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Tests of Partial Dynamical Symmetries and Their Implications

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Abstract. The concept of Partial Dynamical Symmetries (PDSs) is introduced and the first extensive tests of this idea are summarized along with comments about future directions.

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1 Introduction

Symmetries often provide an elegant and simple framework for the elucidation of nuclear structure. Many predictions are parameter–free and give insights into the structure of the nucleus as a many–body system with its own quantum numbers, selection rules, analytic relations for observables, and so on. Unfortunately, very few nuclei manifest any symmetry exactly and, normally, one diagonalizes phenomenological, parameterized Hamiltonians that break the symmetries. One of the most common approaches is that of the Interacting Boson Approximation (IBA) Model [1] with its three benchmark symmetries U(5), SU(3) and O(6) and a simple Hamiltonian written in terms of s and d bosons and their interactions.

The SU(3) situation is illustrated in Figure 1. In SU(3) the levels are grouped into representations. Within each representation there can be one or more axially symmetric quadrupole deformed intrinsic excitations. On top of each intrinsic excitation is a rotational band. States of the same spin within a representation are degenerate. One sees this in the figure for the γ and lowest $K = 0$ bands (sometimes called a β vibrational band). One consequence of SU(3) is that the selection rules allow for strong collective $E2$ transitions between rotational states but transitions between representations, such as from the β and γ bands to the ground band, are forbidden. These two characteristics (degenerate bands and forbidden transitions to the ground state band) differ from the data. Therefore SU(3) is, at best, a first approximation to actual deformed nuclei. In practice, such nuclei are fit in the IBA Model by diagonalizing a Hamiltonian that breaks SU(3)(mixes SU(3) representations). In typical calculations [2] the amplitudes for non-principal SU(3) representations can easily reach 0.4. This approach has,

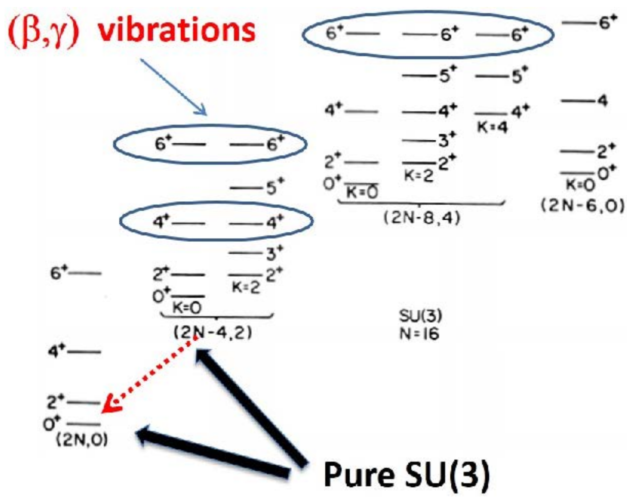

Figure 1. (Color online) Typical level scheme of the lowest levels in SU(3) for the case of 16 bosons. (Other boson numbers are very similar for the excitations shown). Examples of the typical degeneracies are highlighted as is the selection rule of forbidden inter-representation $E2$ transitions. The label "Pure SU(3)" and the black arrows indicate the essence of the PDS we are considering in this paper.

overall, been highly successful and, although it is a parameterized approach, is rather parameter–efficient.

Nevertheless, some years ago the idea of Partial Dynamical Symmetries (PDS) was developed by Leviatan [3]. We refer the reader to this publication for details of the model. In a PDS some of the properties of a symmetry are maintained while others are (often severely) broken. The most appealing PDS was based on SU(3) and described a situation seemingly at complete variance with known data, namely one in which the ground and gamma bands are pure SU(3) while all other bands are mixed.

In particular, in contrast to the selection rule for SU(3), it is well known that transitions from the γ band to the ground band are collective (typically 5-15 W.u.). Nevertheless, Leviatan showed that $E2$ branching ratios from the γ to the ground band in ^{168}Er could be well described by this PDS in a parameter–free manner by utilizing the most general $E2$ operator up to second order [SU(3) utilizes a specific form of this operator that is a generator of SU(3)]. This was a striking result as it seemed to produce results in similar agreement with the data as parameterized diagonalizations even though the structure of the wave functions for these two bands in the PDS is absolutely pure SU(3). However, no further tests of it ensued until recently, when we have carried out the first extensive tests (for more than 60 nuclei) of this PDS throughout the rare earth, actinide, and A $\sim$ 100 regions [4–8]. The results of Refs. [4–6] show significant areas of agreement but also characteristic disagreements for transitions between the γ band and the ground band and within the γ band. They reveal the importance of finite nucleon number effects, and they affect the standard decades-long interpretation of band mixing in deformed nuclei. These results are discussed below. (Refs. [7, 8] apply the PDS to the lowest K = 0 band and re-consider mixing of and ground bands as a further perturbation to the PDS, respectively.)

2 Comparisons of the SU(3) PDS to Experiment

We consider all deformed rare earth nuclei, 47 in all. Of these there were sufficient data to study comparisons with the PDS for 22 nuclei. It is important to stress that the predictions for interband transition ratios from a given level of the γ band to the ground band are completely parameter–free: the PDS either agrees with the data or it does not. We look at these predictions first and then turn to intraband transitions. Figures 2,3 and Table 1 present a sampling of typical comparisons for the interband transitions. In Figure 2 the original ^{168}Er case from Ref. [3] is included as well as several other nuclei. In Table 1, we give detailed predictions for interband predictions for ^{168}Er in comparison to the data. These are updated and slightly corrected for new data from those of Refs. [3, 4]. Figure 3 plots the data for a number of nuclei against the Alaga rules. In this figure the diagonal line indicates agreement between the data and the Alaga rules, and the points are color coded by the spin–changing nature of the transition.

One immediately reaches two conclusions: Overall, the agreement of the PDS

Table 1. Relative interband B(E2) values in ^{168}Er compared to the Alaga rules and the SU(3) PDS. For each initial level one transition is normalized to 100. Updated and corrected from Refs. [3, 4].

$I_i^\pi \to I_f^\pi$	^{168}Er	Alaga	PDS
$2_\gamma^+ \to 0^+$	56.2(2.4)	70	64.3
$2_\gamma^+ \to 2^+$	100(2)	100	100
$2_\gamma^+ \to 4^+$	7.3(3.3)	5	6.3
$3_\gamma^+ \to 2^+$	100(2)	100	100
$3_\gamma^+ \to 4^+$	62.6(7.1)	40	49.3
$4_\gamma^+ \to 2^+$	19.3(1)	34	28.1
$4_\gamma^+ \to 4^+$	100(2)	100	100
$4_\gamma^+ \to 6^+$	13.1(12)	8.64	12.5
$5_\gamma^+ \to 4^+$	100(9.9)	100	100
$5_\gamma^+ \to 6^+$	123(6.3)	57.1	79.6
$6_\gamma^+ \to 4^+$	11.2(6.2)	26.9	20.3
$6_\gamma^+ \to 6^+$	100(6.3)	100	100
$6_\gamma^+ \to 8^+$	37.6(68)	10.6	18.0

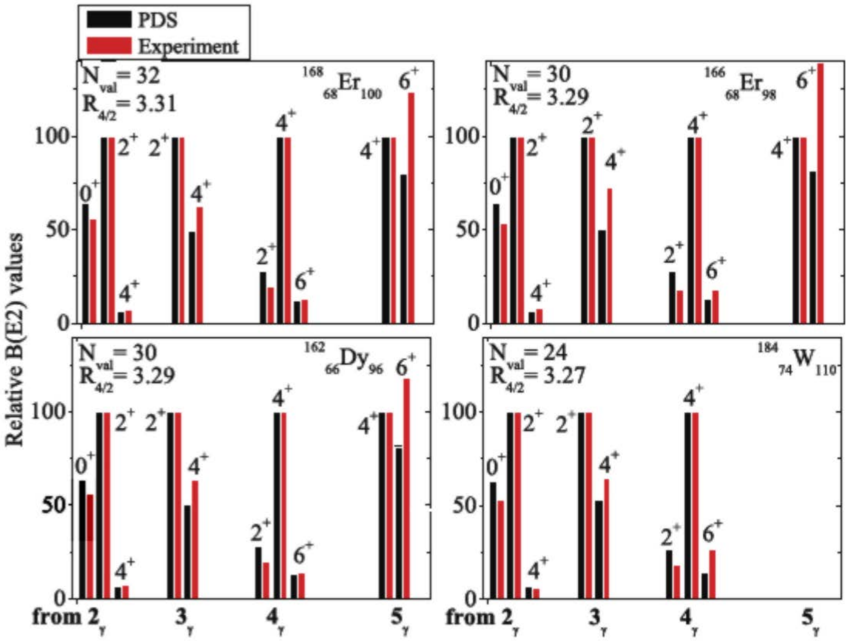

Figure 2. (Color online) Comparison of the data to the PDS for interband transitions in several typical deformed nuclei. [Based on Ref. [4]

with the data is rather good, especially considering the parameter–free nature of the model. Secondly there are clearly a number of disagreements.

Interestingly for understanding the physics, both the data and these disagreements have a characteristic and robust behavior. (See Figures 2, 3 and Table 1.) Namely, for spin increasing transitions the data are always greater than the Alaga rules, as are the PDS predictions; for spin decreasing transitions the opposite is true; the deviations from the Alaga rules increase with spin of the initial level; while the PDS is clearly an improvement on the Alaga rules, its predictions do not go far enough, that is, the data always deviate from the PDS in these characteristic directions (albeit by less than they deviate from the Alaga rules).

There is also a more subtle result. The three trends just cited are exactly characteristic of the effects of γ–ground band mixing, as has been known for decades [9, 10]. Therefore one sees that the PDS somehow mocks up the effects of band mixing even though both the γ band and the ground band are pure SU(3). Inspection of the PDS shows that its deviations from the Alaga rules are solely due to finite valence nucleon number effects. This highlights the importance of considering such effects in collective models (most of which do not). It also raises the question, as yet unanswered, as to why finite nucleon number

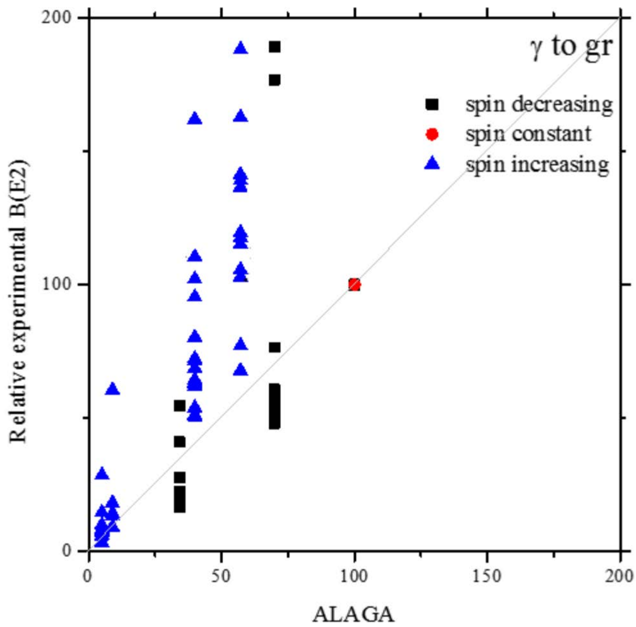

Figure 3. (Color online) Experimental interband relative B(E2) values for several rare earth nuclei plotted against the Alaga rules. Perfect agreement corresponds to points along the diagonal. The transitions are color-coded according to their spin-changing nature. The normalization for each initial level is to the spin-constant (J to J) transition.

effects simulate band mixing. Finally, since the PDS only accounts for part of the deviations of the data from the Alaga rules, this points for the remaining need for a re-introduction of band mixing. However, clearly, that band mixing will be smaller than has been historically considered for half a century. Collective model calculations with the IBA model reproduce the data excellently by including such mixing [This was discussed in Ref. [4]]. A quantitative analysis of this additional (but reduced) mixing is the topic of Ref. [8].

While ratios of interband transitions are parameter-free in the PDS, intraband transitions (within the γ band) depend on one parameter, namely the ratio of the boson effective charges for the two terms in the $E2$ operator. Nevertheless, this is only one parameter per nucleus and it is interesting to see how well the PDS reproduces the data. The one parameter, called θ/α in Ref. [3], is fit to the experimental B(E2) ratio B(E2: $2_\gamma \rightarrow 0_{gr}$)/B(E2: $2_{gr} \rightarrow 0_{gr}$) for each nucleus. For ^{168}Er, θ/α is 3.75. For all other rare nuclei (except a few Os nuclei and ^{156}Dy), the fitted values of θ/α lie in a narrow range near 3.75. In fact, one can, in practice, even use a single value of θ/α for all these nuclei. However, to be as accurate as possible the results we show below use the specific θ/α fitted for each nucleus.

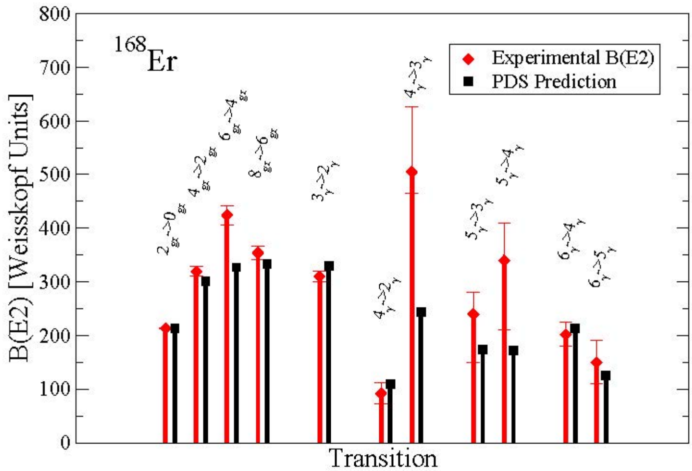

Figure 4. (Color online) Comparison of absolute intra-γ–band B(E2) values (in W.u.) to the PDS for ^{168}Er [based on Ref. [6]. Intra–ground band transitions are also included at the left.

Extensive sets of absolute experimental intra–γ–band B(E2) values are only known for ^{166,168}Er. The results for ^{168}Er are shown in Figure 4, compared to the PDS for $\theta/\alpha = 3.75$. Overall the agreement is rather impressive. However it must be recalled that we have obtained the value of θ/α from another ratio of intra–to–interband transitions and so these calculations mainly test relative intraband B(E2) values and these strongly enhanced transitions are not very sensitive to perturbations to the wave functions. The main disagreements (of about a factor of 2) are for transitions from the 4_γ and 5_γ initial levels.

Is is interesting to look across the entire rare earth region. Here, due to lack of data, one is forced again to use ratios of intra–to–interband B(E2) values. In Figure 5 we show one example, namely for the 5_γ to 3_γ transition normalized to the 5_γ to 4_{gr} transition. This figure is typical of results for other transitions. We chose it because there happens to be data for more nuclei for this transition than for others. In general the agreement in very impressive with the glaring exception of ^{160}Gd. This case is so out of line with all the others that perhaps a re-measurement is called for. We note that, in most of the nuclei in the figure, the PDS predictions are lower than the data. This is misleading. Had we normalized to the 5_γ to 6_{gr} transition we would have seen the opposite. That is, much of the (small) disagreements in Figure 5 are actually due to the interband normalization transition.

Finally, even though the lowest $K = 0$ band in not pure SU(3) in the PDS, branching ratios of two interband transitions from this band to the ground band also turn out to be parameter free. (We thank A. Leviatan for pointing this out to us). We

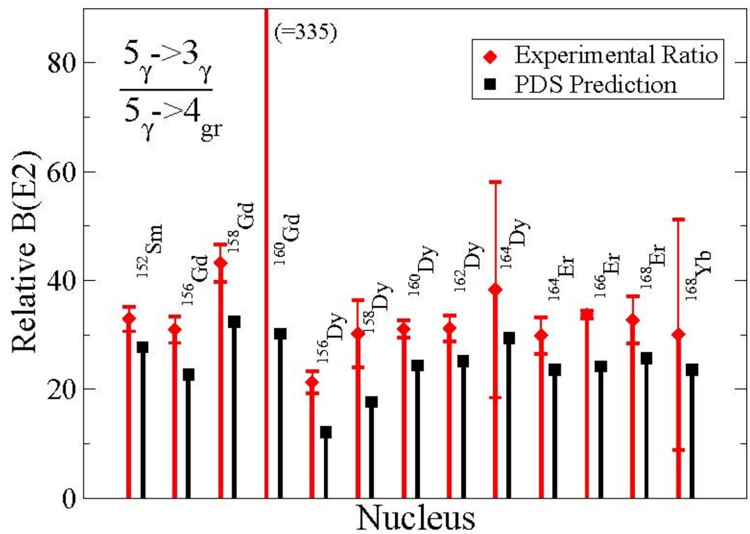

Figure 5. (Color online) Comparison of relative intra-γ to γ to ground interband B(E2) values to the PDS predictions for all the rare earth region nuclei where data are known. Plotted are the 5_γ to 3_γ B(E2) values normalized to the interband 5_γ to 4_{gr} transitions.

are currently testing these PDS predictions. It is premature to show any results and we only note that there seem to be some significant discrepancies between the PDS and the data and even larger ones if one compares to the Alaga rules. It is not yet clear if these can be ameliorated by the introduction of mixing. This topic will be addressed in Ref. [7].

3 Conclusion

We have presented comparisons of data in the rare earth nuclei to the predictions of an SU(3) PDS for both interband and intraband B(E2) values. The PDS, generally, gives good agreement with the data, and, for interband transitions, improved agreement compared to the Alaga rules. This improvement is due specifically to the automatic incorporation of finite nucleon number effects in the IBA-based PDS. Mixing of intrinsic excitations, in particular the γ and ground bands, can lead to improved agreement, at the cost of an additional parameter. Since the PDS predictions deviate from the Alaga rules in the same direction as results including such mixing, the required mixing corrections to the PDS are less than those to the Alaga rules, perhaps suggesting that the historically accepted levels of mixing may be larger than those that may in fact apply. Further research is needed to determine why these two rather different interpretations of deformed nuclei—collective models with mixing of SU(3) representations and the PDS with pure SU(3) structure for the γ and ground bands produce such comparable predictions. Tests for other intrinsic excitations, in particular the lowest $K = 0$ band, are also needed and are in progress.

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References

[1] F. Iachello and A. Arima (1978) *The Interacting Boson Model*, Cambridge University Press, Cambridge.

[2] R.F. Casten and D.D. Warner (1988) *Rev. Mod. Phys.* **60** 389.

[3] A. Leviatan (1996) *Phys. Rev. Lett.* **77** 818.

[4] R.F. Casten, R.B. Cakirli, K. Blaum and A. Couture (2014) *Phys. Rev. Lett.* **113** 112501.

[5] A. Couture, R.F. Casten and R.B. Cakirli (2015) *Phys. Rev. C* **91** 014312.

[6] A. Couture, R.F. Casten and R.B. Cakirli, to be published.

[7] R.F. Casten, A. Couture, R.B. Cakirli and E. Cevik, to be published.

[8] R.B. Cakirli, D. Sen, D. Petrellis and R.F. Casten, to be published.

[9] P. Lipas (1962) *Nucl. Phys.* **39** 468.

[10] L.L. Riedinger, Noah R. Johnson and J.H. Hamilton (1969) *Phys. Rev.* **179** 1214.