscous fluid is given by

$$T_i^j = (\rho + p + \Pi)u_i u^j + (p + \Pi)g_i^j\,, \tag{2}$$

where ρ is the energy density, p is the pressure of the fluid, $\Pi = -\xi\theta$ is the bulk viscous stress and ξ is the bulk viscosity coefficient. The vector represents

u^idescribes the four velocity and x^i represents a direction of anisotropy that is the direction of string, satisfying the standard relation

$$u_i u^i = -x_i x^i = -1 \qquad \text{and} \qquad u_i x^i = 0. \tag{3}$$

Einstein modified field equation in normal gauge for Lyra's manifold obtained by Sen [3] is given by

$$R_i^j - \frac{1}{2} R g_i^j + \frac{3}{2}\varphi_i \varphi^j - \frac{3}{4}\varphi_k \varphi^k g_i^j = -T_i^j\,, \tag{4}$$

where in geometrizes units $8\pi G = 1$, $C = 1$ and $\varphi = (0, 0, 0, \beta(t))$.

For the metric (1), the field equation (4) together with (2) lead to the following system of equations:

$$2\frac{S_{44}}{S} + \frac{S_4^2}{S^2} + \frac{3}{4}\beta^2 = -\left(p + \Pi\right) \tag{5}$$

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{RS} + \frac{3}{4}\beta^2 = -\left(p + \Pi\right) \tag{6}$$

$$2\frac{R_4 S_4}{RS} + \frac{S_4^2}{S^2} - \frac{3}{4}\beta^2 = \rho\,. \tag{7}$$

The energy conservation equation $T_{i;j}^j = 0$ leads to

$$\rho_4 + (\rho + p + \Pi) + \left(\frac{R_4}{R} + 2\frac{S_4}{S}\right) = 0 \tag{8}$$

and conservation of L.H.S of (4) leads to

$$\left(R_i^j - \frac{1}{2} R g_i^j\right)_{;j} + \frac{3}{2}\left(\varphi_i \varphi^j\right)_{;j} - \frac{3}{4}\left(\varphi_k \varphi^k g_i^j\right)_{;j} = 0, \tag{9}$$

which leeds to

$$\frac{3}{2}\varphi_i\Big(\frac{\partial \varphi^j}{\partial x^j} + \varphi^l \Gamma_{lj}^j\Big) + \frac{3}{2}\varphi^j\left(\frac{\partial \varphi_i}{\partial x^j} - \varphi_l \Gamma_{ij}^l\right)$$
$$- \frac{3}{4} g_i^j \varphi_k \Big(\frac{\partial \varphi^k}{\partial x^j} + \varphi^l \Gamma_{lj}^k\Big) - \frac{3}{4} g_i^j \varphi^k \left(\frac{\partial \varphi_k}{\partial x^j} - \varphi_l \Gamma_{kj}^l\right) = 0\,. \tag{10}$$

Equation (10) is automatically satisfied for $i = 1, 2, 3$ and for $i = 4$ it leads to

$$\frac{3}{2}\beta\beta_4 + \frac{3}{2}\beta^2\Big(\frac{R_4}{R} + 2\frac{S_4}{S}\Big) = 0. \tag{11}$$

The average scale factor a for the metric (1) is defined as

$$a = (RS^2)^{\frac{1}{3}}\,. \tag{12}$$

The spatial volume is given by

$$V = a^3 = RS^2\,. \tag{13}$$

The generalized Hubble parameter (H) is defined as

$$H = \frac{a_4}{a} = \frac{1}{3}\Big(\frac{R_4}{R} + 2\frac{S_4}{S}\Big)\,. \tag{14}$$

The expansion scalar (θ), shear scalar (σ) and mean anisotropy parameter (A_m) are defined as

$$\theta = 3H\,; \tag{15}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{3}\right); \tag{16}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{17}$$

3 Solution of the Field Equations

In the above field equations the number of unknowns is more than equations, to be able to obtain an exact solutions of the field equations we assume the solutions of the scale factors of the forms

$$R = S^m, \tag{18}$$

where m is a constant.

From equation (11) we have

$$\beta = \frac{\beta_0}{RS^2}\,, \tag{19}$$

where β_0 is the constant of integration.

Using equation (18) in field equations (5) and (6), we get

$$\frac{S_{44}}{S} + (1+m)\frac{S_4^2}{S^2} = 0\,, \tag{20}$$

which on simplification gives

$$S = (c_1 t + c_2)^{\frac{1}{m+2}}\,, \tag{21}$$

where c_1 and c_2 are constants of integration.

Using equation (21), the metric of our solutions can be written in the form

$$ds^2 = -dt^2 + (c_1 t + c_2)^{\frac{2m}{m+2}}dx^2 + (c_1 t + c_2)^{\frac{2}{m+2}}(dy^2 + dz^2). \tag{22}$$

The mean Hubble parameter (H), expansion scalar (θ), shear scalar (σ), spatial volume (V) and mean anisotropy parameter (A_m) are given by

$$H = \frac{c_1}{3(c_1 t + c_2)}\,, \tag{23}$$

$$\theta = \frac{c_1}{c_1 t + c_2}\,, \tag{24}$$

$$\sigma^2 = \frac{(m-1)^2 c_1^2}{3(m+2)^2 (c_1 t + c_2)^2}\,, \tag{25}$$

$$V = (c_1 t + c_2)\,, \tag{26}$$

$$A_m = \frac{2(m-1)^2}{(m+2)^2}\,. \tag{27}$$

The deceleration parameter (q) for the model (22) is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = 2\,. \tag{28}$$

The deceleration parameter appears with a positive sign. This implies that the model decelerate in the standard way which is not in accordance with the present day scenario of accelerating universe. It may be noted that Bianchi model represent cosmos in its early stage of evolution. However, in spite of the fact that the universe, in this case, decelerates in the standard way it will accelerate in finite time due to cosmic recollapse where the universe in turns inflates “decelerates and then accelerates”.

We assume that the fluid obeys the equation of state of the form

$$p = \gamma\rho\,, \tag{29}$$

where $0 \leq \gamma \leq 1$ is a constant and it termed as equation of state parameter.

Using equations (18), (19) and (21) in (7), one can easily obtain expression for energy density in terms of cosmic time as

$$\rho = \frac{4(2m+1)c_1^2 - 3\beta_0^2(m+2)^2}{4(m+2)^2(c_1 t + c_2)^2}\,. \tag{30}$$

Equations (5), (18), (19), (21), (29) and (30) yield exclusive expression for pressure (p) and bulk viscous stress (Π) as follows:

$$p = \gamma\Big(\frac{4(2m+1)c_1^2 - 3\beta_0^2(m+2)^2}{4(m+2)^2(c_1 t + c_2)^2}\Big), \tag{31}$$

$$\Pi = -\frac{3\beta_0^2(m+2)^2(1-\gamma) + 4\gamma(2m+1)c_1^2 - 4c_1^2(2m+1)}{4(m+2)^2(c_1 t + c_2)^2}. \tag{32}$$

We observe that the model has singularity at $t = -c_2/c_1$, which can be shifted to $t = 0$ by choosing $c_2 = 0$. This is a point type singularity as all scale factors vanish at $t = -c_2/c_1$. The spatial volume becomes zero at $t = -c_2/c_1$ and it increases with time. Figure 1 depicts the energy density (ρ) versus time.

In extended irreversible thermodynamics (EIT), the bulk viscous stress Π satisfies a transport equation given by

$$\Pi + \tau\Pi_4 = -3\xi H - \frac{\varepsilon}{2}\tau\Pi\Big[3H + \frac{\tau_4}{\tau} - \frac{\xi_4}{\xi} - \frac{T_4}{T}\Big], \tag{33}$$

where τ is the relaxation time and T is the temperature. The parameter ε takes the value 0 or 1. Here $\varepsilon = 0$ represents truncated Israel-Stewart theory, $\varepsilon = 1$ represents the full Isreal-Stewart causal theory and $\tau = 0$ recovers the non-causal Eckart theory.

The Gibb's integrability condition (Maarten [28]) is defined as

$$T = \exp\left(\int \frac{dp(\rho)}{\rho + p(\rho)}\right). \tag{34}$$

Using equation (29), equation (34) reduces to

$$T = T_0\rho^{\frac{\gamma}{\gamma+1}}\ , \tag{35}$$

where T_0 is a constant.

Using equation (30) in (35), the temperature (T) is as follows:

$$T = T_0\Big(\frac{4(2m+1)c_1^2 - 3\beta_0^2(m+2)^2}{4(m+2)^2(c_1t+c_2)^2}\Big)^{\frac{\gamma}{1+\gamma}}. \tag{36}$$

From Figure 2 we see that the temperature (T) is a decreasing function of time.

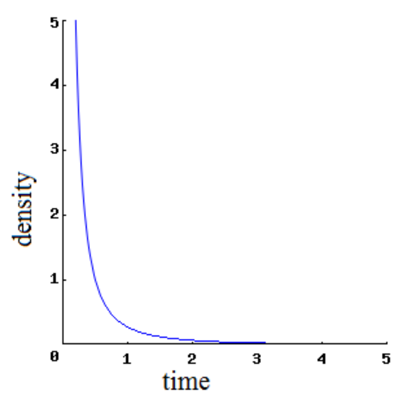

Figure 1. Density (ρ) versus time (t).

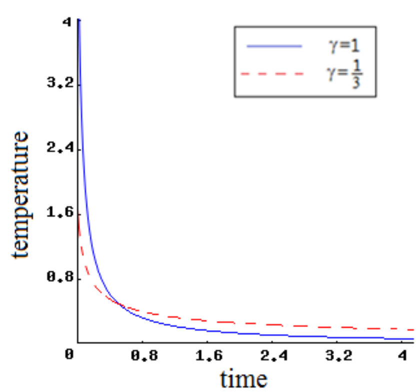

Figure 2. Temperature (T) versus time (t).

Bulk Viscosity in Eckart's Theory: The evolution equation (33) for bulk viscosity in non-causal Eckart's theory (i.e. $\tau = 0$) reduces to

$$\Pi = -3\xi H \tag{37}$$

with the help of equations (23) and (32), we get the bulk viscosity coefficient

$$\xi = \frac{k_1}{4c_1(m+2)^2(c_1 t + c_2)} \,. \tag{38}$$

Bulk Viscosity in Truncated Theory: In truncated theory (i.e. $\varepsilon = 0$), the evolution equation (33) for bulk viscosity reduces to

$$\Pi + \tau \Pi_4 = -3\xi H \,. \tag{39}$$

The relation between τ and the coefficient of bulk viscosity ξ is given by

$$\tau = \frac{\xi}{\rho} \,. \tag{40}$$

Thus the equation (39) reduces to

$$\Pi + \frac{\xi}{\rho}\Pi_4 = -3\xi H \,. \tag{41}$$

Using equations (23), (30) and (32), we obtain

$$\xi = \frac{k_1 k_2}{4c_1(m+2)^2(2k_1 + k_2)(c_1 t + c_2)} \,. \tag{42}$$

Bulk Viscosity in FIS Causal Theory: In FIS theory (i.e. $\varepsilon = 1$), the evolution equation (33) for bulk viscosity reduces to

$$\xi = \frac{k_1 k_2 (1+\gamma)}{2c_1(m+2)^2(2(2k_1 + k_2)(1+\gamma) - k_1(5\gamma + 3))(c_1 t + c_2)} \,, \tag{43}$$

where

$$k_1 = 3\beta_0^2(m+2)^2(1-\gamma) + 4\gamma(2m+1)c_1^2 - 4c_1^2(2m+1)$$

and

$$k_2 = 4(2m+1)c_1^2 - 3\beta_0^2(m+2)^2 \,.$$

To investigate the consistency of the model (22), we measure the physical parameters, such as proper distance, luminosity distance, angular diameter, etc.

Proper distance

The proper distance $d(z)$ is defined as the distance between a cosmic source emitting light at any instant $t = t_1$ located at $r = r_1$ with redshift z and an observer at $r = 0$ and $t = t_0$ receiving the light from the source emitted, i.e.

$$d(z) = a_0 r_1 \,, \tag{44}$$

where

$$r_1 = \int_{t_1}^{t_0} \frac{dt}{a(t)}$$

Hence

$$d(z) = \frac{\big((1+z)^2 - 1\big)}{2H_0(1+z)^2} \tag{45}$$

where $1 + z = a_0/a =$ redshift and a_0 is the present scale factor of the universe.

Luminosity distance

Luminosity distance is the important concept of theoretical cosmology of a light source. The luminosity distance is a way of expanding the amount of light received from a distant object. It is defined in such a way that generalizes the inverse-square law of the brightness in the static Euclidean space to an expanding curved space.

The luminosity distance of a light source is defined as

$$d_L^2 = \frac{L}{4\pi l} \,, \tag{46}$$

where L is the absolute luminosity and l is the apparent luminosity of source. Therefore, one can write

$$d_L = (1+z)d(z) \,. \tag{47}$$

Using equation (45), equation (47) reduces to

$$H_0 d_L = \frac{(1+z)^2 - 1}{2(1+z)} \,. \tag{48}$$

Angular diameter distance

The angular diameter distance is a measure of how large objects appear to be. As with the luminosity distance, it is defined as the distance that an object of known physical extent appears to be at, under the assumption of the Euclidean geometry.

The angular diameter d_A of a light source of proper distance is given by

$$d_A = (1+z)^{-2} d_L \,. \tag{49}$$

Using equation (48), we get

$$d_A = H_0^{-1} \frac{(1+z)^2 - 1}{2(1+z)^3} \,. \tag{50}$$

Look back time

The look back time is defined as the elapsed time between the present age of universe t_0 and the time t when the light from a cosmic source at a particular redshift z has been emitted.

In the context of our model it is given by

$$t_0 - t = \int_a^{a_0} \frac{da}{\dot{a}} \,, \tag{51}$$

which on simplification gives

$$t_0 - t = \frac{1 - (1+z)^{-1}}{3H_0} \,, \tag{52}$$

which can be written as

$$H_0(t_0 - t) = \frac{1 - (1+z)^{-1}}{3} \,. \tag{53}$$

Distance modulus curve

The distance modulus is given by

$$\mu = 5 \log d_L + 25 \,. \tag{54}$$

Using equation (48), we obtain the expression for distance modulus (μ) in terms of redshift parameter (z) as

$$\mu = 5 \log \left(\frac{((1+z)^2 - 1)}{2H_0(1+z)} \right) + 25. \tag{55}$$

The observed value of distance modulus $\mu(z)$ at different redshift parameters (z) given in Table 1 are employed to draw the curve corresponding to the calculate value of $\mu(z)$. The plot of observed $\mu(z)$ (dotted line) and calculated $\mu(z)$ (solid line) versus redshift parameter (z) is shown in Figure 3.

Table 1.

Redshift (z)	Supernovae Ia (μ)	Our model (μ)	Redshift (z)	Supernovae Ia (μ)	Our model (μ)
0.014	33.73	33.81	0.240	40.68	40.15
0.026	35.62	35.14	0.380	42.02	40.89
0.036	36.39	35.84	0.430	42.33	41.20
0.040	36.38	36.06	0.480	42.37	41.15
0.050	37.08	36.54	0.620	43.11	41.82
0.063	37.67	37.03	0.740	43.35	42.01
0.079	37.94	37.50	0.778	43.81	42.12
0.088	38.07	37.73	0.828	43.59	42.30
0.101	38.73	38.02	0.886	43.91	42.24
0.160	39.08	39.19	0.910	44.44	42.34

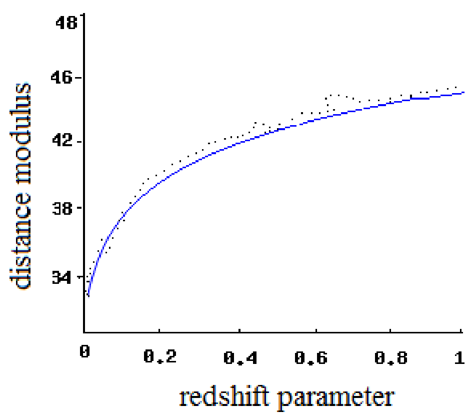

Figure 3. Distance modulus (μ) versus redshift parameter (z).

4 Conclusion

In this paper we have discussed LRS Bianchi type I space-time with bulk viscous fluid in the framework of Lyra geometry with time-dependent displacement vector. It can also be observed that θ, H and σ decrease with time. The spatial volume vanishes at $t = -c_2/c_1$ and increases with time. This shows that at the initial epoch, the universe starts with zero volume and expands uniformly. The displacement vector (β) and the bulk viscosity coefficient (ξ) decreases as time increases. From (30) and (31), we can see that energy density and pressure will vanish with the increase of cosmic time. Hence they represent vacuum cosmological models in general relativity for large values of t. We have also found that the ratio $\sigma/\theta =$ const, which shows that the anisotropy in the universe is maintained throughout. However it becomes isotropic for $m = 1$. We have also taken an account of the consistency of our model with observational parameters, such as proper distance, luminosity distance, angular diameter distance, look back time and also the distance modulus curve of derived model nearly matches with Supernova Ia (SN Ia) observations.

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