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Algebraic Models for Structure of Heavy N=Z Nuclei: IBM-4 Results for Gamow-Teller and α-Transfer Strengths

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Abstract.
Heavy $N = Z$ odd-odd nuclei starting from ^{62}Ga are expected to give new insights into isoscalar vs isovector pairing and also carry signatures of Wigner's $SU(4)$ symmetry. Incorporating these, within the shell model with $L - S$ coupling we have the basic $SO(8)$ algebraic model. With Dyson boson mapping and adding quadrupole degrees of freedom, this model goes over to the algebraic spin-isospin (ST) invariant interacting boson model (called IBM-4) with the bosons carrying $(ST) = (10)$ and (01) degrees of freedom. Using a basis defined by the $SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ limit of IBM-4, reported here are the formulation and numerical results for (i) Gamow-Teller strengths for β^+ decay of $^{4n+2}_{2n+2}X_{2n}$ nucleus with $T = 1$ ground state to the $T = 0$ levels of the odd-odd N=Z $^{4n+2}_{2n+1}Y_{2n+1}$ nucleus and (ii) α-transfer strengths involving N=Z nuclei. All the numerical results are given as a function of a parameter representing the competition between isoscalar and isovector pairing.

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1 Introduction

Nuclei with equal number of protons and neutrons (N=Z) attracted special attention from the early days of nuclear physics as it is possible to imagine N=Z even-even nuclei to be made up of alpha particles and odd-odd N=Z nuclei as a cluster of alpha particles and a deuteron or as a condensate of deuterons. Even in light (A $\leq$ 40) N=Z nuclei there is considerable recent interest. For example: (i) in ^{12}C structure of the Hoyle state at 7.654 MeV excitation with 0^+ and the corresponding Hoyle band are still not fully understood [1]; (ii) alpha clustering with tetrahedral symmetry appears to describe very well ^{16}O and rules out the description as a linear chain of α's [2]; (iii) rod like structures are predicted to appear at high excitation in ^{24}Mg [3].

One important aspect of N=Z nuclei is that protons and neutrons in these nuclei occupy the same orbits. Therefore, isospin (T) is important for these nuclei and combining this with α clustering and $4n$ and $4n+2$ effects seen in binding energies, it is plausible to have Wigner's spin-isospin $SU(4)$ symmetry [4] for these nuclei. For $(2s1d)$ shell nuclei, starting from ^{18}F the ground state isospin of all N=Z nuclei is $T = 0$ and $SU(4)$ symmetry gave good insights into their structure. Moving to the lower $(2p1f)$ shell nuclei, the odd-odd N=Z nuclei ^{42}Sc, ^{46}V, ^{50}Mn and ^{54}Co have ground state isospin $T = 1$ unlike for even-even nuclei. These nuclei are also amenable to detailed shell model studies [5]. Moving beyond ^{56}Ni, we have the heavier N=Z nuclei approaching the proton drip-line and they will start becoming unstable. Except for ^{58}Cu (it has $T = 0$ ground state), all other N=Z odd-odd nuclei have $T = 1$ as ground state. Currently the N=Z line ends at ^{100}Sn. The N=Z nuclei with A > 60 are referred as heavy N=Z nuclei and they start from ^{62}Ga. With the advent of radioactive ion beam (RIB) facilities, considerable interest has been generated in investigating the structure of heavy $N = Z$ odd-odd nuclei as these nuclei are expected to give new insights into neutron-proton (np) correlations. They include [6–8]: (i) isoscalar pairing as a new superfluidity structure; (ii) restoration of $SU(4)$ symmetry in odd-odd N=Z nuclei; (iii) competition between Wigner energy, symmetry energy and isovector pairing energy; (iv) possibility of deuteron condensate states in odd-odd N=Z nuclei; (v) rotations and vibrations in isospace.

In a given shell model space with nucleons occupying both the spin-orbit partners of a ℓ orbit, it is possible to have $L-S$ coupling. Then, a isoscalar pair will have two-particle $L = 0$ and $(ST) = (10)$ and isovector pair will have $L = 0$ with $(ST) = (01)$. In terms of the corresponding pair creation operators $D^\dagger_\mu$ and $P^\dagger_\mu$ respectively, the isoscalar plus isovector pairing Hamiltonian is,

$$H_{\text{pairing}}(x) = -(1-x)\sum_\mu P^\dagger_\mu P_\mu \; - \; (1+x)\sum_\mu D^\dagger_\mu D_\mu \; . \tag{1}$$

Note that $P_\mu = (P^\dagger_\mu)^\dagger$. The pairing Hamiltonian defined by Eq. (1) for any x preserves $SO(8)$ symmetry. The $x = 0, 1$ and -1 define three symmetry limits within $SO(8)$ giving a classification of the many nucleon shell model spaces [9–12]. Although the $SO(8)$ algebras generates some of the features of N=Z nuclei described above, they are mathematically challenging algebras. It is important to add that in general quadrupole deformation being important for low-lying states in nuclei, for more complete shell model description it is necessary to add a $Q.Q$ term to Eq. (1) and also the splitting of the single particle levels but then one has to take resort to numerical shell model calculations; see [13, 14] for recent attempts in this direction. An interesting and important aspect of the shell model $SO(8)$ algebra is that by Dyson boson mapping, the $SO(8)$ generates the interacting boson model (IBM) with scalar (s) bosons carrying spin-isospin degrees of freedom $(ST) = (01)$ and (10). Adding quadrupole (d) bosons also with $(ST) = (01)$ and (10), we have the full sdIBM with $(ST) = (01) \oplus (10)$ degrees of freedom for the bosons. The resulting spin-isospin invariant IBM is called IBM-4 [15]. This model is useful in generating predictions for various properties of heavy N=Z nuclei [16–20] and this is the topic of the present paper.

Spectrum generating algebra (SGA) for IBM-4 is $U_{sdST}(36)$ with six (sd) degrees of freedom and six spin-isospin $[(ST) = (01) \oplus (10)]$ internal degrees of freedom. This SGA generates a large class of dynamical symmetries and at the first level the Lie subalgebras are: (i) $U_{sd}(6) \otimes U_{ST}(6)$; (ii) $U_{sST}(6) \oplus U_{dST}(30)$; (iii) $U_{sdS}(18) \oplus U_{sdT}(18)$; (iv) $SO_{sdST}(36)$. Note that the subscripts here denote the space over which the corresponding algebra is defined. Most significantly, the algebras $U_{sd}(6) \otimes U_{ST}(6)$ and $SO_{sdST}(36)$ admit Wigner's spin-isospin $SU(4)$ algebra and hence relevant for heavy N=Z nuclei. Also, as it is well recognized that heavy N=Z nuclei are soft nuclei (some of them exhibiting shape coexistence and other features [21]), it is argued in [19] that the $SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ limit of IBM-4 provides a good basis for studying the properties of heavy N=Z nuclei. Using this symmetry limit results obtained so far are [19, 20]: (i) number of $T = 0$ pairs (bosons) in the ground states of even-even and odd-odd nuclei exhibiting a staggering structure similar to the one seen in large scale shell model results; (ii) analytical formulas for $B(E2)$'s for the low-lying levels and for the yrast $T = 1$ levels with $B(E2; L \rightarrow L-2)$ for the yrast levels exhibiting a $\Delta L = 4$ staggering; (iii) describing some of the observed properties of the low-lying levels in odd-odd N=Z nucleus ^{74}Rb and in particular the aligned spin to be one for the lowest $T = 0$ band; (iv) predictions for deuteron transfer spectroscopic factors involving heavy N=Z nuclei as a function of the mixing parameter representing the competition between isoscalar and isovector pairing. Following these, formulation for the study of Gamow-Teller (GT) and α-transfer strengths involving heavy N=Z nuclei is developed and for understanding the general structures, numerical calculations are carried out in many examples. These results are reported here. Now we will give a preview.

In Section 2 given is a brief introduction to the $SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ limit of IBM-4 and the basis states employed in the present studies along with a simple mixing Hamiltonian. In Section 3 formulation and numerical results for GT strengths involving ${}^{4n+2}_{2n+2}X_{2n}$ and ${}^{4n+2}_{2n+1}Y_{2n+1}$ nuclei are reported. Similarly, Section 4 gives the formulation and numerical results for α-transfer strengths involving ${}^{4n}_{2n}A_{2n}$ and ${}^{4n+4}_{2n+2}B_{2n+2}$ nuclei. Finally, Section 5 gives conclusions.

2 $SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ Limit of IBM-4 and a Simple Mixing Hamiltonian

One significant dynamical symmetry of IBM-4 is $SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ and it contains spin-isospin $SO(6)$ as a good symmetry not only in the total sd space but also separately in the s and d spaces. Note that the $SO(6)$ is isomorphic to Wigner's spin-isospin $SU(4)$ algebra. The complete group chain and the labels of the irreducible representations (irreps) of the various algebras in the chain are,

$$\begin{array}{l} \left| \begin{array}{ccccc} U_{sdST}(36) \supset SO_{sdST}(36) \supset SO_{S_sT_s}(6) \oplus [SO_{dST}(30) \supset \{SO_d(5) \\ \{N\} \quad [\omega] \quad [\omega_s] \quad [\omega_d] \quad [\omega_1\omega_2] \end{array} \right. \\ \quad \supset SO_L(3)\} \otimes SO_{S_dT_d}(6)] \supset SO_L(3) \otimes SO_{ST}(6) \\ \quad\quad L \quad [\sigma_1\sigma_2\sigma_3] \quad L \quad [\sigma_a\sigma_b\sigma_c] \\ \quad \supset SO_L(3) \otimes [SO_S(3) \oplus SO_T(3)] \supset SO_J(3) \otimes SO_T(3) \\ \quad\quad L \quad S \quad T \quad \vec{J} = \vec{L} + \vec{S} \quad T \Big\rangle \end{array} \tag{2}$$

Methods for obtaining the irrep labels starting with the symmetric irrep $\{N\}$ of a N boson system are given in [19]. For example, for odd-odd N=Z nuclei, the quantum numbers for the yrast levels with $T = 1$ are $S = 0$, $L = 2\omega_d$, $[\omega_s] = [1]$ and $[\sigma_a\sigma_b\sigma_c] = [0]$ for $L/2$ even, $[\omega_s] = [0]$ and $[\sigma_a\sigma_b\sigma_c] = [1]$ for $L/2$ odd, $[\omega_1\omega_2] = [\omega_d]$ and $J = L$ with $\omega_d = 0, 1, 2, \ldots, N$. Similarly for $T = 0$ we have the same quantum numbers except that $S = 1$ giving $J = L, L \pm 1$. This gives for $T = 0$ a 1^+ band with $J^\pi = 1^+, 3^+, 5^+ \ldots$, a 2^+ band with $J^\pi = 2^+, 4^+, 6^+, \ldots$ and another higher lying 1^+ band with $J^\pi = 1^+, 3^+, 5^+ \ldots$. Technical details for analyzing the properties generated by the group chain (2) are described in [19]. With large number of subalgebras in (2), the Hamiltonian in the symmetry limit will contain many terms. More importantly, the $SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ limit implies equal isoscalar and isovector pairing. However, heavy N=Z odd-odd nuclei, exhibiting variation of the energy difference between the lowest $T = 1$ (with $J = 0$) and $T = 0$ (with $J = 1$ or $J = 3$) levels, clearly show that there is competition between isoscalar and isovector pairing, i.e. $x \neq 0$ in Eq. (1). Thus, in practice mixing calculations using the basis defined by (2) are needed. Towards this end a simple mixing Hamiltonian and a simple set of basis states that are physically meaningful are suggested in [19, 20].

For incorporating the competition between $T = 1$ and $T = 0$ pairing, a simple basis for determining the ground states (gs) is,

$$|N, \omega_s, (ST)\rangle = |N, \omega = N, \omega_s, \omega_d = 0, (ST)\rangle \tag{3}$$

where $\omega_s = N$, $N - 2$, ..., 0 or 1. Note that for the gs of even-even nuclei $(ST) = (00)$ and for the gs of odd-odd nuclei $(ST) = (10)$ or (01). A simple mixing Hamiltonian with only liner terms in the number operators but contains the essentials of the competition between $T = 0$ and $T = 1$ pairing is,

$$H_{\text{mix}} = \alpha\{C_2(SO_s(6)) + (\beta/\alpha)\,\hat{n}_{sS} + (\gamma/\alpha)\,\hat{n}_d\} \,. \tag{4}$$

The first term in H_{mix} is diagonal with eigenvalues $\omega_s(\omega_s + 4)$ and the remaining is a one-body Hamiltonian in (sd)-boson space. The β/α term generates mixing and n_{sS}, the eigenvalue of $\hat{n}_{sS}$, gives number of isoscalar ($T = 0$) s-bosons.

Note that the $\hat{n}_d$ term can not change ω_s and therefore contributes to only the diagonal matrix elements. The H matrix is constructed using the expansion,

$$\begin{aligned} |N,\omega_s,(ST)\rangle &= \sum_{n_s(n_d)} C^{N,\omega=N,\omega_s,\omega_d=0}_{n_s,n_d}(6,30)\; |N,n_s,n_d,\omega_s,\omega_d=0,(ST)\rangle\;; \\ |n_s,\omega_s,(ST)\rangle &= \sum_{n_{sS}(n_{sT})} C^{n_s,\omega_s,(S,T)}_{n_{sS},n_{sT}}(3,3)\; |n_s,(n_{sS},n_{sT}),(ST)\rangle\;. \end{aligned} \tag{5}$$

In Eq. (5), $n_s = n_{sS} + n_{sT}$ and $n_d = N - n_s$. The C-coefficients can be calculated easily using the general formula given in [22]. The eigenstates after diagonalization of the H_{mix} will be of the form,

$$|N,\alpha,(ST)\rangle = \sum_{\omega_s} \mathcal{C}^{N,\alpha,(ST)}_{\omega_s}\, |N,\,\omega=N,\,\omega_s,\omega_d=0,\,(ST)\rangle\;. \tag{6}$$

Although $\omega_d = 0$ in Eq. (6), the eigenstates contain on the average $\sim 20-25\%$ d bosons. Significance of the β/α term in the mixing Hamiltonian follows from the single boson (isoscalar and isovector) energies generated by H_{mix}. They are $\epsilon(T_s=0)/\alpha = 5+\beta/\alpha$, $\epsilon(T_s=1)/\alpha = 5$ and $\epsilon(T_d=0)/\alpha = \epsilon(T_d=1)/\alpha = 5+\gamma/\alpha$. Then, for $\alpha > 0$ and $\gamma/\alpha >> 0$, the parameter β/α effects the spacing between $T_s = 0$ and $T_s = 1$ sp levels. As a result, for a N boson system with N odd, $\beta/\alpha < 0$ gives $T = 0$ gs and $\beta/\alpha > 0$ gives $T = 1$ gs and they are degenerate for $\beta/\alpha = 0$. Thus, the β/α term quantifies the competition between $T = 0$ and $T = 1$ pairing and varying this parameter allows us to study various properties as a function of the strength of this competition. Employing Eqs. (3) and (4), deuteron transfer strengths are studied in [20] and it is found that the IBM-4 results are close to those predicted by the fermionic $SO(8)$ model. Here, in the next two Sections we will present the first results for GT strengths and α-transfer strengths.

3 IBM-4 Results for GT Strengths in Heavy N=Z Nuclei

Gamow-Teller (GT) strengths provide a window in the search for the effects of $T = 0$ pairing. The GT operator to the lowest order approximation is $\sum_i \sigma_\mu(i)\tau_\pm(i)$ and the sum is over all particles. Most significant property of the GT operator is that it is a generator of $SU_{ST}(4)$ algebra and therefore the GT operator can not connect states labeled by different $SU(4)$ irreps. This gives a prediction for N=Z nuclei for GT β^+ decay of an even-even nucleus $^{4n+2}_{2n+2}X_{2n}$ to the odd-odd N=Z nucleus $^{4n+2}_{2n+1}Y_{2n+1}$. The $SO(6)$ (or equivalently $SU(4)$) irrep for the gs of the former even-even nucleus is [1] with $(ST) = (01)$ and $J = 0$. Therefore it will decay to the odd-odd N=Z nuclear states with $SO(6)$ irrep [1], $(ST) = (10)$ and $J = 1$. However, with the competition between $T = 0$ and $T = 1$ pairing, the GT strength will spread over many final 1^+ states. Recently there are attempts to measure these GT distributions involving heavy

N=Z odd-odd nuclei [23]. Here below we will discuss the approach to calculate and/or predict GT strengths/distributions using IBM-4. First attempts in this direction are due to Iachello, Barrett and Van Isacker [24–26].

Dyson boson mapping of the fermion GT operator leads to the GT operator in boson space with only s bosons giving,

$$T^{GT} = g_A^{\text{eff}}(s)\left[s^+_{10}\tilde{s}_{01} + s^+_{01}\tilde{s}_{10}\right]^{L=0,S=1:J=1,T=1}_{\mu'} \tag{7}$$

where $\tilde{s}_{sm_s,tm_t} = (-1)^{s+t+m_s+m_t}\, s_{s-m_s,t-m_t}$; $m_s = \pm1, 0$ and $m_t = \pm1, 0$. For β^+ decay we have $\mu' = +1$ and for β^- decay we have $\mu' = -1$. Additional terms to be added to Eq. (7) will be discussed later. The $g_A^{\text{eff}}(s)$ parameter can be in principle determined by a mapping of the shell model states to the IBM-4 states [26].

Let us consider GT strengths for β^+ decay of a even-even nucleus ${}^{4n+2}_{2n+2}X_{2n}$ to the odd-odd N=Z nucleus ${}^{4n+2}_{2n+1}Y_{2n+1}$ from gs of X with $(ST) = (01)$ to the final states of Y with $(ST) = (10)$. Note that the boson number N is odd for both X and Y in this example. Following Eq. (6), the gs of X is of the form,

$$\begin{aligned}&|X, \alpha^{gs}, (ST) = (01), M_T = -1\rangle = \\ &\quad \sum_{\omega_s^x} \mathcal{C}^{N,\alpha^{gs},(ST)=(01)}_{\omega_s^x} |N,\, \omega = N,\, \omega_s^x, \omega_d^x = 0,\, (ST) = (01), M_T = -1\rangle\end{aligned} \tag{8}$$

and similarly, the gs and excited states of Y with $(ST) = (10)$ are of the form,

$$\begin{aligned}&|Y, \alpha, (ST) = (10), M_T = 0\rangle = \\ &\quad \sum_{\omega_s^y} \mathcal{C}^{N,\alpha,(ST)=(10)}_{\omega_s^y} |N,\, \omega = N,\, \omega_s^y, \omega_d^y = 0,\, (ST) = (10), M_T = 0\rangle\,.\end{aligned} \tag{9}$$

Now, the GT strengths $M(GT)$ are given by,

$$\begin{aligned}&M(GT) = \left|\left\langle Y, \alpha, (S_fT_f) = (10) \mid\mid T^{GT} \mid\mid X, \alpha^{gs}, (S_iT_i) = (01)\right\rangle\right|^2 \;; \\ &\left\langle Y, \alpha, (S_fT_f) = (10) \mid\mid T^{GT} \mid\mid X, \alpha^{gs}, (S_iT_i) = (01)\right\rangle = \\ &g_A^{\text{eff}}(s) \sum_{\omega_s} \mathcal{C}^{N,\alpha^{gs},(S_iT_i)}_{\omega_s}\, \mathcal{C}^{N,\alpha,(S_fT_f)}_{\omega_s} \\ &\times\sqrt{(2S_f+1)(2T_f+1)\left\langle C_2(SO_s(6))\right\rangle^{\omega_s}} \left\langle \begin{matrix}[\omega_s] \\ (S_iT_i)\end{matrix} \begin{matrix}[11] \\ (11)\end{matrix} \middle|\middle| \begin{matrix}[\omega_s] \\ (S_fT_f)\end{matrix}\right\rangle \\ &= g_A^{\text{eff}}(s) \sum_{\omega_s} \mathcal{C}^{N,\alpha^{gs},(S_iT_i)}_{\omega_s}\, \mathcal{C}^{N,\alpha,(S_fT_f)}_{\omega_s}\, (\omega_s + 2)\,.\end{aligned} \tag{10}$$

Here we have used the fact that the GT operator is a generator of $SO_{sST}(6)$ algebra and therefore it will not change ω_s. The final result follows from the $SO(6) \sim SU(4)$ algebra given in [27]. Let us mention that in the exact $SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ limit, the $\mathcal{C}$ in Eq. (10) will be unity with $\omega_s = 1$ and then $M(GT)/(g_A^{\text{eff}}(s))^2 = 9$. This verifies the results for $\beta/\alpha = 0$ in Figure 1a.

Numerical results obtained using Eqs (4), (7) and (10), as a function of the mixing parameter β/α are shown in Figures 1a-f. For gs to gs decay, as seen from Figure 1a, the GT strength increases with increasing N value and more importantly with increasing value of $|\beta/\alpha|$. This is because, with increasing $|\beta/\alpha|$ the gs will have larger ω_s as dominant component ($\omega_s = 1$ for $\beta/\alpha = 0$) and then Eq. (10) shows that the GT strength will increase. However, as the Hamiltonian given by Eq. (4) is only an effective Hamiltonian, it is plausible that the β/α value for the nuclei X and Y need not be the same. Keeping this possibility, in Figures 1b and c results are shown for two fixed values of β/α for X and varying β/α for Y. Note that β/α $-$ve means $T = 0$ pairing is stronger than $T = 1$ pairing and similarly $+$ve means $T = 1$ pairing is stronger than $T = 0$ pairing.

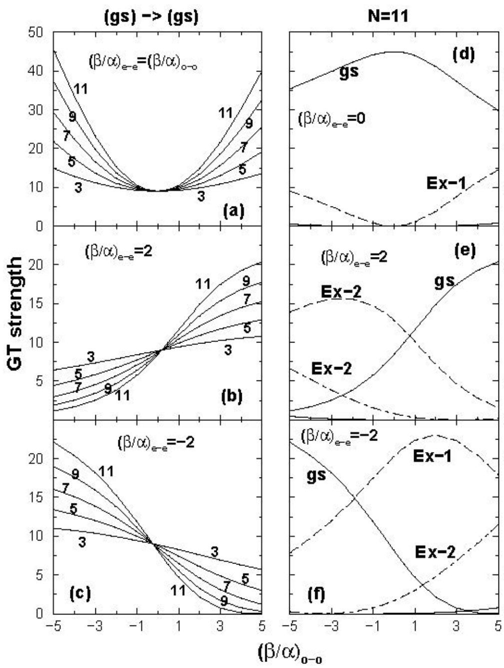

Figure 1. GT strengths for various values of β/α and boson numbers N with $\gamma/\alpha = 1.5$ in Eq. (4). The β/α values are shown in the figure. Results in (a), (b) and (c) are for the gs of a ${}^{4n+2}_{2n+2}X_{2n}$ nucleus with $(ST) = (01)$ decaying to the levels of the odd-odd N=Z nucleus ${}^{4n+2}_{2n+1}Y_{2n+1}$ with $(ST) = (10)$. Similarly, results in (d), (e) and (f) are for gs to gs and gs to excited states for an example with $N = 11$. See text for further details.

This feature explains the results in the figures. The trends seen in Figures 1a-c should be tested in future with experiments for β^+ decay to a series of heavy odd-odd N=Z nuclei such as ^{74}Rb. Besides gs to gs decay, it is also possible that the gs of X decays to excited states of Y. As an example, for $N = 11$ results for GT strengths are shown in Figures 1d-f for the decay from gs of X not only to the gs but also to the excited states of Y. It is seen that there can be decay to the first and second excited states (though not highlighted, there is small strength also to a third excited state). Also, depending on the β/α values, it is possible that the gs to excited state strength may be larger than the gs to gs decay strength. Future experiments are expected to test the results in Figures 1d-f.

It is important to stress that the GT strengths in Figure 1 are only for excited states with $J = S = 1$ and orbital angular momentum $L = 0$ as we are putting $\omega_d = 0$. Thus, the strengths are only for spin excitations. However, for explaining the GT strengths measured [23] for example in the decay of ^{62}Ge gs to ^{62}Ga $J = 1, T = 0$ levels, we need a more general GT operator with the inclusion of terms such as $[d^\dagger_{01}\tilde{s}_{10}]^{L_0=2,S_0=1;J_0=1,T_0=1}$. This will allow for both spin and orbital excitations within IBM-4 (note that the spin and orbital angular momentum in IBM-4 need not be real but the total J should be real). Also, the basis space should have $\omega_d \neq 0$ states and H defined by Eq. (4) has to be extended.

4 IBM-4 results for α-transfer strengths

Another important probe for heavy N=Z nuclei is α-transfer. This will give information on possible [28] quartet structures in these nuclei. Let us consider α addition to a even-even N=Z nucleus and restrict to gs to gs transitions. Then, in IBM-4 the simplest form for the α particle creation operator is

$$T_{\boldsymbol{\alpha}} = \kappa \left[s^\dagger_{10} \cdot s^\dagger_{10} + s^\dagger_{01} \cdot s^\dagger_{01} \right] \; . \tag{11}$$

Note that κ is a parameter. Given an even-even N=Z nucleus with N number of bosons, α-addition will change N to $N + 2$. The ground states of the initial (A) and final (B) nuclei with $(ST) = (00)$ are of the form

$$\begin{array}{l} |A : N, \alpha^{gs}, (00)\rangle = \displaystyle\sum_{\omega_s} \mathcal{C}^{N,\alpha^{gs},(00)}_{\omega_s} \, |N,\, \omega = N,\, \omega_s, \omega_d = 0,\, (00)\rangle \; , \\ |B : N+2, \beta^{gs}, (00)\rangle = \\ \displaystyle\sum_{\omega_s} \mathcal{C}^{N+2,\beta^{gs},(00)}_{\omega_s} \, |N+2,\, \omega = N+2,\, \omega_s, \omega_d = 0,\, (00)\rangle \; . \end{array} \tag{12}$$

As N is even, ω_s will be even. Now, the α-transfer strength is

$$S_{\boldsymbol{\alpha}}(N \to N+2) = \left| \langle B : N+2, \beta^{gs}, (00) \mid\mid T_{\boldsymbol{\alpha}} \mid\mid A : N, \alpha^{gs}, (00)\rangle \right|^2 \; . \tag{13}$$

Here $\langle ---||-||---\rangle$ is the reduced matrix element with respect to both spin and isospin $SO(3)$ algebras (orbital angular momentum is zero for both the initial nucleus and final nucleus). The reduced matrix element is given by,

$$\begin{array}{l} |B:N+2,\beta^{gs},(00)\ ||\ T_{\boldsymbol{\alpha}}\ ||\ A:N,\alpha^{gs},(00)\rangle = \\ \displaystyle\sum_{n_s(n_d),\omega_s} \mathcal{C}^{N,\alpha^{gs},(00)}_{\omega_s} \mathcal{C}^{N+2,\beta^{gs},(00)}_{\omega_s} C^{N,N,\omega_s,\omega_d=0}_{n_s,n_d}(6,30) \\ \times\, C^{N+2,N+2,\omega_s,\omega_d=0}_{n_s+2,n_d}(6,30)\langle n_s+2,\omega_s,(00)\ ||\ T_{\boldsymbol{\alpha}}\ ||\ n_s,\omega_s,(00)\rangle\ ; \\ \langle n_s+2,\omega_s,(00)\ ||\ T_{\boldsymbol{\alpha}}\ ||\ n_s,\omega_s,(00)\rangle = \kappa\ \sqrt{(n_s-\omega_s+2)(n_s+\omega_s+6)}\,. \end{array} \quad (14)$$

Here we have used the result that $T_{\boldsymbol{\alpha}}$ is a generator of $SU(1,1)$ algebra complimentary to the $SO_{sST}(6)$ algebra and Eq. (A7) of [19]. Combining Eqs. (12), (13) and (14) will allow one to calculate α addition strengths $S_{\boldsymbol{\alpha}}(N \to N+2)$.

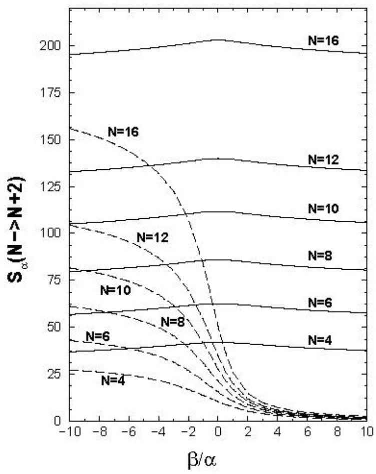

Figure 2. Alpha transfer strengths between gs of even-even N=Z nuclei for various values of β/α. The boson number N of the initial even-even nucleus is shown in the figure. The continuous curves are for the strengths generated by $T_{\boldsymbol{\alpha}}$ with $\kappa = 1$ and dashed curves are for $T^{(d)}_{\boldsymbol{\alpha}}$ with $\kappa' = 1$. See text for further details.

Some numerical results for $S_{\boldsymbol{\alpha}}(N \to N+2)$ obtained using the Hamiltonian given by Eq. (4) with $\gamma/\alpha = 1.5$ and varying β/α are shown in Figure 2. Before discussing these results, let us point out that for $\beta/\alpha = 0$ it is easy to show,

$$S^{\beta/\alpha=0}_{\boldsymbol{\alpha}}(N \to N+2) = \kappa^2\ \frac{(N+2)(N+6)(N+30)(N+34)}{4(N+17)(N+18)}\ . \quad (15)$$

Formula given by Eq. (15) is well tested in numerical calculations via matrix diagonalization. Turning to Figure 2, it is seen that the variation in the α-transfer strength is quite small as β/α changes from -10 to $+10$. Interestingly, there is a small peak at $\beta/\alpha = 0$. The structures seen in Figure 2 are identical to the results reported in [10] for the fermionic $SO(8)$ model. It is also seen from Eq. (15) that the strengths have an approximate N^2 scaling.

It is important to recognize that the operator $T_{\boldsymbol{\alpha}}$ is symmetric in $(ST) = (10)$ and (01). However, it is possible to consider a more general asymmetric combination and this may happen with a shell model to IBM-4 mapping. Then,

$$T_{\boldsymbol{\alpha}}^{(\text{eff})} = \left[\kappa' \, s_{10}^{\dagger} \cdot s_{10}^{\dagger} + \kappa'' \, s_{01}^{\dagger} \cdot s_{01}^{\dagger}\right] \tag{16}$$

where κ' and κ'' are constants. One extreme of this (if the α-particle is effectively a system of two deuteron like particles) gives the transfer operator to be

$$T_{\boldsymbol{\alpha}}^{(d)} = \kappa' \, s_{10}^{\dagger} \cdot s_{10}^{\dagger} \tag{17}$$

The operator $T_{\boldsymbol{\alpha}}^{(d)}$ will not preserve ω_s and but it is a generator of $SU(1,1)$ that is complimentary to the $SU_{sS}(3) \supset SO_{sS}(3)$ algebra. This property gives,

$$\begin{array}{l} S_{\boldsymbol{\alpha}}^{(d)}(N \to N+2) = (\kappa')^2 \left| \displaystyle\sum_{n_s(n_d),\omega_s^i,\omega_s^f} \mathcal{C}_{\omega_s^i}^{N,\alpha^{gs},(00)} \, \mathcal{C}_{\omega_s^f}^{N+2,\beta^{gs},(00)} \right. \\ \times\, C_{n_s,n_d}^{N,N,\omega_s^i,\omega_d=0}(6,30) \, C_{n_s+2,n_d}^{N+2,N+2,\omega_s^f,\omega_d=0}(6,30) \\ \times \left. \displaystyle\sum_{n_{sS}+n_{sT}=n_s} C_{n_{sS},n_{sT}}^{n_s,\omega_s^i,0,0}(3,3) \, C_{n_{sS}+2,n_{sT}}^{n_s+2,\omega_s^f,0,0}(3,3) \, \sqrt{(n_{sS}+2)(n_{sS}+3)} \right|^2 . \end{array} \tag{18}$$

Transfer strengths generated by the operator $T_{\boldsymbol{\alpha}}^{(d)}$ are calculated using Eq. (18) and some results are shown in Figure 2. As expected, for β/α $-$ve, the strength is larger and it goes to zero for β/α value positive and large. Note that for β/α $-$ve, the $T=0$ pairing will be stronger compared to $T=1$ pairing and the $T_{\boldsymbol{\alpha}}^{(d)}$ operator creates a condensate of two $T=0$ pairs. Extension of the above formulation for odd-odd N=Z nuclei is straightforward. Although we haven't shown, the results for odd-odd N=Z nuclei are similar to those in Figure 2. Finally, strengths for gs to excited states will be considered elsewhere.

5 Conclusions

Formulation and some numerical results, within the framework of the $SO_{sdST}(36) \supset SO_{sST}(6) \oplus SO_{dST}(30)$ limit of IBM-4, are presented for GT strengths and α-transfer strengths involving heavy N=Z nuclei. These are in addition to the results for deuteron transfer strengths that are reported in [20]. It will be interesting and important to test these IBM-4 results in future experiments. Finally, though d bosons are included, it is assumed that $\omega_d = 0$. In future studies it is important to expand the basis space to include $\omega_d \neq 0$ states and also include in the Hamiltonian terms that correspond to the $SU(3)$ limit in the orbital $[U_{sd}(6) \supset SU_{sd}(3) \supset SO_L(3)]$ space so that the effects of deformation are fully taken into account. In addition, in future it is also important to derive results for more general forms of the transfer operators.

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