

*Dedicated to the memory of
Academician Christo Ya. Christov on the 100-th anniversary of his birth*

Symmetry Energy and Structure of Exotic Nuclei

**M.K. Gaidarov¹, A.N. Antonov¹, P. Sarriguren²,
E. Moya de Guerra³**

¹Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Tsarigradsko Chaussee 72, BG-1784 Sofia, Bulgaria

²Instituto de Estructura de la Materia, IEM-CSIC, Serrano 123, E-28006 Madrid, Spain

³Departamento de Física Atomica, Molecular y Nuclear, Facultad de Ciencias Fisicas, Universidad Complutense de Madrid, E-28040 Madrid, Spain

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Abstract. The symmetry energy, the neutron pressure and the asymmetric compressibility of spherical Ni, Sn, Pb and deformed Kr, Sm neutron-rich even-even nuclei are calculated within the coherent density fluctuation model using the symmetry energy as a function of density within the Brueckner energy-density functional. The correlation between the thickness of the neutron skin and the characteristics related with the density dependence of the nuclear symmetry energy is investigated for isotopic chains of these nuclei in the framework of the deformed self-consistent mean-field Skyrme HF+BCS method. The mass dependence of the nuclear symmetry energy and the neutron skin thickness are also studied together with the role of the neutron-proton asymmetry. The studied correlations reveal a smoother behavior in the case of spherical nuclei than for deformed ones. We also notice that the neutron skin thickness obtained for ^{208}Pb with SLy4 force is found to be in a good agreement with the recent data. In addition to the interest that this study may have by itself, we give some numerical arguments in proof of the existence of peculiarities of the studied quantities in Ni and Sn isotopic chains that are not present in the Pb chain.

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1 Introduction

Great attention has been paid to the nuclear equation of state of isospin asymmetric nuclear matter (ANM), in particular the nuclear matter symmetry energy. The density-dependent symmetry energy governs numerous isospin-dependent properties of nuclei such as the binding energy, the location of the drip lines, the

density distributions, as well as the reactions: giant resonances, heavy ion collisions, isospin diffusion, and multifragmentation. At the same time, the nuclear symmetry energy is crucial in the astrophysical calculations of neutron stars, supernova explosions and stellar nucleosynthesis. The neutron skin thickness is one of the observables where symmetry energy shows up in the ground state of nuclei [1]. Neutron skin defined through the rms radii of protons and neutrons depends on the properties of the nuclear surface. The relative differences of the neutron and the proton distributions in this region are sensitive to the symmetry energy at the subsaturation densities.

The main aim of this work is to investigate the relation between the neutron skin thickness and some nuclear matter properties in finite nuclei, such as the symmetry energy at the saturation point s , symmetry pressure p_0 (proportional to the slope of the bulk symmetry energy), and asymmetric compressibility ΔK , considering nuclei in given isotopic chains and within a certain theoretical approach. For this purpose, the Brueckner energy density functional for infinite nuclear matter [2,3] was applied to calculate s , p_0 , and ΔK of medium-heavy and heavy spherical Ni ($A = 74 - 84$), Sn ($A = 124 - 152$), Pb ($A = 202 - 214$) nuclei [4] and deformed Kr ($A = 82 - 120$), Sm ($A = 140 - 156$) isotopes [5] that include surface effects. A theoretical approach that combines the deformed HF+BCS method with Skyrme-type density-dependent effective interactions (e.g., [6]) and the coherent density fluctuation model (CDFM) [7] was used. We would like to note the capability of the CDFM to be applied as an alternative way to make a transition from the properties of nuclear matter to the properties of finite nuclei. Finally, we have analyzed in details the existence of kinks in Ni and Sn isotopic chains and the lack of such a kink for the Pb chain. The kinks are produced because of the sensitivity of the symmetry energy and neutron pressure to the shell structure (see, for instance, the discussion in Refs. [4, 8]).

2 Theoretical Framework

The quantity $s^{ANM}(\rho)$, which refers to the infinite system and therefore neglects surface effects, is related to the second derivative of the energy per particle $E(\rho, \delta)$ using its Taylor series expansion in terms of the isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$ (ρ , ρ_n and ρ_p being the baryon, neutron and proton densities, respectively) (see, e.g., [4]):

$$\begin{aligned} s^{ANM}(\rho) &= \frac{1}{2} \left. \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \right|_{\delta=0} \\ &= a_4 + \frac{p_0^{ANM}}{\rho_0^2} (\rho - \rho_0) + \frac{\Delta K^{ANM}}{18\rho_0^2} (\rho - \rho_0)^2 + \dots \end{aligned} \quad (1)$$

In Eq. (1) the parameter a_4 is the symmetry energy at equilibrium ($\rho = \rho_0$). In ANM the pressure p_0^{ANM} , the "slope" parameter L^{ANM} and the curvature

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ΔK^{ANM} are:

$$p_0^{ANM} = \rho_0^2 \left. \frac{\partial s^{ANM}(\rho)}{\partial \rho} \right|_{\rho=\rho_0}, \quad (2)$$

$$L^{ANM} = \frac{3p_0^{ANM}}{\rho_0}, \quad (3)$$

$$\Delta K^{ANM} = 9\rho_0^2 \left. \frac{\partial^2 s^{ANM}(\rho)}{\partial \rho^2} \right|_{\rho=\rho_0}. \quad (4)$$

For instance, using the Brueckner theory, the symmetry energy $s^{ANM}(x)$ has the form:

$$s^{ANM}(x) = 41.7\rho_0^{2/3}(x) + b_4\rho_0(x) + b_5\rho_0^{4/3}(x) + b_6\rho_0^{5/3}(x). \quad (5)$$

Under some approximation the symmetry energy for *finite nuclei* and the related quantities can be written within the CDFM as infinite superpositions of the corresponding ANM quantities weighted by $|f(x)|^2$:

$$s = \int_0^\infty dx |f(x)|^2 s^{ANM}(x), \quad (6)$$

$$p_0 = \int_0^\infty dx |f(x)|^2 p_0^{ANM}(x), \quad \Delta K = \int_0^\infty dx |f(x)|^2 \Delta K^{ANM}(x). \quad (7)$$

The neutron skin thickness is usually estimated as the difference of the rms radii of neutrons and protons:

$$\Delta R = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}. \quad (8)$$

In our calculations the following Skyrme force parametrizations are used: SLy4, Sk3, SGII, and LNS (see, e.g., [4]). Also, we use the proton and neutron densities obtained from self-consistent deformed Hartree-Fock calculations with density-dependent Skyrme interactions [6] and pairing correlations.

3 Results and Discussion

The correlation of the neutron skin thickness ΔR in the Ni isotopic chain with the s , p_0 and ΔK parameters is shown in Figure 1. It is seen from Figure 1(a) that there exists an approximate linear dependence between ΔR and s for the even-even Ni isotopes with $A = 74 - 84$. We observe a smooth growth of the symmetry energy till the double-magic nucleus ^{78}Ni ($N = 50$) and then a linear decrease of s while the neutron skin thickness of the isotopes increases. This behavior is valid for all Skyrme parametrizations used in the calculations, in particular, the average slope of ΔR for various forces is almost the same. We also find a similar approximate linear correlation for Ni isotopes between

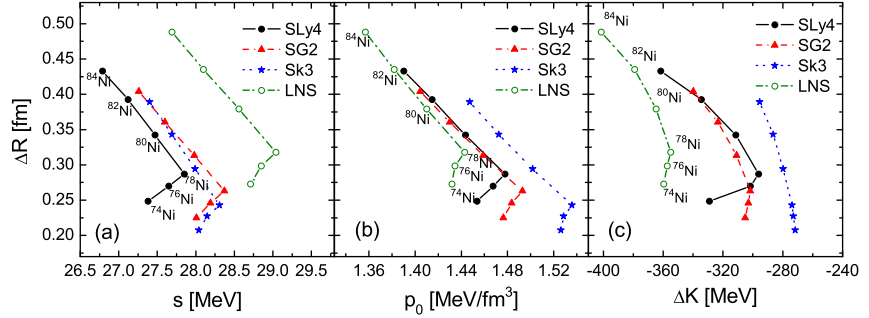


Figure 1. HF+BCS neutron skin thicknesses ΔR for Ni isotopes as a function of the symmetry energy s (a), pressure p_0 (b), and asymmetric compressibility ΔK (c) calculated with SLy4, SG2, Sk3, and LNS forces.

ΔR and p_0 [Figure 1(b)] and less strong correlation between ΔR and ΔK [Figure 1(c)]. As in the symmetry energy case, the behavior of the curves drawn in these plots shows the same tendency, namely the inflexion point transition at the double-magic ^{78}Ni nucleus. We would like to note that the predictions for the difference ΔR between the rms radii of neutrons and protons with Skyrme forces obtained in Ref. [9] exhibited a steep change at the same place in which the number of neutrons starts to increase in the chain of nickel isotopes.

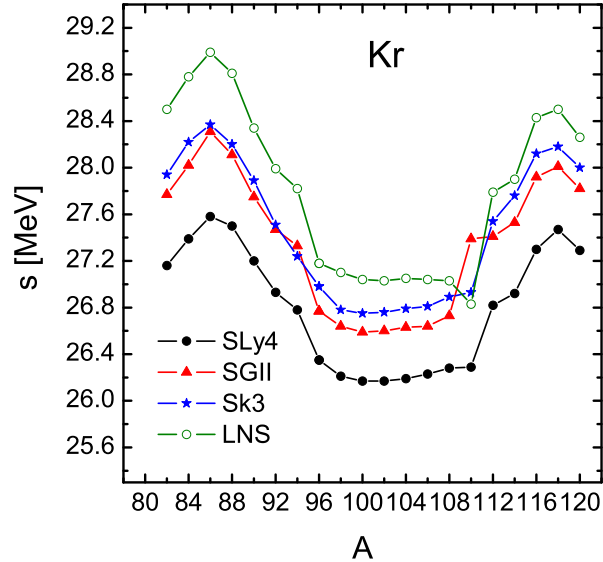


Figure 2. The symmetry energies s for Kr isotopes ($A = 82 - 120$) calculated with SLy4, SGII, Sk3, and LNS forces.

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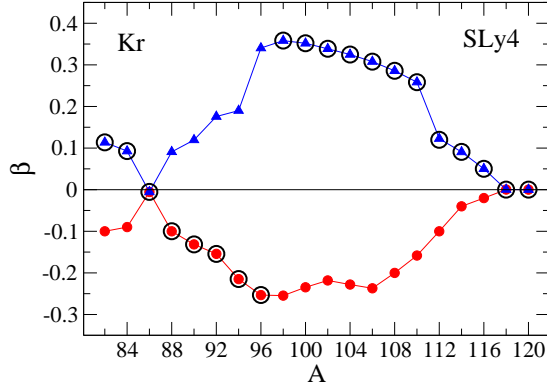


Figure 3. The quadrupole parameter β as a function of the mass number A for the even-even Kr isotopes ($A = 82 - 120$) in the case of SLy4 force.

In our work we consider a chain of deformed neutron-rich even-even Kr isotopes (including, as well, the case of some extreme neutron-rich nuclei up to ^{120}Kr). The results for the symmetry energy s as a function of the mass number A for the whole Kr isotopic chain are presented in Figure 2. We observe peaks of the symmetry energy at specific Kr isotopes, namely at semi-magic ^{86}Kr ($N = 50$) and ^{118}Kr ($N = 82$) nuclei. In addition, a flat area is found surrounded by transitional regions $A = 88 - 96$ and $A = 110 - 116$. The results shown in Figure 2 are closely related to the evolution of the quadrupole parameter β as a function of the mass number A that is presented in Figure 3. First, one can see from Figure 3 that the semi-magic $A = 86$ and $A = 118$ Kr isotopes are spherical, while the open-shell Kr isotopes within this chain possess two equilibrium shapes, oblate and prolate. In the case of open-shell isotopes, the oblate and prolate minima are very close in energy and the energy difference is always less than 1 MeV. Nevertheless, we specify in Figure 3 which shape corresponds to the ground state of each isotope by encircling them. Thus, the trend that the evolution of the symmetry energy shown in Figure 2 follows can be clearly understood. The peaks of the symmetry energy correspond to the closed-shell nuclei that are spherical. Mid-shell nuclei ($A = 96 - 110$) are well deformed and exhibit a stabilized behavior with small values of s . The transitional regions from spherical to well deformed shapes correspond to transitions from the peaks to the valley in the symmetry energy.

4 Conclusions

The results of the present work can be summarized as follows:

- i) There exists an approximate linear correlation between ΔR and s , as well as p_0 , while the relation between ΔR and ΔK is less pronounced. A behavior

containing an inflexion point transition at specific shell closure is observed for these correlations (^{78}Ni , ^{132}Sn , ^{86}Kr , and ^{144}Sm).

ii) Our HF+BCS calculations lead to s in the range of 27–30 MeV, which is in agreement with the empirical value of 30 ± 4 MeV. The calculated values of $p_0=1.36\text{--}1.68$ MeV/fm³ lead to values of the slope parameter $L=26\text{--}32$ MeV, in agreement with other theoretical predictions.

iii) The kinks displayed by the Ni and Sn can be understood as consequences of particular differences in the structure of these nuclei and the resulting densities and weight functions. It is shown that for the Pb isotopes the different signs of the relative deviations corresponding to the range of integration on x that contains the peak of $|f(x)|^2$ (being sensitive to the different density profiles of the considered isotopes around double-magic ones) is in favor of the absence of kink in the Pb chain.

Concluding, we would like to note that the capability of the present method can be further demonstrated by taking into consideration Skyrme-type and relativistic nuclear energy density functionals.

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