Quasipartices, Phonons and Beyond. Nuclear Structure Calculations in a Large Domain of Excitation Energies

Ch. Stoyanov

Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Tsarigradsko Chaussee 72, BG-1784 Sofia, Bulgaria

Abstract. The complex structure of low-lying as well as those of high-lying states is discussed within multiphonon approach. The approach is based on Quasiparticle-Phonon Model. This microscopic model goes beyond the quasiparticle random-phase approximation by treating a Hamiltonian of separable form in a microscopic multiphonon basis. It is therefore able to describe the anharmonic features of collective modes. In the case of low-lying part of excitations the model has close correspondence with the proton-neutron interacting boson model. Within the model highly-excited single-particle states in nuclei are coupled with the excitations of a more complex character, first of all with collective phonon-like modes of the core. Although, on the level of one and two-phonon admixtures, the fully chaotic GOE regime is not reached, the eigenstates of the model carry significant degree of complexity that can be quantified with the aid of correlational invariant entropy.

PACS codes: 21.60.Jz, 21.65.Ef, 21.10.Gv

1 Introduction

Multiphonon collective modes in nuclei have been predicted within the BohrMottelson model [1]. Their evidence has grown considerably in the last two decades. At low energy, fluorescence scattering experiments have detected low-lying quadrupole, octupole and mixed quadrupoleoctupole multiplets in nearly spherical heavy nuclei [2, 3]. At high energy, reaction experiments have established strong distribution of Giant Multipole Resonances and high-lying single-particle mode [4, 5]. Particle emission of high-lying states have been measured [6–8]. These remarcalle founds have been sucsessfuly interpreted within multiphonon approach.

The main approximation in this approach is based on the idea that fermions are coupled in pairs and the new objects are "boson"-like ones. The "boson"

picture is studied in the framework of algebraic [9] as well as within the microscopic [10, 11] approaches. The microscopic way of the "bosonization" of the fermion motion in nuclei is given within Quasiparticle-Phonon Model (QPM) [12]. The building blocks of the QPM are RPA-phonons. The important feature of the model is that the phonons are not only collective mode but they are involved as a tool to describe the non-collective motion. The structure of the exited states results in as an interplay of the collective and non-collective modes. This approach, to be efficient, requires the account for the Pauli principle. It is shown, that the effects due to the violation of the Pauli principle are possible to be taken into account in a simple way [12].

In spite of the usefulness of the "boson" idea the QPM reveals very approximate picture. The new experimental information obtained by means of high-resolution multidetector systems points out to a very complicated structure of the low-lying states [2]. The recent data about nucleon decay of highly excited states enable to study in detail the damping process [13]. An opportunity to make QPM adequate to the new experimental information gives the extension of the model basis. The basis is used to calculate the structure of excited states in large domain of exitation energy by means of unique set of parameters [17]. In a number of papers [14–19] the low-lying excited states in medium and heavy even spherical nuclei have been studied in the framework of the QPM. The main interest was concentrated on the increase of the collective basis of the model and its impact on the properties of the low-lying sates.

The QPM has been used to study the properties of the high-lying excited states. The model describes the distributions of spectroscopic strengths in medium and heavy nuclei in satisfactory agreement with experimental data extracted from transfer reactions [5]. The coupling to the continuum is incorporated in the formalism of QPM by means of projection operator method. It permits to calculate the partial cross sections and branching ratios for the particle decay of high-lying states in medium and heavy spherical nuclei [20].

2 Low-Lying Excited States

2.1 Formalism

The most general form of the QPM Hamiltonian is

$$H = H_{sp} + H_{pair} + H_M^{ph} + H_{SM}^{ph} + H_M^{pp} \qquad (1)$$

The term H_{sp} describes the motion of the independent nucleons in a self-consistent mean field (to generate mean field the Woods-Saxon potential is used in QPM); H_{pair} represents the monopole pairing interaction in the particle-particle channel; H_M^{ph} is the sum of isoscalar and isovector separable multipole interactions in the particle-hole channel; H_{SM}^{ph} is the same for the spin-multipole

interaction and H_M^{pp} is the sum of the multipole interaction in the particle-particle channel (multipole pairing). For example, the ph separable multipole pieces have the structure

$$H_M^{ph} = \sum_{\lambda\mu}^{\tau\tau'} \kappa_\lambda(\tau\tau') M_{\lambda\mu}^\dagger(\tau) M_{\lambda\mu}^\dagger(\tau') \tag{2}$$

where

$$M_{\lambda\mu}^\dagger(\tau) = \sum_{qq'} \langle q|F_{\lambda\mu}|q'\rangle a_q^\dagger a_{q'} \tag{3}$$

where $\tau = \pi, \nu,$ and

$$F_{\lambda\mu} = R(r)Y(\hat{r})_{\lambda\mu}$$

To ensure selfconcistecy with mean field usually radial dependence $R(r)$ is taken as derevative of mean field potencial

$$R(r) = \frac{V_{WS}}{dr}$$

A method for separabelization of $p-h$ hole Skyrme interaction is published [21]. The proposed finite rank approximation for $p-h$ interactions of Skyrme type is incorporated in QPM [21].

The phonon creation operator $Q_{\lambda\mu i}^+$ is a superposition of a bi-linear forms of the quasiparticle creation α_{jm}^+ and annihilation α_{jm} operators

$$Q_{\lambda\mu i}^+ = \frac{1}{2} \sum_\tau^{n,p} \sum_{jj'} \left\{ \psi_{jj'}^{\lambda i} [\alpha_j^+ \alpha_{j'}^+]_{\lambda\mu} - (-1)^{\lambda-\mu} \varphi_{jj'}^{\lambda i} [\alpha_{j'} \alpha_j]_{\lambda-\mu} \right\}, \tag{4}$$

where jm denote a single-particle level of the average field for neutrons (or protons) and the notation $[\cdots]_{\lambda\mu}$ means coupling to the total momentum λ with projection μ: $[\alpha_j^+ \alpha_{j'}^+]_{\lambda\mu} = \sum_{mm'} C_{jmj'm'}^{\lambda\mu} \alpha_{jm}^+ \alpha_{j'm'}^+$; the quantity $C_{jmj'm'}^{\lambda\mu}$ is the Clebsch-Gordon coefficient. Quasiparticles themselves are the result of the linear Bogoliubov transformation from the particle creation a_{jm}^+ and annihilation a_{jm} operators: $a_{jm}^+ = u_j \alpha_{jm}^+ + (-1)^{j-m} v_j \alpha_{j-m}$. In the QPM, quasiparticle energies and Bogoliubov's coefficients u_j and v_j are obtained by solving the BSC equations. Taking into account the fermion structure of the phonon operator the commutation relations read

$$[Q_{\lambda\mu i}, Q_{\lambda'\mu'i'}^+]_- = \frac{\delta_{\lambda,\lambda'}\delta_{\mu,\mu'}\delta_{i,i'}}{2} \sum_{jj'} [\psi_{jj'}^{\lambda i}\psi_{jj'}^{\lambda i'} - \varphi_{jj'}^{\lambda i}\varphi_{jj'}^{\lambda i'}]$$

$$- \sum_{\substack{jj'j_2 \\ mm'm_2}} \alpha_{jm}^+ \alpha_{j'm'} \left\{ \psi_{j'j_2}^{\lambda i}\psi_{jj_2}^{\lambda'i'} C_{j'm'j_2m_2}^{\lambda\mu} C_{jmj_2m_2}^{\lambda'\mu'} \right.$$

$$\left. - (-)^{\lambda+\lambda'+\mu+\mu'} \varphi_{jj_2}^{\lambda i}\varphi_{j'j_2}^{\lambda'i'} C_{jmj_2m_2}^{\lambda-\mu} C_{j'm'j_2m_2}^{\lambda'-\mu'} \right\}. \tag{5}$$

The first term of Eq. (5) corresponds to the boson approximation while the second one takes into account the internal fermion structure of phonons. The second term is important in considering multi-phonon states which may violate the Pauli principle. A phonon basis is constructed by diagonalizing the QPM Hamiltonian on the set of one-phonon states [12]. The procedure yields the RPA equations, and solving these equations one obtains the phonon spectrum and the internal phonon structure, i.e., the coefficients $\psi_{jj'}^{\lambda i}$ and $\varphi_{jj'}^{\lambda i}$ of Eq. (4) for any multipolarity λ under consideration. The index i in the definition of the phonon operator (4) gets the meaning of the RPA root number. The phonons are of different degree of collectivity, from collective ones (e.g. $[2_1^+]_{RPA}$) to pure two-quasiparticle configurations.

2.2   The low-lying isovector mode

In the case of even-even nuclei the Hamiltonian (1) is diagonalized in a basis of wave function constructed as a superposition of one-, two-, and three-phonon components [18, 19].

$$
\begin{aligned}
\Psi_\nu(JM) = \Bigg\{ & \sum_i R_i(J\nu) Q^+_{JMi} + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) [Q^+_{\lambda_1 \mu_1 i_1} \times Q^+_{\lambda_2 \mu_2 i_2}]_{JM} \\
& + \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 I}} T^{\lambda_1 i_1 \lambda_2 i_2 I}_{\lambda_3 i_3}(J\nu) 2\Big[[Q^+_{\lambda_1 \mu_1 i_1} \times Q^+_{\lambda_2 \mu_2 i_2}]_{IK} \times Q^+_{\lambda_3 \mu_3 i_3} \Big]_{JM} + \cdots \Bigg\} \Psi_0
\end{aligned}
$$

$$(6)$$

where Ψ_0 represents the phonon vacuum state and R, P, and T are unknown amplitudes. The index ν specifies the particular excited state.

The foregoing formalism was applied to study the low-lying excited states in even-even nuclei having neutron number $N = 80$ and $N = 84$ and domain around neutron number $N = 50$. The results are published in [22–26].

The low-lying quadrupole states in the $N = 80$, and $N = 84$ isotones have been subject to several experimental and theoretical investigations in connection with the so-called isovector or mixed-symmetry states [27,28,31–33]. The small mixing ratios $\delta(E2/M1; 2_3^+ \to 2_1^+)$ and relatively large value of $B(M1; 2_3^+ \to 2_1^+)$ correspond to a classification of the first 2^+ state as a symmetric state and of the third 2^+ state as a mixed-symmetry state in the $U(5)$-limit of IBM-2 [27, 28]. In the frame of the extended vibrational model [31], the isospin dependence of the collective coordinates is used and the 2_1^+ state is regarded as an in-phase (isoscalar) vibration and the 2_3^+ as an out-of-phase (isovector) vibration of protons and neutrons . Recent experimental data, obtained by direct lifetime measurements [33], show that the $B(M1; 2_3^+ \to 2_1^+)$ value is close to the value of the single particle estimation for this transition. This means that the low-lying

states in ^{144}Nd contain significant two-particle components. It points out to the complex structure of the low-lying 2^+ states.

A conclusive evidence was provided by an experiment on ^{94}Mo exploiting β-decay as a populating mechanism [34]. Recently, this and other experiments using different techniques [35–37] have produced an impressive body of data which greatly enriched our knowledge about the low-energy spectrum of this nucleus.

The structure of the 2^+ states calculated in RPA reveals a difference of the relative sign of the main neutron and proton components, the $[2_1^+]_{RPA}$ state is symmetric and the $[2_2^+]_{RPA}$ state is anti-symmetric [14, 15]. A relevant quantity [38] to check the nature of a RPA phonon, taking into account its structure, is the ratio

$$B = \frac{|\langle 2_i^+ \| \sum_k^p r_k^2 \, Y_{2\mu}(\Omega k) - \sum_k^n r_k^2 \, Y_{2\mu}(\Omega k) \| g.s. \rangle|^2}{|\langle 2_i^+ \| \sum_k^p r_k^2 \, Y_{2\mu}(\Omega k) + \sum_k^n r_k^2 \, Y_{2\mu}(\Omega k) \| g.s. \rangle|^2}. \tag{7}$$

As shown in Ref. [38], in the case of $B > 1$, the 2^+ state under consideration is an isovector state, otherwise – an isoscalar one. According to the calculations for ^{94}Mo and for $N = 80$ and $N = 84$ isotones, the $[2_1^+]_{RPA}$ state is an isoscalar state (the value of B is $\approx 10^{-5}$) and the $[2_2^+]_{RPA}$ state is an isovector one (the value of B is larger than 1) [14, 15].

The calculations [22–26] reveal that the lowest isoscalar and isovector 2^+ states have a dominant one-phonon component. The others, are mainly two-phonon states with a dominant $\left[2_{is}^+ \otimes 2_{is}^+\right]_{RPA}$ or $\left[2_{is}^+ \otimes 2_{iv}^+\right]_{RPA}$. The phonon structure of the states combined with the isospin properties of the phonons leads to well defined $E2$ and $M1$ selection rules. The $E2$ transitions among states differing by one phonon are strong. More specifically, the $E2$ strengths are quite large for transitions between $p - n$ symmetric states, and fairly large for transitions between mixed symmetry states. They are promoted by the phonon-exchange term of the transition operator. Weak $E2$ transitions occur between symmetric and mixed symmetry states, only if they differ by an even number of phonons.

A reverse pattern holds for the $M1$ transitions. These occur only between states with an equal number of phonons or differing by two phonons, like in the $g.s. \rightarrow 1_{1,iv}^+$ transition. The transitions between mixed symmetry and symmetric states are enhanced while those linking states of the same $p-n$ symmetry are suppressed.

Some results for ^{94}Mo are shown in Figures 1 and 2 and results for ^{94}Mo and ^{136}Ba are shown in Table 1 and Table 2. As it is seen the QPM energies and strengths are in overall good agreement with the experimental data and are consistent [10] with the picture provided by the IBM. The transitions allowed in the $IBM2$ are strong in QPM. Those forbidden in $IBM2$ are week in QPM. This correspondence is almost one to one, with few exceptions.

Ch. Stoyanov

Table 1. $E2$ and $M1$ transition strengths in ^{94}Mo. The experimental data are taken form Refs. [34–37]

| $J_i \to J_f$ | B(E2) (e^2b^2) | | | B(M1) (μ_N^2) | | | |
	EXP	QPM	IBM2	EXP	QPM $g_s=0.7$	$g_s=0$	IBM2
$0^+_{gs} \to 2^+_{1,is}$	2030(40)	1978	2333				
$0^+_{gs} \to 2^+_{2,is}$	32(7)	35	0				
$0^+_{gs} \to 2^+_{1,iv}$	230(30)	150	151				
$0^+_{gs} \to 2^+_{2,iv}$	27(8)	18	0				
$0^+_{gs} \to 2^+_{3,iv}$	83(10)	10	0				
$1^+_{1,iv} \to 0^+_{gs}$				0.16(1)	0.14	0.09	0.16
$1^+_2 \to 0^+_{gs}$				0.046(18)	0.04	0.009	
$1^+_{1,iv} \to 2^+_{1,is}$	30(10)	13	49	0.007^{+6}_{-2}	6.10^{-4}	5.10^{-3}	0
$1^+_{1,iv} \to 2^+_{2,is}$				0.43(5)	0.75	0.22	0.36
$1^+_{1,iv} \to 2^+_{1,iv}$	< 690	374	556	< 0.05	3.10^{-6}	2.10^{-5}	0
$2^+_{2,is} \to 2^+_{1,is}$	720(260)	673	592	0.06	0.006	0.004	0
$2^+_{1,iv} \to 2^+_{1,is}$				0.48(6)	0.72	0.23	0.30
$2^+_{2,iv} \to 2^+_{1,is}$				0.07	0.001	0.002	0
$2^+_{2,iv} \to 2^+_{2,is}$					0.10	0.034	
$2^+_{3,iv} \to 2^+_{1,is}$				0.03	0.013	0.005	0
$2^+_{3,iv} \to 2^+_{2,is}$				0.35(11)	0.24	0.072	0.1
$3^+_{1,iv} \to 2^+_{1,is}$	9^{+25}_{-8}	12		$0.01^{+0.012}_{-0.006}$	0.006	0.0025	0
$3^+_{1,iv} \to 2^+_{1,iv}$	250^{+310}_{-210} $(1.5^{+1.2}_{-0.6}) \times 10^3$	368	582	$0.021^{+0.035}_{-0.014}$ $0.09^{+0.07}_{-0.03}$	2.10^{-5}	9.10^{-6}	0
$3^+_{1,iv} \to 2^+_{2,is}$				$0.24^{+0.14}_{-0.07}$	0.34	0.10	0.18
$3^+_{1,iv} \to 4^+_{1,is}$				$0.074^{+0.044}_{-0.019}$	0.26	0.08	0.13
$4^+_{1,is} \to 2^+_{1,is}$	670(100)	661	592				
$4^+_{1,iv} \to 4^+_{1,is}$				0.8(2)	0.75	0.23	

Table 2. $E2$ and $M1$ transitions connecting some excited states in ^{136}Ba calculated in QPM. The experimental data are taken form Ref. [34–37]

| $J_i \to J_f$ | B(E2) (e^2b^2) | | B(M1) (μ_N^2) | |
	EXP	QPM	EXP	QPM
$0^+_{gs} \to 2^+_{1,is}$	0.400(5)	0.39		
$0^+_{gs} \to 2^+_{2,is}$	0.016(4)	0.08		
$0^+_{gs} \to 2^+_{1,iv}$	0.045(5)	0.05		
$0^+_{gs} \to 1^+_{1,iv}$			0.13(2)	0.17
$2^+_{2,is} \to 2^+_{1,is}$	0.09(4)	0.15		
$2^+_{1,iv} \to 2^+_{1,is}$		0.003	0.26(3)	0.25
$1^+_{1,iv} \to 2^+_{2,is}$		0.0004	0.6(1)	0.18

Figure 1. $B(E2; 0_1^+ \rightarrow 2_k^+)$ strength distributions in ^{94}Mo.
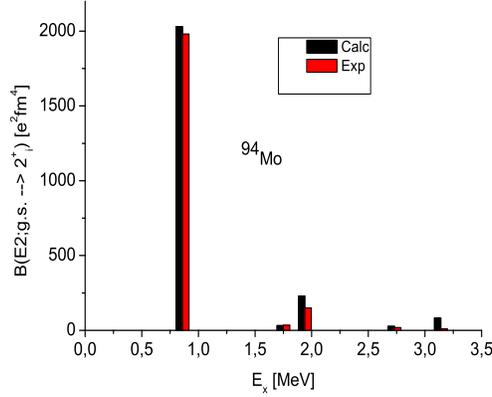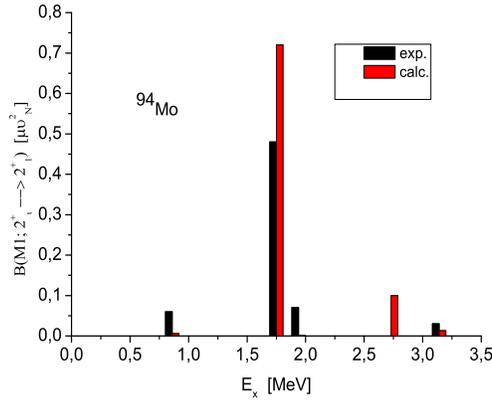
Figure 2. $B(M1; 2_k^+ \rightarrow 2_1^+)$ strength distributions in ^{94}Mo

3 Highly Excited States

The complexity of stationary many-body states can be quantified with the aid of such characteristics as information entropy or inverse participation ratio [39–42] of generic wave functions.

Different characteristics can be used in order to probe the sensitivity of the system to external perturbations. In classical case such sensitivity is known to be the main property of dynamical chaos. A special entropy-like quantity, called invariant correlational entropy (ICE), was suggested [43] as a measure of complexity related to the response of a system to a random noise included in the Hamiltonian. This quantity is by construction invariant with respect to the basis transformations. It also reflects the correlations and phase relationships between the components of the wave functions insofar as they are revealed in the response of the system to a perturbation. Taking the strength of the interaction as a control

parameter, one can clearly see the quantum phase transitions as it was demonstrated in the interacting boson model [44] and in the evolution of pairing in the shell model [45].

The problem of the interaction of a quasiparticle with an even-even core is of considerable interest for nuclear physics. It was quite well studied for low-lying excited states where the interactions of the quasiparticle predominantly with quadrupole and octupole surface vibrations are important. At higher excitation energy, a large number of other nuclear modes influence the damping of the quasiparticle motion. The mixing of the simple mode with the states of the next levels of complexity leads to the fragmentation of the single-particle strength over a wide domain of excitation energy - the single-particle state obtains a spreading width. It was shown [5] that the spreading occurs in two stages. At the first stage of fragmentation, the single-particle state is spread over several doorway states. At the second step, the doorway states are spread through the mixing with many complex excitations related to other degrees of freedom. The limiting case of self-consistent hierarchy of multi-step spreading was discussed [46].

3.1 Correlational entropy

To study the sensitivity of the excited states upon variation of external parameters, we use the invariant correlational entropy (ICE) [43]. The ICE method presumes that Hamiltonian $H(\lambda)$ of a system depends on a random parameter λ. The parameter λ ("noise") is considered as a member of an ensemble characterized by the normalized distribution function $P(\lambda)$,

$$\int d\lambda P(\lambda) = 1.$$

In an arbitrary primary basis $|k\rangle$ evolution of any stationary state $|\alpha, \lambda\rangle$ is a function of λ. At a given value of λ, the state can be decomposed as

$$|\alpha, \lambda\rangle = \sum_k C_k^\alpha(\lambda)|k\rangle.$$

The ICE is defined as
$$S^\alpha = -\text{Tr}\{\rho^\alpha \ln(\rho^\alpha))\}, \tag{8}$$

where ρ^α is the density matrix of the state $|\alpha\rangle$ averaged over the noise ensemble. In the basis $|k\rangle$ prior to the averaging, the matrix for a given value of α has to be constracted

$$\rho_{k,k'}^\alpha = C_k^\alpha(\lambda)C_{k'}^\alpha(\lambda)^*$$

and then average over the ensemble.

The ICE is basis-independent von Neumann entropy that reflects the correlations between the wave function components which are subject to fluctuations determined by the parameter λ. The value S^α for a given state typically increases with the complexity of the state and reaches the maximum at the point where the change of the parameter around some average value implies the most radical change of the structure of the system.

3.2   The model of excited states in odd nuclei

Quasiparticle-Phonon Model is used to describe the properties of highly-excited states in odd spherical nuclei. According to the model, the Hamiltonian of the system of an odd number A+1 particles has the form

$$H = h + H_{core} + H_{coup}\,.$$

The first term, h, describes the motion of a quasiparticle in a mean field potential U created by the even-even core. In the spirit of the QPM, the Hamiltonian H_{core} is treated in the random phase approximation (RPA), i.e. the particle-hole configurations are built with the subsequent RPA diagonalization. The properties of the $(A + 1)$-nucleus can be described in terms of the quasiparticle states $\alpha_\beta^\dagger \mid 0\rangle$, quasiparticle-plus-phonon states $[\alpha_\beta^\dagger \otimes Q_\mu^\dagger] \mid 0\rangle$ and quasiparticle-plus-twophonon states $[\alpha_\beta^\dagger \otimes [Q_\nu^\dagger \otimes Q_\mu^\dagger]] \mid 0\rangle$, where all combinations have the same total spin and parity quantum numbers JM. Here $\alpha_\beta^\dagger$ is the quasiparticle creation operator with shell-model quantum numbers $\beta \equiv (n, l, j, m)$, whereas $Q_\nu^\dagger \equiv Q_{\lambda,\mu,i}^\dagger$ denotes the phonon creation operator with the angular momentum λ, projection μ and RPA root number i. This basis completely determines the two steps of damping process where the single-particle state acquires spreading width.

The following wave functions describe in the QPM the ground and excited states of the odd nucleus with angular momentum J and projection M:

$$\mid d_i\rangle \equiv d_i^\dagger \mid 0\rangle = \Big(\sum_\beta C_\beta^{(i)}\alpha_\beta^\dagger + \sum_{\beta,\mu} D_{\beta,\mu}^{(i)}[a_\beta^\dagger \otimes Q_\mu^\dagger]$$

$$+ \sum_{\beta\nu\nu'} F_{\beta\nu\nu'}^{(i)}(I, J\beta)[\alpha_\beta^\dagger \otimes [Q_\nu^\dagger \otimes Q_{\nu'}^\dagger]] \Big) \mid 0\rangle\,. \quad (9)$$

The ground state $\mid 0\rangle$ is the ground state of the neighboring even-even nucleus (also quasiparticle and phonon vacuum) and i stands for the number within a sequence of states of given J^π. The coefficients C, D, F determine the quasiparticle, quasiparticle + phonon and quasiparticle + two-phonons amplitudes, respectively, for the state i.

3.3 Complexity of states and correlational entropy

The calculation is for the highly-excited states in ^{209}Pb. For studying the ICE the single-particle $1k_{17/2}$ state is selected. The properties of this state are studied in detail [5, 20]. This orbital is quasi-bound in the Woods-Saxon potential being located at 4.88 MeV, energy much higher than the Fermi level of ^{209}Pb. Because of its high energy, the state is surrounded by many quasiparticle-plus-phonon and quasiparticle-plus-two-phonon states.

At high excitation energy, the level density is large and it is convenient to calculate the single-particle strength distribution by means of the strength function [5] using an averaging Lorentzian function. The distribution of the single-particle strength (coefficient C^2 of eq. 9) of the $1k_{17/2}$ state is shown in Figure 3 The number of quasiparticle-plus-phonon components included in the wave function (9) is 420, while the number of quasiparticle-plus-two-phonon components is 1116. The Lorentzian smoothing parameter was chosen to be 0.2 MeV. It is seen from Figure 3 that the single-particle strength is spread over a broad interval of excitation energy. The largest fraction, 81% of the strength, is concentrated between 5 and 13 MeV excitation energy, and the state acquires the large spreading width in this domain, $\Gamma^{\downarrow} = 1.5$ MeV ($\Gamma^{\downarrow}$ is calculated as the second moment of the distribution). It was shown [5] that the main mechanism leading to the spreading is the interaction of the particle with the vibrations of the even core.
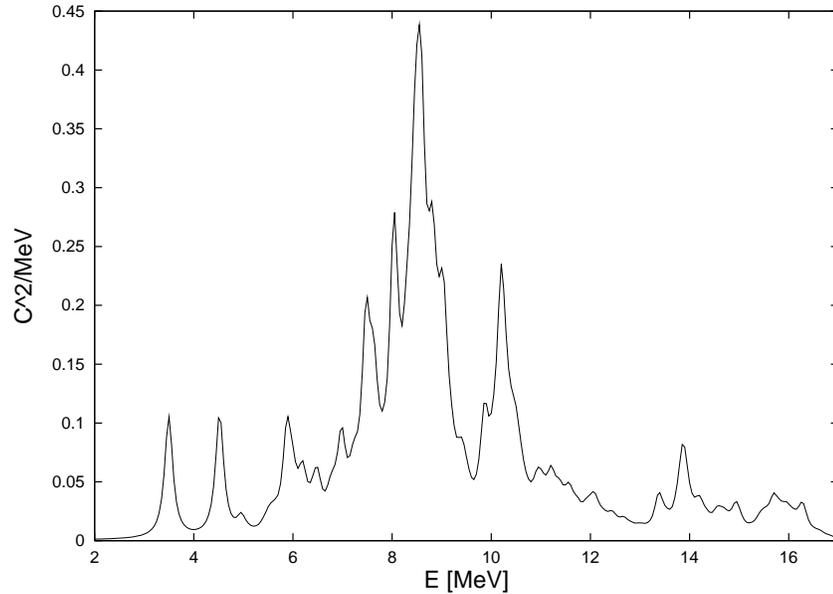

Figure 3. Distribution of the single-particle strength ($C^2 Eq.9$) of the state $1k_{17/2}$ in ^{209}Pb.
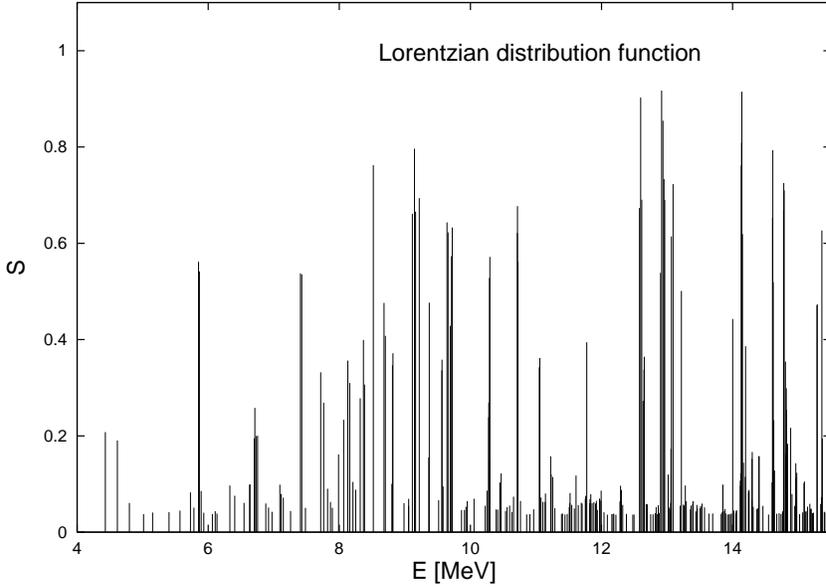
Figure 4. Correlation entropy of the excited states $17/2^+$ in ^{209}Pb. The Lorentzian distribution is used. The wave function includes only quasiparticle-onephonon components.

The correlation entropy (8) connected with the excited states was calculated using the Lorentzian distribution function:

$$P(\lambda) = \frac{1}{2\pi} \frac{\Delta}{\lambda^2 + \frac{\Delta^2}{4}}$$

The distribution is shown on Figure 4. It is seen that there is a background of small values of the ICE due to the large amount of weakly interacting states. The greater values of entropy are located in the vicinity of the peaks of the single-particle strength distribution pointing to the regions where the proximity of many strongly coupled states leads to enhanced sensitivity of the wave functions. One can see the tendency to the enlargement of entropy when the excitation energy increases and a larger density of the states at higher excitation energy enters the game.

The two steps of damping process are well pronounced in the comparison when respectively quasiparticle-plus-phonon and quasiparticle-plus-twophonons components are included in the wave function (9). The nearest-neighbor spacing distribution of $17/2^+$ excitations for the case when the quasiparticle-plus-twophonons components are included in the wave function (9) is fitted by the Brody distribution with the Brody parameter equal to 0.6 (see Figure 5). If only the quasiparticle-plus-phonon compunents are taken into account in the wave
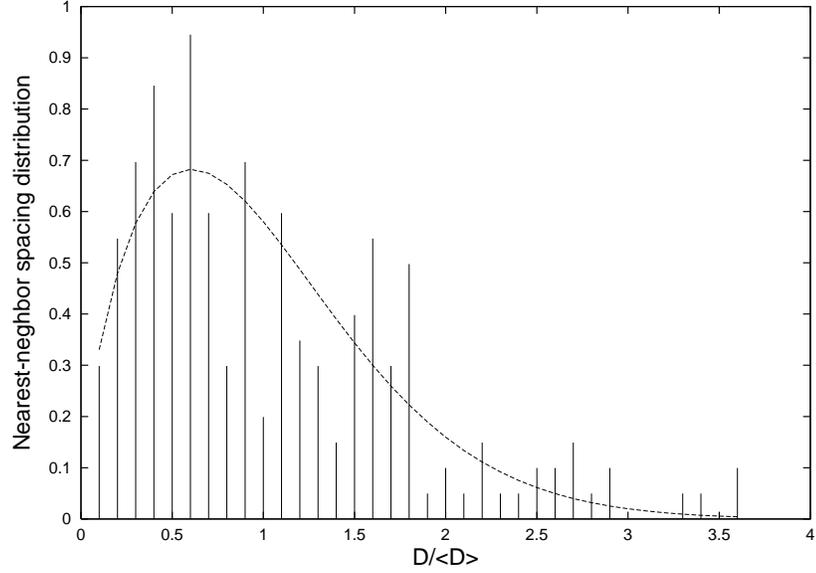
Figure 5. Nearest-neighbor spacing distribution of $17/2^+$ states in ^{209}Pb. The calculated values are fit by the Brody distribution with the Brody parameter equal to 0.6.

function (9) the nearest-neighbor spacing distribution is fitted by the Brody distribution with the Brody parameter equal to 0.4. The higher degree of chaoticity when the quasiparticle-plus- twophonon components are taken into account indicates the importance of more complex components and their influence on the damping process. At the first stage, when the single-particle states interact only with quasiparticle-plus- one-phonon components, the regularities induced by the mean field, such as the structure of the level density, are not completely destroyed. At the next stage, when the interaction with the components of the next level of complexity is switched on, these regularities are partly smeared, but the wave function (9) is still relatively simple.

The neighboring level spacing distribution is a conventional indicator of quantum chaos. It is shown that this distribution is correlated with ICE moving in the direction of the Wigner distribution characteristic for the Gaussian orthogonal ensemble (but not reaching this limit) in parallel to the increase of the value of ICE. Although the model wave function truncated on the level of the quasiparticle-plus-two-phonon components is not sufficiently chaotic to manifest entirely the complicated structure of high-lying excited states, it can be used to describe the distribution of a few quasiparticle components. These components in fact play the role similar to the scars in quantum chaos.

More details about the connection of ICE and properties of highly-excited states can be found in [47]

4　Conclusions

The presented examples illustrate the opportunities and limitations of QPM. The model could be used in the low-lying sector of nuclear excitations to predict complicated motion of neutrons versus protons. The complex structure of high-lying states is estimated by means of single number given by Invariant Correlational Entropy. In spite of the simplicity of QPM the calculated properties of excited states are in good agreement with the measured values.

Acknowledgements

References

[1]　A. Bohr and B. Mottelson (1975) "*Nuclear Structure*" vol. 2, New York: Benjamin.

[2]　M. Kneissl, H.H. Pitz and H. Zilges (1996) *Progr. Part. Nucl. Phys.* **37** 349.

[3]　N. Pietralla, P. von Brentano and A.F. Lisetskiy (2008) *Prog. Part. Nucl. Phys*. **60** 225.

[4]　G.F. Bertsch, P.F. Bortignon, R.A. Broglia et al. (1983) *Rev. Mod. Phys.* **55** 287.

[5]　S. Galès, Ch. Stoyanov and A.I. Vdovin (1988) *Phys. Rep*. **166** 11.

[6]　S. Fortier et al. (1990) *Phys. Rev. C* **41** 2689.

[7]　D. Beaumel et al. (1994) *Phys. Rev. C* **49** 2444.

[8]　S. Fortier et al. (1995) *Phys. Rev. C* **52** 2401.

[9]　A. Arima and F. Iachello (1976) *Ann. Phys. (N.Y.)* **63** 253; *Phys. Rev. C* **14** 761.

[10]　N. Lo Iudice, V.Yu. Ponomarev, Ch. Stoyanov, A.V. Sushkov, and V.V. Voronov (2012) *J. Phys. G: Nucl. Part. Phys.* **39** 043101.

[11]　D. Bianco, N. Lo Iudice, F. Andreozzi, et al. (2012) *Phys. Rev. C* **86** 044325.

[12]　V.G. Soloviev (1992) "*Theory of Atomic Nuclei: Quasiparticle and Phonons*", Bristol, IOP.

[13]　S. Fortier et al. (1995) *Phys. Rev. C* **52** 2401.

[14]　M. Grinberg, Ch. Stoyanov and N. Tsoneva (1998) *Sov. J. Part. Nucl*. (*Elementarnie Tchastitsi i Atomnie Jadra*) **29** 1456.

[15]　Thai Thac Dinh, M. Grinberg and Ch. Stoyanov (1992) *J. Phys.* **G 18** 329.

[16]　J. Adam, J. Dobeš, B. Kracík et al. (1992) *Z. Phys.* **A 343**381.

[17]　M. Grinberg, Thai Thac Dinh et al. (1993) *J. Phys. G* **19** 1179.

[18]　M. Grinberg, Ch. Stoyanov, Nucl (1994) *Phys. A* **573** 231.

[19]　V.Yu. Ponomarev et al. (1998) *Nucl. Phys. A* **635** 470.

[20]　N. Van Giai, Ch. Stoyanov, V.V. Voronov, S. Fortier (1996) *Phys. Rev. C* **53** 730.

[21]　N. Van Giai, Ch. Stoyanov, V.V. Voronov (1998) *Phys. Rev. C* **57** 1204.

[22]　N. Lo Iudice and Ch. Stoyanov (2000) *Phys. Rev. C* **62** 047302.

[23]　N. Lo Iudice and Ch. Stoyanov (2004) *Phys. Rev. C* **69** 044312.

[24] N. Lo Iudice and Ch. Stoyanov (2006) *Phys. Rev. C* **73** 037305.

[25] N. Lo Iudice, Ch. Stoyanov and D. Tarpanov (2008) *Phys. Rev. C* **77** 044310.

[26] N. Lo Iudice, Ch. Stoyanov and N. Pietralla (2009) *Phys. Rev. C* **80** 024311.

[27] W.D. Hamilton, A. Irbäck and J.P. Elliott (1984) *Phys. Rev. Lett.* **26** 2469.

[28] P. van Isacker, K. Heyde, J. Jolie et al. (1986) *Ann. Phys.* **171** 253.

[29] R.A. Meyer, O. Scholten, S. Brant, V. Paar (1990) *Phys. Rev. C* **41** 2386.

[30] J. Copnell, S. Robinson, K. Heyde, et al. (1989) *Phys. Lett.* **222B** 1.

[31] R. Nojarov and A.J. Faessler (1987) *Nucl. Phys. G* **13** 337.

[32] A. Richter (1991) *Nucl. Phys. A* **522** 139.

[33] S.J. Robinson, J. Jolie, H.G. Börner et al. (1994) *Phys. Rev. Lett.* **73** 412.

[34] N. Pietralla et al. (1999) *Phys. Rev. Lett*. **83** 1303.

[35] N. Pietralla, C. Fransen, P. von Brentano, A. Dewald et al. (2000) *Phys. Rev. Lett*. **84** 3775.

[36] C. Fransen, N. Pietralla, P. von Brentano, A. Dewald et al. (2001) *Phys. Lett. B* **508** 219.

[37] C. Fransen et al. (2003) *Phys. Rev. C* **67** 024307.

[38] R. Nikolaeva, Ch. Stoyanov and A.I. Vdovin (1989) *Europhys. Lett.* **8** 117.

[39] V. Zelevinsky, B.A. Brown, N. Frazier, and M. Horoi (1996) *Phys. Rep. C* **54** 1665.

[40] V.K.B. Kota (2001) *Phys. Rep*. **347** 223.

[41] F.M. Izrailev (1990) *Phys. Rep*. **196** 299.

[42] V. Zelevinsky, M. Horoi, and B.A. Brown (1995) *Phys. Lett. B* **350** 141.

[43] V.V. Sokolov, B.A. Brown, and V. Zelevinsky (1998) *Phys. Rev. E* **58** 56.

[44] P. Cejnar, V. Zelevinsky, and V.V. Sokolov (2001) *Phys. Rev. E* **63** 036127.

[45] A. Volya and V. Zelevinsky (2003) *Phys. Lett. B* **574** 27.

[46] B. Lauritzen, P.F. Bortignon, R.A. Broglia, and V.G. Zelevinsky (1995) *Phys. Rev. Lett*. **74** 5194.

[47] Ch. Stoyanov and V. Zelevinsky (2004) *Phys. Rev. C* **70** 014302.