Bulg. J. Phys. **42** (2015) 554–564

E0 Transitions in Even-Even Nuclei

V.P. Garistov[1], **A.I. Georgieva**[2]

[1]Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

[2]Institute of Solid State Physics, Bulgarian Academy of Sciences,
Sofia 1784, Bulgaria

Received *25 November 2015*

Abstract. The symplectic realization of the phenomenological algebraic model is employed for the description of the structure of different low lying collective excited states in heavy even-even nuclei. First we illustrate the parabolic distribution of the energies of low lying collective states and further within the Interacting Vector Boson Model (IVBM) we analyze the structure of these states. The analysis is performed with the fixed set of parameters extracted from the fitting of the model energies with the experimental data for different K^π rotational bands. The estimation of the transition probabilities is also carried out.

PACS codes: 03.65.Fd, 02.20.Sv, 21.60.Fw

1 Introduction

Small vibrations of nuclear shapes around equilibrium can give rise to physical states at low to moderate excitation energies. The description of the vast amount of experimental data on the low-lying collective spectra of even-even nuclei in the rare-earth and actinide regions is still a problem of particular interest in the nuclear structure physics. The classifications of this data is mainly done from a "horizontal perspective" in sequences of nuclei where the investigated nuclear characteristics are empirically studied as functions of the numbers of their valence nucleons. Many well studied nuclei can be listed around the nuclear chart. The theoretical approaches that are able to explain and correctly describe all the data in the same nucleus in this respect are seemingly in debt to the experiment. In this paper we intend to diminish our debt at least with respect to the 0^+ excited states and the bands built on them.

It is well known that the Interacting Vector Bosons Model (IVBM) [1] provides a very good description of the energies of different rotational bands with positive and negative parities in even-even nuclei and up to rather high values of the angular momentum. See for example the left panel of Figure 1.

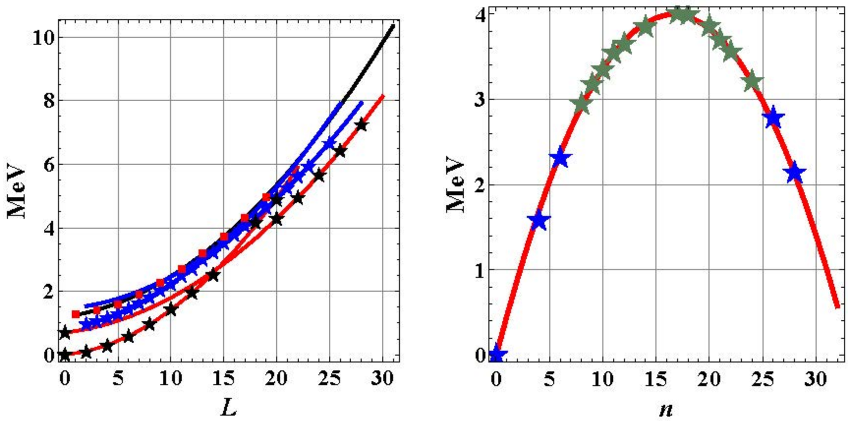

Figure 1. (Color online) Left: IVBM description [2] of the ground, first β, first γ and octupole rotational bands in ^{160}Dy.; Right: Distribution of the 0^+ excited states energies in ^{136}Ba. Blue stars are the experimental data for the states with exactly defined spin, while the green stars are the states with the energies presented in Table 1.

On the other hand the empirical classification of energies of low-lying 0^+ states has shown that practically all the energies in heavy even-even nuclei follow the parabolic distribution law: $E(0_i^+) = an_i + bn_i^2$ in the space of the number of monopole bosons n_i [2]. Our previous investigations have shown that sometimes this type of classification can be useful in the experimental search of new 0^+ excited states [3] and also could be very helpful for the exact determination of the spin and parity of the collective states. See for example Table1 and the right panel of Figure 1.

The obvious success in using the parabolic classification of the energies of the low-lying collective states together with the precise description of collective rotational bands within IVBM engaged our interest in the parallel study of the classification of the energies of the low-lying excited states and the estimation of the nuclear matrix elements for E0 transitions between different 0^+ states within these two approaches.

Table 1. Ambiguous spin data for ^{136}Ba

Energy (MeV)	Spin	Energy (MeV)	Spin
2.9460	$0(+), 1, 2, 3+$	3.5795	$0+, 1, 2, 3, 4+$
3.1789	$0(+), 1, 2, 3(+)$	3.6500	$(0+), 1, 2, 3, 4(+)$
3.2120	$0(+), 1, 2, 3+$	3.6985	$(0+), 1, 2, 3, 4(+)$
3.3476	$0(+), 1, 2, 3+$	3.8485	$0(+), 1, 2, 3+$
3.5050	$0(+), 1, 2, 3+$	3.8595	$0(+), 1, 2, 3(+)$
3.5425	$(0+), 1, 2, 3, 4(+)$	3.9925	$0(+), 1, 2, 3+$
3.5590	$0(+), 1, 2, 3(+)$	4.0086	$0(+), 1, 2, 3(+)$

2 Classification of the Energies Of Low-Lying Collective 0^+ States within IVBM

The algebraic IVBM is realized in terms of 2 types of boson creation (annihilation) operators $u^\dagger_m(\alpha)(u^m(\alpha))$, in a 3- dimensional space $l = 1, m = 0, \pm 1$ (vectors). They differ by the value of the projection $\alpha = 1/2$ (for protons) or $\alpha = -1/2$ (for neutrons) of an additional quantum number, called "pseudo-spin " T. The pseudo-spin has the properties of the F-spin in IBM-2 [4]. These operators satisfy the commutation relations $\left[u^m(\alpha), u^\dagger_n(\beta)\right] = \delta(\alpha,\beta)\,\delta_{m,\,n}$ and Hermitian conjugation rules $\left[u^\dagger_m(\alpha)\right]^\dagger = u^m(\alpha)$, $\left[u^m(\alpha)\right]^\dagger = u^\dagger_m(\alpha)$.

The successful application of the symplectic extension of the IVBM is achieved by exploring the new subgroup chains that reduce its dynamical symmetry group Sp(12,R) to the angular momentum subgroup SO(3) [1]. The corresponding exactly solvable limiting cases are applied for the description of different types of nuclear collective spectra. The first reduction that one exploits is the one through the maximal compact subgroup U(6)

$$\mathrm{U}(6) \supset \mathrm{SU}(3) \otimes \mathrm{U}(2) \supset \mathrm{SO}(3) \otimes \mathrm{U}(1), \tag{1}$$

$$[N] \quad\; (\lambda,\mu) \quad\; (N,T) \;\; K \quad\; L \qquad\; T_0, \tag{2}$$

where the labels below the subgroups are the quantum numbers (2) corresponding to their irreducible representations. Since the reduction from U(6) to SO(3) is carried out by the mutually complementary groups SU(3) and U(2), their quantum numbers are related in the following way: $T = \lambda/2$ and $N = 2\mu + \lambda$. Making use of the latter we can write the basis as

$$\mid [N]_6; (\lambda,\mu); K, L, M; T_0\rangle = \mid (N,T); K, L, M; T_0\rangle, \tag{3}$$

which provides a very good description of the collective bands of some even-even rare earth and actinide nuclei up to very high spins. The Hamiltonian, corresponding to the considered limit of the IVBM, is expressed in terms of the first and second order invariant operators of the different subgroups in the chain (1). Its eigenvalues are the energies of the basis states (3) of the boson representations of Sp(12,R)

$$E((N,T), L, T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2. \tag{4}$$

In order to confirm the empirical observation, that sets of states with fixed angular momentum L fall on parabolas with respect to the variable N, we obtain their theoretical energy distributions, exploring the reduction chain Sp(12,R)$\supset$Sp(4,R)$\otimes$SO(3). The following correspondence between the latter and the chain (1) through u(6) was established

$$\begin{array}{ccccc} \text{sp(12,R)} & \supset & \text{sp(4,R)} & \otimes & \text{so(3)} \\ \cup & & \cup & & \cap \\ \text{u(6)} & \supset & \text{u(2)} & \otimes & \text{su(3).} \end{array} \tag{5}$$

Because of the correspondence (5) and the relation between the SU(3) and SU(2) second order Casimir operators, the Hamiltonian and bases are equivalent in both chains and the eigenvalues of the Halmitonian (4) can be used in both cases for evaluation of the energy distribution of states with a fixed L [5] and for calculation of the energies of states from a given band [6].

These results of the applications are dependent in the first place on the proper identification of the experimentally observed bands with the sequences of basis states from the even and odd representation of Sp(12,R) [6]. In this work we are concerned with the energy distribution of the 0^+ states, which are bandheads of the ground and $K^\pi = 0^+, \beta-$bands. Here we use the following identification of the experimentally observed bands

- ground state band, $\mu_0 = 0$, $(\lambda, \mu) = (\mu_0 + 2i, \mu_0), i = 0, 1, 2, ... \; L = 2i,$
- excited $(K^\pi = 0^+), \beta-$bands, $\mu_0 = \lambda_0 \neq 0$-even,
  $(\lambda, \mu) = (\mu_0 + 2i, \mu_0)$ $L = 2i$, $N = N_0 + L, T = (N_0 + 3L)/2.$

Using the above assignments of the experimentally observed $K^\pi = 0^+$, β-bands to sequences of SU(3) basis states and the relations between the quantum numbers labeling the representations of the subalgebras in the reduction scheme (5) we evaluate the parameters of the Hamiltonian H by fitting the energies in (4) to the experimentally observed states in the considered ground, $\beta-$, $\gamma-$ and octupole bands in a given nucleus [6]. In most cases we obtain a very good agreement with the experiment up to very high spins [7]. An example is given in the left panel in Figure 1.

Further from the same assignments we obtain the following dependencies of the energies of the considered collective bands on the numbers of phonon excitations building their bandhead configurations $N_{0\beta}$, where $N_{0gr} = 0$

$$E_{\text{ground}}(L) = aL + bL^2 + \beta_3 L(L+1), \quad L = N, \quad N_{0gr} = 0. \tag{6}$$

The energies of the states in the $\beta-$ bands as functions of the number of phonons $N_{0\beta}$ are

$$\begin{aligned} E_\beta &= a\,(L + N_{0\beta}) + b\,(L + N_{0\beta})^2 \\ &\quad + \alpha_3 \frac{1}{2}\left(\frac{1}{2}\,(L + N_{0\beta}) + 1\right)(L + N_{0\beta}) + \beta_3 L(L+1) + \alpha_1. \end{aligned} \tag{7}$$

Obviously for the bandheads of the β bands, which have $L = 0$, we have the following parabolic in $N_{0\beta}$ expression

$$E_\beta(L = 0, N_{0\beta}) = bN_{0\beta}^2 + aN_{0\beta} + \frac{1}{2}\left(\frac{N_{0\beta}}{2} + 1\right)\alpha_3 N_{0\beta} + \alpha_1. \tag{8}$$

Since the parameters in the eigenenergies of the states in the bands depend on the same parameters as its bandheads we can use (8) to evaluate the distribution of

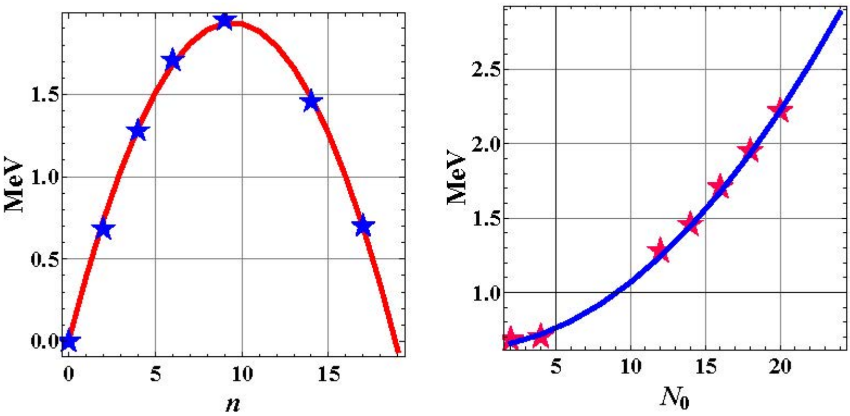

Figure 2. (Color online) Left: Empirical Parabolic Distribution $E = an + bn^2$ of the 0^+ excited states energies in ^{160}Dy. Here $a = 0.4105$, $b = -0.0217586$ and $\Delta = 0.9$ keV; Right: IVBM distribution of the 0^+ excited states energies in ^{160}Dy.

all the 0^+ states in respect to the number of phonons $N_{0\beta}$ that build them. This distribution determines as well the quantum numbers of all the states belonging to the respective excited collective band with $K^\pi = 0^+$. Illustrating this result we give in Figures 2 and 3 the energy distributions of the excited 0^+ states, obtained with the empirical formula $E = an + bn^2$ and by means of the IVBM expression (8).

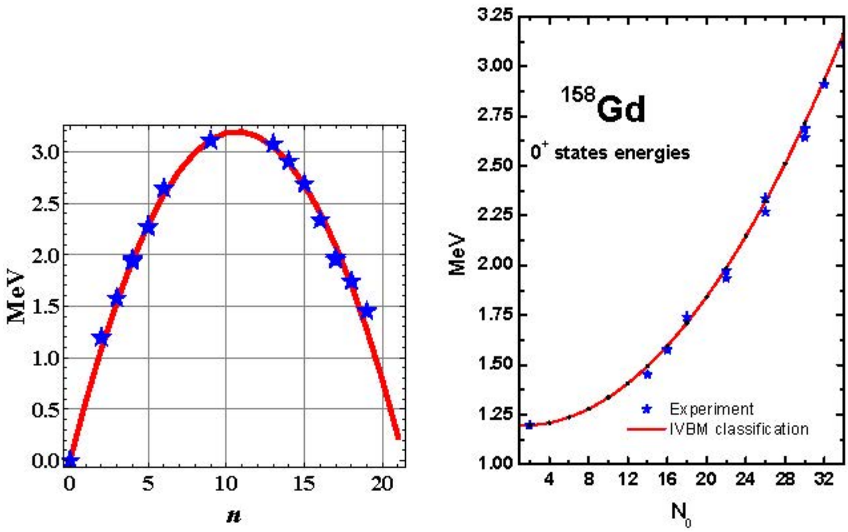

Figure 3. (Color online) Left: Distribution of the 0^+ excited states energies in ^{158}Gd; Right: IVBM distribution of the 0^+ excited states energies in ^{158}Gd

From these figures it could be seen that both expressions give rather accurate results and could be used for the prediction of some low lying excited 0^+ states as well, as the bands are built on them.

3 Matrix Elements for $E0$ Transition

The return to the investigations of the energies of the 0^+ states and in particular the $E0$ transitions between them is provoked by the new and more precise experimental technics that not only made revision of the previous data but also gave a possibility to obtain a great amount of energies of new 0^+ states and conversion electrons data. The nature of the low lying 0^+ excited states and the bands built on them in heavy even-even nuclei remains a mystery under debate. The improvements in technology have improved the situation by enabling spectroscopy, reactions, and life-time measurements of a large number of $K^\pi = 0^+$ bands that were previously inaccessible in nuclei [8]. Some authors point out the importance of studying anharmonic effects in the microscopic approaches in deformed nuclei [9], quadrupole and pairing vibrational modes in conversion electrons and internal pair decay [10], or exact diagonalization in the restricted space of collective phonons of different types [11]. The energies and electromagnetic decay properties of the excited 0^+ states are important in determining the applicability and as tests of the models, like the Shell Model, the Cluster-Vibrational model, The Quasiparticle Phonon model, the Interacting Boson Approximation, the Pairing Quadrupole correlations etc. There are some, rather poor, calculations [10, 12–14] devoted to the estimation of $E0$ nuclear matrix elements ρ^2 between different 0^+ states in the same nucleus. For instance in [14] we find that $\rho^2(0_2^+ \to 0_1^+)$ is very small in comparison with $\rho^2(0_3^+ \to 0_1^+)$, which indicates the more collective nature of the 0_2^+ state. It should be very important to determine the half-lives of the 0^+ states that would allow more definite conclusions on the structures of the excited 0^+ states [13] and [14] . Often the first excited 0^+ state in nuclei is considered as less collective than the next ones, by increasing energy. In some even-even nuclei the first in energy excited state is not necessarily the lowest state by degree of collectivity [14]. For instance the 0^+ state with the excitation energy 0.2548 MeV (n = 20) [15] is also much more collective than the 0^+ state with energy 0.5811 MeV (n = 1), observed in ^{158}Gd. The separation of the $E0$ conversion probability into electronic and nuclear factors is not as well defined as for the conversion of higher multipoles, nor is the electronic factor Ω_e, completely independent of the nuclear properties. Physically, the monopole transition interaction vanishes, except while the electron is within the nuclear charge distribution, and hence it is the electron wave functions within the nucleus which enter into the calculation of Ω_e. These in turn, depend on the average static nuclear charge distribution. In this paper we will estimate the transition probability without taking into account the mutual influence of the electronic and nuclear wave functions (that really may be very important).

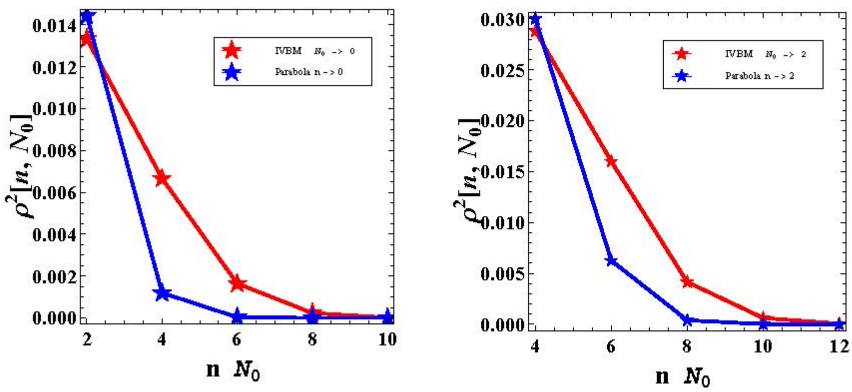

Figure 4. Behavior of calculated within both approaches nuclear matrix elements for $E0$ transition from 0^+ excited states N_0 to (left) the ground state; (right) to two bosons excited state.

Nevertheless using this approximation we can feel the gross behaviour of the transition probabilities. Here we suggest two phenomenological models for the estimation of the matrix elements of the $E0$ nuclear transitions. In accordance with the reduction rules for the chain Sp(12,R) $\supset$ Sp(4,R) $\otimes$ SO(3), the connection between pseudo-spin and number of bosons is $T = N_0/2$. Taking $T_0 = 0$ we can write for the normalized to unit wave function of the collective monopole state $|m\rangle$

$$|m\rangle = \left(\frac{2}{3}\right)^{m/2} \prod_{T=1}^{m-1} \frac{(T+1)\sqrt{\frac{2(T+1)+1}{4T^3+12T^2+11T+3}}}{\sqrt{(2m)!}} (u^\dagger.u^\dagger)^m \mid 0>, \quad (9)$$

where the scalar product $(u^\dagger.u^\dagger)$ of the vector boson creation operators is one of the Sp(4,R) algebra generators. For the estimation of the nuclear matrix elements between different 0^+ states in the IVBM approach we propose a transition operator in the form

$$\widehat{O_{IVBM}} = \alpha e^V + \alpha^* e^{V^\dagger}, \quad (10)$$

where

$$V = \sum (1\,k\,1-k \mid 0\,0)(\frac{1}{2}\,l\frac{1}{2}-l \mid 1\,0) u\,u. \quad (11)$$

Hence, the explicit expression of (10) is

$$\widehat{O_{IVBM}} = \alpha\, e^{\sqrt{\frac{2}{3}}u.u} + \alpha^*\, e^{\sqrt{\frac{2}{3}}u^\dagger.u^\dagger}.$$

Here u and $u^\dagger$ are the defined above building blokes of the IVBM generators. The matrix elements $\rho^2 = |< m \mid \widehat{O_{IVBM}} \mid n >|^2$, in the framework of the

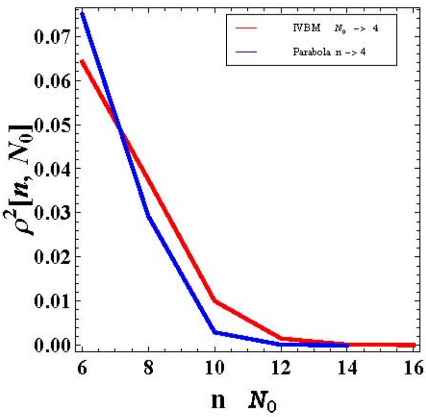

Figure 5. Behavior of calculated within both approaches nuclear matrix elements for $E0$ transition from 0^+ excited states N_0 to four bosons excited state.

IVBM, calculated for arbitrary values of m and n are

$$\rho^{\mathbf{2}} = \frac{\left(\frac{\mathbf{3}}{\mathbf{2}}\right)^{\mathbf{m-n}} \alpha^{\mathbf{2}}(\mathbf{2n})!}{(\mathbf{2m})!((\mathbf{n}-\mathbf{m})!)^{\mathbf{2}}}. \tag{12}$$

Within the empirical approach, that gives the parabolic distribution of the 0^+ states, we use again the exponential form of the E0 transition operator

$$O_{par} = \widehat{\alpha\ e^{b} + \alpha^{*}\ e^{b^{\dagger}}}, \tag{13}$$

and the states functions

$$|n\rangle = \frac{1}{\sqrt{n!}}(b^{\dagger})^{n}|0\rangle.$$

The ideal bosons b and $b^{\dagger}$ are built by boson creation and annihilation operators R_+ , R_-, and R_0 representing fermion pairs from the same subshell and coupled to zero total momentum

$$\mathbf{R}_{+} = \frac{\mathbf{1}}{\mathbf{2}}\sum_{\mathbf{m}}(-\mathbf{1})^{\mathbf{j-m}}\mathbf{a}^{+}_{\mathbf{jm}}\mathbf{a}^{+}_{\mathbf{j-m}},$$

$$\mathbf{R}_{-} = \frac{\mathbf{1}}{\mathbf{2}}\sum_{\mathbf{m}}(-\mathbf{1})^{\mathbf{j-m}}\mathbf{a}_{\mathbf{j-m}}\mathbf{a}_{\mathbf{jm}},$$

$$\mathbf{R_0} = \frac{\mathbf{1}}{\mathbf{4}}\sum_{\mathbf{m}}\left(\mathbf{a}^{+}_{\mathbf{jm}}\mathbf{a}_{\mathbf{jm}} - \mathbf{a}_{\mathbf{j-m}}\mathbf{a}^{+}_{\mathbf{j-m}}\right),$$

$$[\mathbf{R_0}, \mathbf{R}_{+}] = \mathbf{R}_{+}, \quad [\mathbf{R_0}, \mathbf{R}_{-}] = -\mathbf{R}_{-}, \quad [\mathbf{R}_{+}, \mathbf{R}_{-}] = \mathbf{2R_0},$$

$$\mathbf{R_-} = \sqrt{\mathbf{2\Omega - bb^+}}\,\mathbf{b}, \tag{14}$$

$$\mathbf{R_+} = \mathbf{b^+}\sqrt{\mathbf{2\Omega - bb^+}}, \tag{15}$$

$$\mathbf{R_0} = \mathbf{b^+b - \Omega}. \tag{16}$$

It is easy to see that in "zero" approximation the operator $b \sim R$. As a result we obtain for the matrix elements of the transition operators (13)

$$\rho^{\mathbf{2}} = \frac{\alpha^{\mathbf{2}}\mathbf{n}!}{\mathbf{m}!((\mathbf{n-m})!)^{\mathbf{2}}}. \tag{17}$$

In Figures 4 and 5 we compare the results for ρ^2 values obtained in these two approaches, (12) and (17). In spite of the different mechanism of the excitation of 0^+ states in these two suggested approaches, the behavior of the nuclear matrix elements is functionally alike.

4 Conclusions

In this talk we review the empirical [2] and IVBM [5] approaches to the investigation of the energy distribution of the 0^+ states. This renewed interest in this subject is provoked from one side by the increased amount of newer and more precise experimental data on these states, and from the other side by the need to understand better the nature of these states from theoretical point of view. Until now it is clear that the theoretical treatments of these states are all model dependent and it is hard to establish any relations between all these approaches, which probably concern different aspects of the observed phenomena. Up till now we have gained rather useful results on the description of the 0^+ states with their parabolic empirical energy distribution [2], the main one being the prediction of some low lying 0^+ states in ^{160}Dy [3]. The effect that some lower in energy states are more collective than the higher lying ones can also be observed in some cases. These empirical results were later motivated in the investigation of the Sp(4,R)$\otimes$ SO(3) dynamical symmetry of the IVBM [5]. In the framework of the model, within the dynamical symmetry (1), additionally an accurate description and ordering of the bands built on them was achieved. Nevertheless we need to clarify the problem of the restrictions to the used parabolas, imposed by the number of existing experimental states in the empirical approach. Hence we turned to the more direct application of the already established relation between the two dynamical symmetries of the IVBM (5), using the same parameters in the eigenvalues of the equivalent Hamiltonians of the two chains. In the present paper we use a more appropriate assignment [6] of the bands of states to the respective SU(3) irreps of the 0^+ states in the symplectic space. These improvements lead to a more correct distribution of the states on a parabola, which is not restricted by the number of experimentally observed 0^+ states, since in the

IVBM the number of phonon excitations that build the states is infinite dimensional in the symplectic space. Furthermore, the exact placement of an experimentally observed state on the IVBM parabolas is a proof that it has the collective character prescribed by the model, and not some other origin. Further we made some first steps towards involving the absolute values of the monopole matrix elements $\rho^2(0^+_m \to 0^+_n)$ into understanding the structures of the excited 0^+ states. We introduced again two approaches, one based on the IVBM vector bosons and another simplified approach based on ideal scalar bosons representing fermion pairs in the same j-orbit. Within them we analyzed the behaviour of ρ^2 as a function of the collective parameter $\Delta N = n - m$. In Figures 4 and 5 are shown the values of ρ^2 between different nuclear excited 0^+ states. From our analysis it could be seen that both functional dependencies are alike. At present it is obvious that in order to prove the validity of our results, we need more experimental investigations on the half-lives of the 0^+ states in a given nucleus.

Acknowledgments

This work was supported by the Bulgarian National Foundation for Scientific Research under Grant Number DFNI-E02/6/12.12.2014.

References

[1] A. Georgieva, P. Raychev, and R. Roussev (1983) *J. Phys. G: Nucl. Phys.* **9** 521; A.I. Georgieva, H.G. Ganev, J.P. Draayer, and V.P. Garistov (2009) *Phys. Part. Nucl.* **40** 461.

[2] V.P. Garistov (2003) *Rearrangement of the Experimental Data of Low Lying Collective Excited States*, Proceedings of the XXII International Workshop on Nuclear Theory, ed. V. Nikolaev, Heron Press Science Series, Sofia, p. 305; A.A. Solnishkin et al. (2005) *Phys. Rev.* **C72** 064321.

[3] J. Adam *et al*. (2014) *Bulg. J. Phys.* **41** 10.

[4] F. Iachello and A. Arima (1978) *The Interacting Boson Model*, Cambridge University Press, Cambridge.

[5] H.G. Ganev, V.P. Garistov, A.I. Georgieva, and J.P. Draayer (2004) *Phys. Rev.* **C70** 054317.

[6] V.P. Garistov, A.I. Georgieva, and T.M. Shneidman (2013) *Bulg. J. Phys.* **40** 1.

[7] A.A. Solnyshkin, V.P. Garistov, A. Georgieva, H. Ganev, and V.V. Burov (2005) *Phys. Rev.* **C72** 064321.

[8] S.R. Lesher, A. Aprahamian, *et al*. (2002) *Phys. Rev.* **C66** 051305(R); A. Aprahamian (2002) *Phys. Rev.* **C**65 031301(R).

[9] B. Silvestre-Brac and R. Piepenbring (1977) *Phys. Rev.* **C16** 1638; (1978) *Phys. Rev.* **C17** 1978.

[10] A. Passoja, J. Kantele, M. Luontama, J. Kumpulainen, R. Julin, P. O. Lipas and P. Toivonen (1983) *Phys. Lett.* **124B** 157.

[11] J. Kantele, M. Luontama, W. Trzaska, R. Julin, and A. Passoja (1986) *Phys. Lett.* **171B** 151.

[12] A. Passoja, R. Julin, J. Kantele, J. Kumpulainen, M. Luontama, and W. Trzaska (1985) *Nucl. Phys.* **A438** 413.

[13] A. Passoja, R. Julin, J. Kantele, M. Luontama, and M. Vergnes (1985) *Nucl. Phys.* **A441** 261.

[14] J. Kantele, *et al*. (1979) *Z. Phyz.* **A289** 157; R. Julin, *et al*. (1980) *Z. Phyz.* **A295** 315; R. Julin, *et al*. (1981) *Z. Phyz.* **A303** 147.

[15] K. Heyde and R.A. Meyer (1988) *Phys. Rev.* **C37** 2170.