Bulg. J. Phys. 42 (2015) 565–571

Clustering – Possible Origin of Deformation in ^{32}S?

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Received *30 October 2015*

Abstract. The formation of large size clusters, namely two doubly magic ^{16}O, and their relative motion, as possible excitation mode, are suggested to be a probable solution of the longstanding problem of missing quadrupole collectivity in shell-model calculations aiming to describe electromagnetic B(E2) transition strengths and quadrupole moments of the low-lying states in ^{32}S.

PACS codes: 21.10.Ft, 21.10.Ky, 21.10.Re, 21.60.Gx

1 Introduction

The nucleus ^{32}S has been thoroughly investigated experimentally using as tools different reactions to excite its states and to measure their properties (see e.g [1] and references therein). More precisely, being stable, this nucleus is accessible to Coulomb excitation experiments which have been used to determine E2 transition matrix elements as well as the quadrupole moment of the first excited 2^+ state which is negative and interestingly indicates a large prolate deformation as noticed in [2]. Correspondingly, considerable theoretical efforts have been undertaken to describe all these experimental data. Since the number of protons and neutrons (both of them 16) is relatively small and does not represent an obstacle to advanced shell model calculations, many examples of the latter are available with a tendency to go deeper and deeper into the structure of ^{32}S as does the latest work [3]. However, a specific problem arises, observed also in neighboring ^{30}S and more general in the whole mass region, namely the lack of enough collectivity in the theoretical E2 transition matrix elements and quadrupole moments of the 2_1^+ levels when compared to the experimental data. Thus, in ^{32}S the B(E2,$2_1^+ \to 0_1^+$) value is predicted [3] to be only about 60% of the experimental one, even after a renormalization leading to the maximal possible increase of the theoretical value.

This situation motivated us to investigate additional sources of quadrupole collectivity as possible clustering of the constituent protons and neutrons within large agglomerations. The simplest variant of such clustering would be the formation of two slightly distant in space ^{16}O clusters. The present work is dedicated to the study of the impact of such clustering on the quadrupole properties of the lower-lying 0^+ and 2^+ states. It should be mentioned that a similar picture was suggested in the 80-ies in terms of the possible formation of nuclear molecules and their effect on the properties of nuclear reactions, shape isomers and the Giant Quadrupole Resonance (GQR) (see e.g. [4]). However, here we will consider the simplest collective excitations of even-even ^{32}S at much lower energy above the ground state.

2 Geometry of the Clustering and Its Effect on the Quadrupole Properties

Let us consider two spheres representing the ^{16}O clusters with equal radii which overlap and whose centers lie on the line being the symmetry axis of the whole ^{32}S nucleus as shown in Figure 1. The distance between the centers is equal to $2d$. Postponing the precise calculation of the intrinsic quadrupole moment of this system to a forthcoming work [5], one can roughly approximate it by that of an axially symmetric ellipsoid encompassing the two spheres (also shown in Figure 1) which is given by the well known expression

$$Q_0 = \frac{2}{5} Ze(c^2 - a^2) \,, \tag{1}$$

where c and a are its long and short axes, respectively. They can be approximated in turn as follows:

$$c \sim R_0(^{16}O) + d, a \sim \sqrt{R_0(^{32}S)^3/c} \tag{2}$$

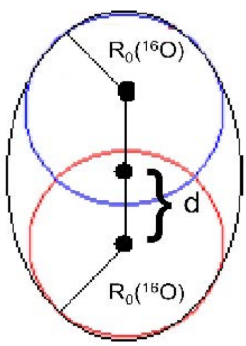

Figure 1. (Color online) Schematic representation of the 2x^{16}O cluster excitation in ^{32}S.

by using the formula for the volume of an ellipsoid $V = 4\pi ca^2/3$ and the effective radius of ^{32}S. In principle both rigid deformation and/or vibrations may be responsible for the establishment of a mean distance $2d$ between the two spheres. In the following, we shall employ the formulae of the rigid rotor model [6] to calculate the reduced matrix elements for the E2 transitions and quadrupole moments associated with states belonging to a class called by us "cluster states". Thus, a comparison with data may lead to the determination of Q_0 and finally, of d, providing a geometrical picture of the large-size clustering.

3 Two-Level Mixing Calculation Involving Shell Model and Cluster States

In this section, we consider a two-level mixing calculation for the lowest two 0^+ and 2^+ levels in ^{32}S whose ingredients belong to the Shell model space (abbreviation index "sh") and to the deformed (statically or dynamically) two-cluster space (abbreviation index "cl"). Since the experimental level energies are known, a given interaction strength V_I between the unperturbed states for each spin I uniquely determines the mixing amplitudes. In Figure 2 we show the low-lying states in ^{32}S and experimental data on the qudrupole properties together with the wave functions of the physical (perturbed) levels as a superposition of the two discussed structures (unperturbed states). The corresponding amplitudes are calculated in the following using the standard formulae for the two-level mixing case.

The intra-structure reduced E2 matrix elements ($sh \to sh$, $cl \to cl$) are "theoretical" taken from the literature or calculated by us within some simplifications for fixed average distance between the two magic clusters (see previous section) while the inter-structure were subject to variation. Later it was found that without the latter the entire date set cannot be consistently reproduced. A computer code was developed which performs the fitting procedure by varying the interaction strengths and the unknown E2 matrix elements between the unperturbed states with the aim to minimize a χ^2 built by comparing the experimental reduced transition matrix elements with the theoretical ones (as well as the quadrupole moment of the first excited 2^+ level).

The final results obtained by the fitting procedure are presented in Table 1 where also theoretical matrix elements from the literature are displayed.

Obviously the presently deduced by the two-level mixing calculation values of the observables reproduce better the data than the previous shell model calculation [1] used for comparison. We note that there are more recent calculations which are much more advanced and considering the nuclear many body problem at a deeper level while their results for the quadrupole properties are even worse. For example, in [3] the question of the weaker collectivity predicted by the Shell-model in ^{32}S with reasonable (not specially normalized) effective E2

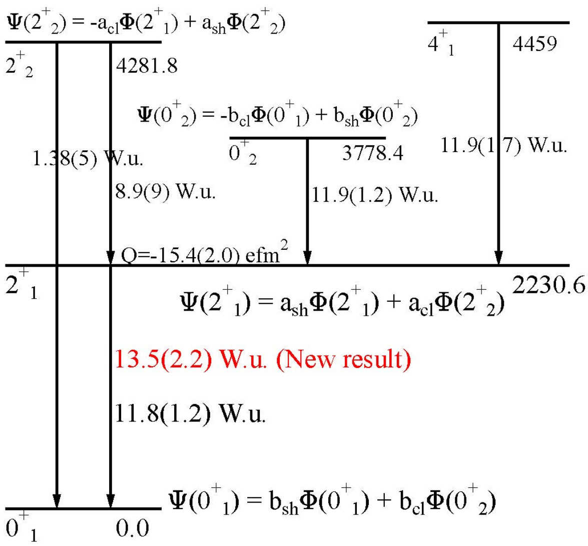

Figure 2. (Color online) The experimental energies of the low-lying I^{π}=0^+, 2^+ and 4^+_1 states in ^{32}S together with experimental B(E2) transition strengths and, when known, quadrupole moments. The structure of the mixed wave functions is also indicated. See also text.

charges was again risen. Therefore our results may point to one possible solution of that problem in providing the missing collectivity by a mixing with states associated with much more deformed nuclear shape, namely, the two-cluster states based on 2x^{16}O configuration. Thereby, the deduced value for the half-distance parameter $d \approx 2$ fm correlates reasonably with the result for $R_0(^{32}S)$ = 4.2 fm from the literature. An additional result of the two-level mixing calculation is the interaction strength $V_{0^+,2^+}$ between the unperturbed states. It varies in the range 600-750 keV being stronger for the 0^+ states. The situation with the mixing percentage is the inverse – the mixing amounts to 4% at the 0^+ states and to 10% at the 2^+ states.

Table 1. Experimental and theoretical values of quadrupole observables in ^{32}S.

Quantity	Experimental value $[efm^2]$	Deduced value $[efm^2]$	Value from [1] $[efm^2]$
$Q(2^+_1)$	-15.4 (2.0)	-14.4	
$\|<0^+_1\|\|E2\|\|2^+_1>\|$	17.5 (6)	18.5	15.7
$\|<0^+_1\|\|E2\|\|2^+_2>\|$	6.4 (3)	-4.9	7.8
$\|<2^+_1\|\|E2\|\|2^+_2>\|$	15.4 (7)	15.8	16.3
$\|<2^+_1\|\|E2\|\|0^+_2>\|$	8.4 (4)	8.1	6.7

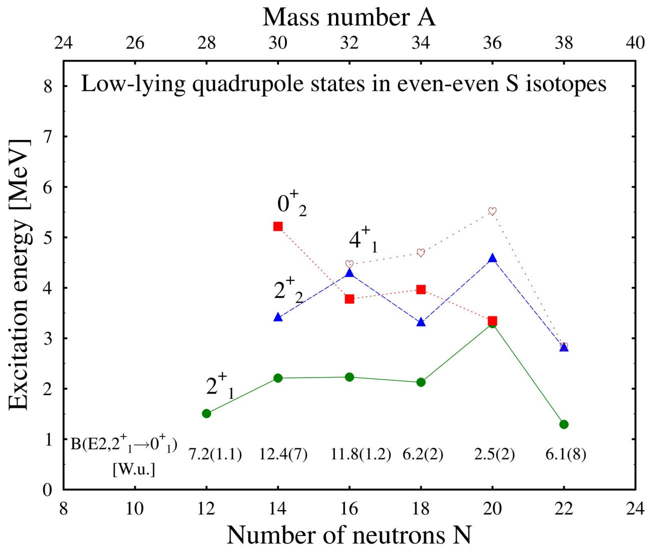

Figure 3. (Color online) The experimental and theoretical energies of the ground band $J^\pi = 0^+$ and $J^\pi = 2^+$ states and the non-yrast $K^\pi = 0^+_2$ and $K^\pi = 2^+_\gamma$ states of deformed nuclei in the $F_0 = 0$ multiplet of Sp(4,R). The experimental values are indicated with bars and the calculated numbers with shapes.

4 Systematics of the Quadrupole Properties of Even-Even S Isotopes

In Figure 3, we show the experimental excitation energies of the 2^+_1, 0^+_2, 2^+_2 and 4^+_1 levels in some even-even sulfur isotopes. The 2^+_1 levels are also characterized by the B(E2) transition strengths to the ground state 0^+_1 in Weisskopf units. The active single-particle orbitals in the mass region A $\sim$ 30 are presented in Figure 4. An inspection of the latter immediately associates the gross features of the systematics in Figure 3 with the subsequent filling of the sd subshells. Namely, at N=14, the subshell $1d_{5/2}$ is filled while in ^{32}S, at N=16, the $2s_{1/2}$ subshell is completely occupied by two neutrons.

These effects lead to the increase of the excitation energy of the 2^+_1 state in ^{30,32}S, the new increase at N=20 being associated with magicity. In the same time, the behavior of the 0^+_2 state is peculiar, it comes abruptly down at N=16, and its excitation energy **decreases** when approaching the magic number N=20 in contrast to all other excited states. Therefore intruder features may be associated with this state, and it might be related to the presently discussed large-size cluster states. In this way, the nuclear structure description proposed in the present paper might be appropriate also for the other S isotopes around ^{32}S, each time within a specific interplay of the single particle and clustering effects.

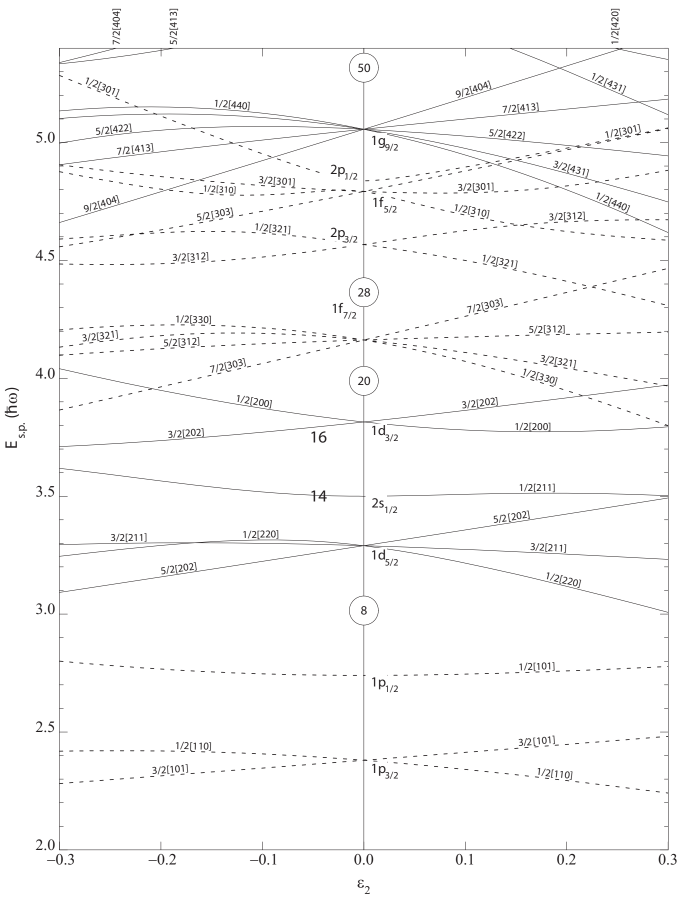

Figure 4. (Color online) Nilsson level scheme illustrating the formation of energy gaps each time an sd subshell is filled.

5 Conclusions

Interesting and sometimes crucial nuclear structure information can be obtained by measuring lifetimes and static moments and comparing experimental and theoretical transition and diagonal matrix elements, respectively. Large scale clustering, especially the one involving magic clusters, may lead to a new type collective motion (relative displacement/vibrations) at relatively lower excitation energy above the ground state. This effect was shown to be a possible origin of the enhanced quadrupole collectivity in ^{32}S within some correlation with filling of subshells i.e. with the single particle properties. Further investigations are in progress.

Acknowledgments

The authors are thankful to A.Astier, M.-G. Porquet, P.Schuck, J.Jolie and O.Yordanov for fruitful discussions and idea exchange.

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