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General Relativity, Quantum Gravity and All That: Time Machines in Perspective by Singularity Functions*

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Abstract. This article presents a theoretical model that establishes a framework to describe natural time machines in certain particular circumstances. Time machines are viewed as resulting from deformations of the spacetime itself, being such deformations originated only in the spatial part of the continuum. Spacetime is treated as the very target of quantization, instead of gravitational field, undergoing shrinkage in determinate conditions of gravitational compression. Singularity functions are applied in the formal representation of the invariant element of spacetime shrinkage in commoving coordinates. Time travel without displacement in space is considered possible in odd regions under gravitational compression. Space compression is discussed from the model of interaction between two black holes. Geodesic equation is rewritten in accordance with the proposal of representation by singularity functions. Standard Friedmann equation is recovered in its version for loop quantum gravity.

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1 Introduction

If we had a survey classifying the most visited issues in fictional literature since the nineteenth century, time travel would occupy one of the highest positions in the ranking. Even in the early nineties, exceptional novels have been written with amazing speculations concerning the enigmatic nature of the time [1]. Over the decades, the scientific progress arising from general relativity and supersymmetric theories made possible, in addition to technological advances, the

*This work is dedicated to Professor Antônio Fernandes da Fonseca Teixeira, DSc, Corresponding Member of the Braziian Center of Physics Research.

resumption of the subject of time travel on more solid foundations from a theoretical point of view. Black holes and wormholes are now commonplace objects in heated debates about the riddles of gravity, space and time, just as it has become commonplace to talk about other solar systems.

Time travel is still a very speculative conception, mainly because it is supported by assumptions about astrophysical objects, and Astrophysics is not an experimental science but fundamentally an observational science (with severe restrictions). Although the equations of general relativity allow us to predict certain phenomena, we cannot reproduce in laboratory a cosmic black hole or wormhole. Even with all the predictive ability we have, only empirical verifications would lead us to the certainty of the facts. The issue becomes even more complex if we consider quantum mechanical effects in the core of the theory of natural time machines, when we know that we are far from an effective quantum theory of gravity.

The search for a quantum theory of gravity has not been very fruitful and conclusive (this is something that clearly can be seen from important works with titles in the form of questions). On one hand, as noted Carlip, "it could be that gravity is simply not quantum mechanical" [2]. On the other hand, as concluded Woodard in his paper of 2009 (basically with the same uncertainties of the nineties), to quantize gravity is a logical necessity "because part of any force field is entirely determined by its sources, and the matter fields which source gravity are indisputably quantum mechanical, whether or not gravitons are quantized or even exist" [3]. Seeking to permanently solve the dilemma, a great endeavor has been undertaken to build alternative theories. Non-local gravity modeling, generically defined by an action like $S = \int d^D x \sqrt{-g}\left[R + R\mathcal{F}(\Box) R\right]$ with $\mathcal{F}(\Box)$ expressing an analytic function of the d'Alembertian operator and D the dimension in which the theory lives, looks for ghost-free solutions that can be renormalizable, however they cannot fully address the unquantized frame of Einstein's general relativity at the ultraviolet [4]. Perhaps the answer begins with non-quantized gravitons considered within a metafield theory of gravity [5], or in a new criticism on our concepts in quantum cosmology and quantum gravity. In recent times, considerable effort was required to harmonize different viewpoints and writing styles to compile the significant contributions. Renowned physicists as Baez and Rovelli [6–10], and many others, have joined their intellects with philosophers interested in scientific issues, bringing to mind discussions in the old style of Reichenbach [11] and attempting to find insights that shed new light on the subject. Those philosophers have contributed to making the discussion better than it might have been without their participation.

This paper seeks to address time machines in a somewhat different perspective, notwithstanding the above limitations. It is a work partially based on approaches of Woodard [12] about quantum cosmological gravity (QCG), and inspired in researches of Rovelli on loop quantum gravity [7, 8, 13], taking into account the ultra-short distances featured by the quantum nature of spacetime. After Einstein, time and space came to be understood as a single, inseparable construct, amalgamated under the universal gravitation; however, time and space are very different dimensions, a fact that leads us to wonder what kind of gravitational aberration would originate phenomena or pathologies affecting only the space or the time. Could it exist time flow without spatial displacement? Would it be possible to have a quantum description of gravitation without contradiction with the relativistic conception of gravity? These are questions to which we want to propose answers in theoretical level, or at least to indicate alternative ways to find answers, in the light of what is currently known with the aid of the mathematical model that we shall propose.

2 Preliminaries on Gravitation

That the unification mechanism is open to question is a generic problem in quantum gravity. The great objection which stands between quantum field theory and gravitation is that general relativity is not properly a theory of the gravitational field but rather a theory of the spacetime itself. Therefore, such a theory should describe something like “the quantum structure of spacetime”, a very obscure notion waiting for convincing explanations. Concrete predictions at low energies were studied by Bjerrum-Bohr [14], for renormalizable or non-renormalizable theories, based on the quantum gravitational correction applied to the Newtonian potential

$$V\left(r\right)=-\frac{Gm_1m_2}{r}\left(1+3\frac{G\left(m_1+m_2\right)}{rc^2}+\frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right), \tag{1}$$

where the last term is the quantum gravity correction, but many questions remain unanswered.

Even so, on some kind of theory of quantum gravity, leastwise in the context of cosmology, we observe that the only way to be effective in the subject is to forget the idea of quantizing the gravitational field, trying to find a manner to treat the spacetime metric from a new operatorial point of view with regard to evaluate possible states of spacetime itself; stated another way, we must try to establish an invariant measure of the expansion/shrinkage rate of the spacetime, looking for descriptions which may be compatible with a representation of the gravitational interactions. This is precisely the sort of difficulty we must to confront in order to have a hope of finding a representation of gravity in quantum level.

One may think that the assumption of the aforementioned compatibility of descriptions starts from the introduction of the perturbed metric, which we take as

$$\tilde{g}_{\mu\nu}=g_{\mu\nu}+\gamma h_{\mu\nu}, \tag{2}$$

where $g_{\mu\nu}$ is the classical background, $h_{\mu\nu}$ is the graviton field and $\gamma^2\equiv 16\pi G$ is the loop counting parameter of perturbative quantum gravity. Here, is appropriate to assume a de Sitter geometry, since, as noted by Woodard, it allows coordinates in such a way that the invariant element is proportional thereto in flat spacetime [12], from which we have the metric leading to

$$g_{\mu\nu}dx_{\mu}dx_{\nu} = \Omega^2 \left(-du^2 + d\vec{x}d\vec{x}\right), \tag{3}$$

with

$$\Omega \equiv \frac{1}{Hu} \equiv \exp\left(Ht\right). \tag{4}$$

In fact, a conformally invariant Lagrangian with rescaled fields in this coordinate system reduces to the same Lagrangian in flat spacetime.

Many physicists still see in perturbative theories a source of clarifications for the quantum approach of gravity, as Upadhyay and Mandal applying Green's functions in a non-linear gauge theory [15]. In any case, to treat the metric perturbatively brings some significant drawbacks and virtually restricts the theory to the deep infrared region. An interesting approach, but which deserves careful considerations, is the work of Agullo and colleagues [16]. Based on the fact that cosmological perturbations – commonly described by quantum fields on classical spacetimes – cannot be in general justified at the quantum gravity era (during which matter densities were at Planck scale), they applied techniques from loop quantum gravity, widening the standard theory of cosmological perturbations to overcome this limitation. Hence, they discussed the quantum state of the background geometry and the quantum state of the perturbation in order to get the full state from the tensor product

$$\Psi\left(v, \mathcal{T}_{\vec{k}}, \phi_0\right) = \Psi_0\left(v, \phi_0\right) \otimes \psi\left(\mathcal{T}_{\vec{k}}, \phi_0\right), \tag{5}$$

where Ψ_0 is the quantum state of the background geometry and ψ the quantum state of the perturbation,$\mathcal{T}_{\vec{k}}$ is the tensor mode giving the test quantum perturbation $\hat{\mathcal{T}}_{\vec{k}}$, ϕ_0 is any fixed initial instant of the internal time ϕ, and v is a dynamical variable that determines the volume of the universe [16]. So, ψ evolves on the background quantum geometry Ψ_0, so that quantum trajectory $\Psi_0\left(v, \phi\right)$ is raised up to trajectory $\Psi_0\left(v, \phi_0\right) \otimes \psi\left(\mathcal{T}_{\vec{k}}, \phi_0\right)$. They showed accurately the distinction between true and apparent trans-Planckian difficulties, explaining how to ride out those true difficulties. Meanwhile, these authors emphasized that it would be a misleading to understand certain simple results of the theory, in a technical level, as a confirmation that standard quantum theory of perturbations on a classical FLRW solution of Einstein's equations works in the Planck scale, drawing attention to the comparatively very large quantum corrections embedded in a smooth FLRW metric. In addition, those results do not even satisfy the equations which track the peak of the wave function $\Psi_0\left(v, \phi\right)$ of the background geometry (the reader will find more details in the reference [16]).

In a general way, however, it is known that perturbative quantum gravity is inconsistent at the quantum level, since it leads to an infinite number of non-renormalizable ultraviolet divergences. Furthermore, the number of additional technical artifices has increasingly distanced the theory from its physical content, a fact that makes difficult the evaluation of the effectiveness of a model.

2.1 Some ideas on time machines during the twentieth century and after

After these preliminaries the reader is prepared to enter the subject of natural time machines. Among the first works referencing cosmic physical objects as possible time machines, we have those pioneering papers dated back to the late eighties by Morris and Thorne [17] and to the early nineties by Frolov and Novikov [18, 19], followed by the classic work of Visser [20], all of which, about wormholes, have since been very influential. More even, we have the recent work from Sharif and Rani on traversable wormholes in the framework of generalized teleparallel gravity [21]. From these works, we shall keep the focus on a spherically symmetric (3+1)-dimensional intra-universe wormhole, assuming that their two mouths are embedded in the same asymptotically flat universe. In addition, there is no reason to suppose that time is running at the same rate on either side of the wormhole. Finally, we pay attention on the (1+1)-dimensional surface swept out by the central geodesic matching the mouths.

Under a microphysical look, space is revealed irregular and consists primarily of void, as well as time. The so-called "quantum foam" would contain a plethora of amazingly tiny holes connecting infinitesimal time intervals. These little "time tunnels" are called wormholes. For our purposes, although the reader can retrieve good approaches in references [22] and [23], we call "quantum foam" a construct that represents how spacetime appears at Planck length scale (of the order of one divided by a thousand quadrillion quadrillion centimeters). At this scale, spacetime would be fulfilled with fluctuating regions in which geometry oscillates very rapidly. This constantly changing geometry is compared to a foam. Looking through the prism of our scale, however, this intense "boiling" averages out to seem like a smooth bend on a thin brane.

Considering those wormholes consisting of normal matter, they are unstable due to the radiative feedback that leads them to collapse quickly, a way that nature found to avoid time paradoxes. It is thought that, conjecturally, these micro tunnels could be expanded up to a reasonable size to enable any type of material flow in time, if it was possible stabilize them using some exotic matter. But, as discussed by Woodward (do not confuse with the author referred at the introduction of this article), such a thing would be possible only in presence of an acceptable background independent quantum theory of gravity, that remains to be conceived, and there is no evidence of a "spacetime foam" of microscopic wormholes [24].

No observational evidence for cosmic wormholes currently exists, although mathematical solutions of Einstein's field equations describing wormholes have long been known in general relativity. As in the quantum foam, cosmic wormholes made of normal matter with positive energy density would be unstable, collapsing by feedback in presence of matter trying to traverse it, in the same sense that sound feedback would lead to the destruction of the amplifiers if not interrupted. Thus, stable traversable wormholes would require some kind of exotic matter with a negative energy density, for instance, the quantum field configuration responsible for the Casimir effect.

Only to clarify the example, let us take a somewhat more adequate approach to the spherically symmetric wormhole model we mentioned at the beginning of this section. We return to that (3+1)-dimensional intra-universe wormhole; considering the space-time metric on the (1+1)-dimensional surface related to the connection of the mouths given by a simple Lorentzian form in the (t, l) plane, we get

$$ds^2 = -e^{2\phi(l)}dt^2 + dl^2. \tag{6}$$

We have that the l coordinate ranges from $-L/2$ to $+L/2$; the coordinates $l = -L/2$ and $l = +L/2$ are identified, so that the total way around distance between the two mouths is equal to L. Assuming that the metric is smooth (not necessarily its components) at the junction, we have

$$d\tau = e^{\phi_-}dt_- = e^{\phi_+}dt_+. \tag{7}$$

Identifying the points $(0, -L/2) \equiv (0, +L/2)$ at the time coordinate origin, the discontinuity of the time is

$$t_+ = t_- e^{(\phi_- - \phi_+)} = t_- e^{(-\Delta\phi)}, \tag{8}$$

providing time is not running at the same rate on both wormhole sides. From this,

$$(t_-, -L/2) \equiv \left(t_- e^{(-\Delta\phi)}, +L/2\right). \tag{9}$$

Thereby, time t_+ on one side of the wormhole mouth is identified with time t_- on the other side. For a null geodesic starting out from $l = -L/2$ at the initial time $[t_i]_-$, we have

$$\frac{dl}{dt} = \pm e^{+\phi(l)}, \tag{10}$$

where the symbol "+" is assigned to a right-moving null geodesic and the symbol "–" to a left-moving one. For instance, integrating a right-moving geodesic, it arrives at $l = +L/2$ at coordinate time

$$[t_f]_+ = [t_i]_- + \int_{-L/2}^{+L/2} e^{-\phi(l)}dl. \tag{11}$$

From equation (8), the matching condition under the coordinate discontinuity at $l = \pm L/2$ forces the light ray returns to the starting point at time

$$[t_f]_- = [t_f]_+ \exp(\Delta\phi) = \left[[t_i]_- + \oint e^{-\phi(l)}dl\right] \exp(\Delta\phi). \tag{12}$$

Note that all the explanation is based on the fact that time flow rates are different at both sides of the wormhole.

More recent works get the geometry of pseudo Schwarzschild spacetime in warped product form joining the base-cylinder $Z = S^1 \times \mathbb{R}^+$ with the fiber H^2 (the hyperbolic plane) to treat a viable time machine whose background metric is

$$g = \left(1 - \frac{2m}{r}\right) dv^2 + 2dvdr + r^2 \left(d\theta^2 + \sinh^2(\theta) d\phi^2\right), \qquad (13)$$

where v is a circular coordinate on S^1 and r is the radial coordinate [25]. To provide the reader with background for his study, there are interesting works from Bagrov [26] and Monton [27], and the very theoretical works from Manchak [28, 29].

2.2 Compression of spacetime

Interestingly, when one thinks of contraction of space the immediate impulse is to consider the same center point of the spherical symmetry that led to the expansion, something like a gravity vortex dragging all the spacetime. Instead, we shall assume singularities caused by spherical pressure from the outside in; unlike a single source, we have now several sources for each singularity. Theoretically, such bubbles of compression, or anti-expansion, may be generated by gravitational pressure caused by great masses (this is readily understood by considering two massive bodies close to each other). The most likely image corresponding to this situation is when two massive black holes interact with one another (Figure 1). Similar effects in the strong-gravitational-field limit can also be expected from two supermassive pulsars in close binary systems [30]. On one hand, by the fact that bodies are not perfectly rigid, when two bodies interact gravitationally with one another their facing surfaces are more distorted because they are attracted more strongly than the opposite sides. But, on the other hand, the gravitational attraction between the bodies tends to squeeze or compress them as they pull one another toward their center points; so, the strong attraction between massive bodies leads to a compression of space contained among them. We call those bubbles of compressed space “G-closures”. However thus far they are only theoretical objects, gravitational bubbles of spacetime shrinkage within an expanding universe are plausible, featuring local inhomogeneities. As above mentioned, we can speculate on the spacetime behavior in such physical conditions from the study of the interaction of two black holes with similar masses forming a binary system.

The understanding that is the spacetime itself which moves as long as it expands brings very interesting metaphysical consequences. If space or spacetime expands, what is the conclusion we have attained? Provided we are made of spacetime, we are expanding along with the universe, even if our expansion is not perceptible. In fact, we can only move in because the space itself remains

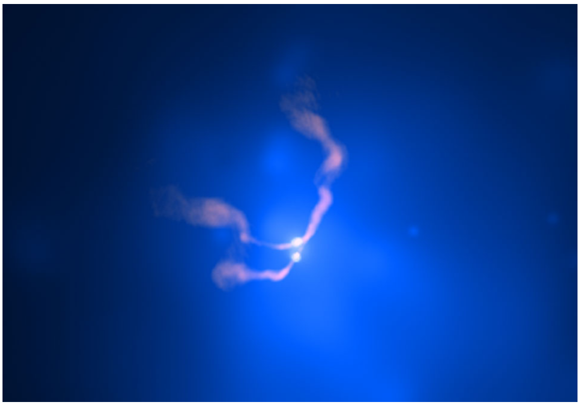

Figure 1. Co-orbiting supermassive black holes powering the giant radio source 3C75. Credit: X-Ray (blue), NASA/CXC/D. Hudson, T.Reiprich et al. (AIfA); Radio (pink), NRAO/VLA/ NRL.

constantly expanding; otherwise, we would have to return to the idea of an absolute, empty space, within which the bodies move independently. A simple one-dimensional analogy is to imagine that the world line of a particular individual is a treadmill that expands from a zero point at which it has no dimension. The individual lies on the mat, and in so far as it expands he gets a degree of freedom to walk on it. But the body-space of the individual also expands, so in our analogy we must bear in mind that in reality the expansion would be in all directions, thereby maintaining the structural proportions of the objects. If the individual stops, the treadmill continues to expand and, likewise, continues to expand the body of the individual. Now imagine that the treadmill contracts — as well as the individual body —, and that each individual step is such that its virtual progress is precisely counterbalanced by the contraction. The individual continues his march ahead, but goes nowhere as in a sportive mat. Thus, there is no displacement in space; only time persists marking the moments of each contraction. This is time travel without going anywhere, and we assume that this could happen in a G-closure. Compressed by gravity, the individual remains in the same place over time. In a sense, time appears again as a fundamental dimension without which there would be no physical world. It is even possible to think physics with no space, however, impossible to conceive it without time.

Now, we can summarize this discussion as a lawlike statement:

Proposition 2.1. *Under strong gravitational compression, time dilates and space ceases to be a degree of freedom in the direction of compression.*

Just to avoid misunderstandings, recalling Bunge [31], this type of proposition is a corrigible statement (or hypothesis); *a posteriori*, as a lawlike statement, "it requires corroboration and systemicity in order to be ranked as a law statement"

[31]. Of course, this statement presupposes the well known time dilatation due to gravity, as presumed in milestone works on black hole and wormhole physics [18–20, 32, 33].

2.3 Singularity functions

Our references and motivations for the use of the so-called "singularity functions" came from the analysis of distinct spatial segments under the action of different deforming strains in a beam. The mathematical operator of singularity functions have formerly been applied for structural engineering analysis of beams beneath complex loads [34]. Their representations in "brackets" are due to English mathematician William Macauley (1853-1936) [35], although credits for the method are assigned to both German mathematician Alfred Clebsch (1833-1872) and German civil engineering Otto Föppl (1854-1924). However, it seems that preconceptions in scientific circles obscured the merits of the true author of singularity functions, the English mathematician Oliver Heaviside (1850-1925). Subsequently, these functions were applied in a variety of situations including productivity scheduling analysis [36]. Now, we apply them to construct an analytical model to study gravity within a framework to understand a kind of hypothetical natural time machine. The principal advantages of singularity functions to treat spacetime are in the facts that a)- they describe phenomena based on geometry (very interesting for our proposal), b)- they capture any changes in time evolution, c)- they can include infinitely many spacetime segments in different states, d)- they can be rescaled by any factor, e)- they are independent of units, and f)- they are continuous, differentiable and integrable like common functions.

Within our scope, singularity functions serve to build a geodesic equation that makes possible to analyze any particular interval of the curve, in view of the concept of G-closure, defining the nature of the geodesic itself in each particular situation. A singularity function, given in Macauley kets [35] as $\langle x - x_0 \rangle^n$, obeys the rule

$$\langle x - x_0 \rangle^n = \begin{cases} (x - x_0)^n, x > x_0 \\ 0, x \leq x_0 \end{cases} \tag{14}$$

In addition, making $\langle x - x_0 \rangle = \mathrm{X}$, we write

$$\int \langle x - x_0 \rangle^n d\mathrm{X} = \frac{1}{n+1} \langle x - x_0 \rangle^{n+1} + C; \tag{15}$$

$$\frac{d\langle x - x_0 \rangle^n}{d\mathrm{X}} = n\langle x - x_0 \rangle^{n-1}. \tag{16}$$

We shall deduct the first form of the geodesic equation with singularity functions, while recalling the classical deduction also found in Eddington [37], in order to reach a time-like geodesic description for real time paths with no space paths. Later, based on the initial hypothesis that such path is physically possible, we shall investigate what motivators could lead to predict that phenomenon. In principle, one can think of some kind of gravitational singular aberration able to distort the spatial part of the spacetime continuum, leaving the time as the only independent dynamic variable regulating a succession of events without space paths. At this point, we have no interest about the direction of the time-arrow, just focusing on the model of an evolutionary step free from space contributions. Obviously, it is not the case to vanishing the spatial components of the metric tensor[1], which would be trivial, nor to disregard by second order transformations the infinitesimal space variables, but to null the participation of them in the geodesic path; the space still exists in the singularity, however, as it was "frozen". This means that the geometry of spacetime fluctuates (or undergoes excitations) over "non-space", apart from the trivial case of the $g_{\mu\nu} = 0$ solution.

3 The Geodesics in Singularity Functions

Let us begin with the expression of the invariant commoving element in singularity functions,

$$ds^2 = g_{\mu\nu} d\left\langle x_\mu - \varepsilon_\mu \right\rangle d\left\langle x_\nu - \varepsilon_\nu \right\rangle, \tag{17}$$

where ε_μ and ε_ν are fixed distances from a point on the spherical boundary of the G-closure. Putting $d\left\langle x_\mu - \varepsilon_\mu \right\rangle d\left\langle x_\nu - \varepsilon_\nu \right\rangle = d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\nu$ and applying the variational principle on the commoving element, we get

$$\delta\int_A^B ds = \delta\int_A^B \sqrt{g_{\mu\nu} d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\nu}\,\delta\int_A^B \frac{g_{\mu\nu} d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\nu}{\sqrt{g_{\mu\nu} d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\nu}}$$

$$= \delta\int_A^B g_{\mu\nu}\frac{d\langle x-\varepsilon\rangle_\mu}{ds}\frac{d\langle x-\varepsilon\rangle_\nu}{ds}ds. \tag{18}$$

Now, we know that

$$\delta\left(g_{\mu\nu}\frac{d\langle x-\varepsilon\rangle_\mu}{ds}\frac{d\langle x-\varepsilon\rangle_\nu}{ds}\right) = \delta g_{\mu\nu}\frac{d\langle x-\varepsilon\rangle_\mu}{ds}\frac{d\langle x-\varepsilon\rangle_\nu}{ds}$$

$$+ g_{\mu\nu}\left[\frac{d\langle x-\varepsilon\rangle_\mu}{ds}\delta\left(\frac{d\langle x-\varepsilon\rangle_\nu}{ds}\right) + \frac{d\langle x-\varepsilon\rangle_\nu}{ds}\delta\left(\frac{d\langle x-\varepsilon\rangle_\mu}{ds}\right)\right]. \tag{19}$$

[1] In fact, the issue for us is not to establish the physical elimination of space, but rather to demonstrate its metric irrelevance in certain physical conditions. Nor do we start with spatial rigidity assumptions, especially since we know that rules of length "l", initially supposed rigid in systems said "in free fall", act according to the expression

$$dl^2 = \left(-g_{\mu\nu} + \frac{g_{\mu 0} g_{\nu 0}}{g_{00}}\right) dx_\mu dx_\nu.$$

This shows that if the metric is non-stationary ($g_{\mu\nu} \neq 0$), even if $g_{\mu\nu,0} = 0$, a rigid rule taken as distance unit have lengths $dl_1 \neq dl_2$ at different points in space. In this study, we consider that any region in space is continually being expanded (or compressed), so that there are no rigid structures at all.

Also, the last two terms of the above equation are equal, as we may see by simple interchanging of μ and ν, so that, doing

$$\delta g_{\mu\nu} = \frac{\partial g_{\mu\nu}}{\partial \langle x - \varepsilon \rangle_k} \delta \langle x - \varepsilon \rangle_k, \tag{20}$$

we gain

$$\delta \Big(g_{\mu\nu} \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \frac{d\langle x - \varepsilon \rangle_\nu}{ds} \Big) = \frac{\partial g_{\mu\nu}}{\partial \langle x - \varepsilon \rangle_k} \delta \langle x - \varepsilon \rangle_k \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \frac{d\langle x - \varepsilon \rangle_\nu}{ds} + 2 g_{\mu k} \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \delta \left(\frac{d\langle x - \varepsilon \rangle_k}{ds} \right). \tag{21}$$

There is no danger of confusion if, for convenience, we replace μ by k. Since

$$\delta \left(\frac{d\langle x - \varepsilon \rangle_k}{ds} \right) = \frac{d}{ds} \left(\delta \langle x - \varepsilon \rangle_k \right), \tag{22}$$

we may integrate between A and B for the geodesics

$$\int_A^B \frac{\partial g_{\mu\nu}}{\partial \langle x - \varepsilon \rangle_k} \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \frac{d\langle x - \varepsilon \rangle_\nu}{ds} \delta \langle x - \varepsilon \rangle_k ds + 2 \int_A^B g_{\mu k} \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \frac{d}{ds} \left(\delta \langle x - \varepsilon \rangle_k \right) ds = 0. \tag{23}$$

Integrating by parts, disregarding any possible contributions from the boundaries putting δ equal to zero at the endpoints A and B, we get

$$\int_A^B \Big[\frac{\partial g_{\mu\nu}}{\partial \langle x - \varepsilon \rangle_k} \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \frac{d\langle x - \varepsilon \rangle_\nu}{ds} - 2 \frac{d}{ds} \Big(g_{\mu k} \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \Big) \Big] \delta \langle x - \varepsilon \rangle_k ds = 0. \tag{24}$$

But the variations $\delta \langle x - \varepsilon \rangle_k$ are arbitrary, from which it follows that

$$\frac{\partial g_{\mu\nu}}{\partial \langle x - \varepsilon \rangle_k} \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \frac{d\langle x - \varepsilon \rangle_\nu}{ds} - 2 \frac{d}{ds} \left(g_{\mu k} \frac{d\langle x - \varepsilon \rangle_\mu}{ds} \right) = 0. \tag{25}$$

This is the first form of the geodesics equation. Now, according to the principle of equivalence, there must be a coordinate system χ^η in which particles move in a "straight" line and therefore satisfying

$$\frac{d^2 \chi^\eta}{d\tau^2} = 0, \tag{26}$$

where τ is the proper time of the particle. If we choose a different coordinate system considering singularity functions, say $\langle x-\varepsilon\rangle_\mu$, we may use the chain rule and write

$$0=\frac{d}{d\tau}\left(\frac{\partial\chi^\eta}{\partial\langle x-\varepsilon\rangle_\mu}\frac{d\langle x-\varepsilon\rangle_\mu}{d\tau}\right); \tag{27}$$

$$0=\frac{\partial\chi^\eta}{\partial\langle x-\varepsilon\rangle_\mu}\frac{d^2\langle x-\varepsilon\rangle_\mu}{d\tau^2}+\frac{\partial^2\chi^\eta}{\partial\langle x-\varepsilon\rangle_\mu\partial\langle x-\varepsilon\rangle_\nu}\frac{d\langle x-\varepsilon\rangle_\mu}{d\tau}\frac{d\langle x-\varepsilon\rangle_\nu}{d\tau}. \tag{28}$$

Multiplying last equation by the inverse Jacobian $\partial\langle x-\varepsilon\rangle_\mu/\partial\chi^\eta$ we end up with

$$\frac{d^2\langle x-\varepsilon\rangle_\xi}{d\tau^2}+\Gamma^\xi_{\mu\nu}\frac{d\langle x-\varepsilon\rangle_\mu}{d\tau}\frac{d\langle x-\varepsilon\rangle_\nu}{d\tau}=0, \tag{29}$$

where

$$\Gamma^\xi_{\mu\nu}=\frac{\partial\langle x-\varepsilon\rangle_\xi}{\partial\chi^\eta}\frac{\partial^2\chi^\eta}{\partial\langle x-\varepsilon\rangle_\mu\partial\langle x-\varepsilon\rangle_\nu} \tag{30}$$

is the affine connection. This is the second form of the geodesic equation. The reader must understand that while no calculation is performed, it will remain the representation in singularity functions. Thus, we gain a valid and general representation for any particular spacetime interval, no matter the temporal or spatial nature of the geodesic.

Now, due to gravitational compression, the contraction of space compensates any displacement, so that the geodesic equation reduces to

$$\frac{d^2\langle x-\varepsilon\rangle_\xi}{d\tau^2}+\Gamma^\xi_{00}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}=0; \tag{31}$$

$$\frac{d^2\langle x-\varepsilon\rangle_\xi}{d\tau^2}=-\Gamma^\xi_{00}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}. \tag{32}$$

Further, since no effective displacement occurs, field becomes static in space, so that

$$\Gamma^\xi_{00}=\frac{1}{2}g^{\xi\lambda}\left(\partial_0 g_{\lambda0}+\partial_0 g_{0\lambda}-\partial_\lambda g_{00}\right) \tag{33}$$

reduces to

$$\Gamma^\xi_{00}=\frac{1}{2}g^{\xi\lambda}\left(\partial_0 g_{\lambda0}+\partial_0 g_{0\lambda}\right). \tag{34}$$

One must understand that contracting bubbles, or G-closures, are not conventional "bubbles" entrained by the expanding outer space; they are regions of this space itself where there is no expansion but compression. If they were not so, we would go back to think of an absolute expanding space within which a small moving volume shrinks.

When space contracts, so reducing scale, it appears what we call "translational coupling": according to the explanatory construction presented in subsection

2.2, for the individual walking faster, the treadmill runs faster backwards. Thus, he shall not go anywhere and the geodesic becomes timelike. These are the reasons for the use of intervals (not points) and singularity functions. Firstly, an interval defines a portion of the geodesic to be analyzed; but as the whole spacetime is expanding we have to discount the expansion from the apparent extent. Since there is scale variation, there is interval variation too. Secondly, as we are interested in measurable mechanical displacements along a geodesic, when there is no displacement the variations of space intervals do not matter, a fact that happens precisely in translational coupling regions; but translational coupling regions occur in G-closures where the rate of expansion counterbalances any displacement. In this case, by definition, although space intervals exist (in contraction) they are ignored when translational coupling runs. In other words, a space interval is always non-stationary, however not characterizing displacements in translational coupling regions. In practice, being $\langle x-\varepsilon\rangle_\mu \leq 0$, this means that or the displacement is compensated by compression, or it is largely exceeded by compression. In both cases, the spatial terms disappear accordingly singularity function's definition (in fact, this is intuitively evident since compression faster than any step and from all directions would not allow displacement).

4 The Measure Operation on the Spacetime Invariant Element

The basics of Woodard's work provided some insights for our modeling, but the reader must be mindful to the fact that we are not inferring conclusions from a perturbative model. Let us assume, without loss of generality, an initial state of the G-closure characterized by homogeneity and local isotropy. We suppose, in accordance with the local isotropy, a locally de Sitter classical background on a manifold which admits flat 3-sections. Thus, it is possible to find the invariant measure of the spacetime contraction rate, within any interval of the geodesic line, first writing the invariant element in commoving coordinates by the correlation function

$$\begin{aligned}\langle 0|\, g_{\mu\nu} d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\eta \,|0\rangle &= -d\langle t-\varepsilon\rangle_0^2 \\ &+R^2_{\langle t-\varepsilon\rangle_0} d\,\langle\vec{x}-\vec{\varepsilon}\rangle\, d\,\langle\vec{x}-\vec{\varepsilon}\rangle\,, \qquad (35)\end{aligned}$$

with the effective Huble constant as the logarithmic derivative of the scale factor

$$H_{eff} \equiv \frac{d}{d\,\langle t-\varepsilon\rangle_0} \ln\left(R_{\langle t-\varepsilon\rangle_0}\right). \qquad (36)$$

At this point, we compare expression (35) with the quantum-corrected invariant element in conformal coordinates

$$\begin{aligned}&\langle 0|\, g_{\mu\nu} d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\eta \,|0\rangle \\ &= \Omega^2\left\{-\left[1-C(u)\right]du^2+\left[1+A(u)\right]d\,\langle\vec{x}-\vec{\varepsilon}\rangle\, d\,\langle\vec{x}-\vec{\varepsilon}\rangle\right\}, \qquad (37)\end{aligned}$$

where u is a time function that corresponds to $1/H$ for time coordinate equal to 0 and to 0 for time coordinate equal to ∞, $A(u)$ and $C(u)$ are defined from the retarded Green's functions of the massless minimally coupled and conformally coupled scalars, so that

$$A(u) = -4G_A^{ret}\left[a\right](u) + G_C^{ret}\left[3a + c\right](u), \tag{38}$$

$$C(u) = G_C^{ret}\left[3a + c\right](u). \tag{39}$$

Therefore, we can deduce the scale factor from the quantum-corrected invariant element in conformal coordinates [12], and relate u to t

$$R_{\langle t-\varepsilon\rangle_0} = \Omega\sqrt{1 + A(u)}, \tag{40}$$

$$d\left\langle t-\varepsilon\right\rangle_0 = -\Omega\sqrt{1 - C(u)}du. \tag{41}$$

In view of what has been summarized in proposition 2.1, we can say that for the perspective of finding a G-closure, we must write

$$\langle 0|\, g_{\mu\nu} d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\nu \,|0\rangle = -d\langle t-\varepsilon\rangle_0^2, \tag{42}$$

and so

$$\langle 0|\, g_{\mu\nu} d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\nu \,|0\rangle = \Omega^2\left\{-\left[1 - C(u)\right]du^2\right\}. \tag{43}$$

Accordingly the above arguments, having in mind proposition 2.1, all we can say is the expectation value measure of the rate in which the invariant element evolves only in time mode, once a G-closure is manifested.

5 Discussion

The overall formalism considered thus far are important to what shall be discussed below. One might object that, since Woodard's theory is based on perturbative methods, we would be trying to reconcile two incompatible scales (Planck scale and deep infrared scale). We shall see why the authors do not find this objection to be convincing. Firstly, we note that the reason to consider Woodard's approach is precisely the fact that retarded Green's function inherited from Schwinger's formalism and applied in the expressions of $A(u)$ and $C(u)$ may be freely evaluated nom-perturbatively[2], as Woodard himself pointed out in his discussion on the relaxation in perturbative time evolution [12]. A good check on this last observation is feasible by means of more complete descriptions if we assume $\Omega = 1/Hu$ in de Sitter geometry. Then,

$$R_{\langle t-\varepsilon\rangle_0} = \Omega\sqrt{1 + A(u)} = \frac{1}{Hu}\sqrt{1 + A(u)}, \tag{44}$$

[2] As a matter of fact, Schwinger's article "The Theory of Quantized Fields", submitted to Physical Review (1951) was one in which non-perturbative methods play a central role [38]. In addition, he introduced a description in terms of Green's functions not depending on perturbative expansions.

and

$$\ln R_{\langle t-\varepsilon\rangle_0} = \ln\left[\frac{1}{Hu}\sqrt{1+A(u)}\right]. \tag{45}$$

Now, we can write

$$\frac{d}{d\langle t-\varepsilon\rangle_0}\left(\ln R_{\langle t-\varepsilon\rangle_0}\right) = \frac{Hu}{\sqrt{1+A(u)}}\left[-\frac{1}{Hu^2}\frac{du}{d\langle t-\varepsilon\rangle_0}\sqrt{1+A(u)}\right. \\ \left. + \frac{1}{2Hu}\frac{1}{\sqrt{1+A(u)}}\frac{dA(u)}{du}\frac{du}{d\langle t-\varepsilon\rangle_0}\right]; \tag{46}$$

$$\frac{d}{d\langle t-\varepsilon\rangle_0}\left(\ln R_{\langle t-\varepsilon\rangle_0}\right) = \frac{du}{d\langle t-\varepsilon\rangle_0}\left[-\frac{1}{u}+\frac{1}{2}\frac{1}{1+A(u)}\frac{dA(u)}{du}\right]. \tag{47}$$

From expressions (40) and (41) we obtain

$$\frac{du}{d\langle t-\varepsilon\rangle_0} = -\frac{1}{\Omega\sqrt{1-C(u)}} = -\frac{Hu}{\sqrt{1-C(u)}}, \tag{48}$$

and so

$$\frac{d}{d\langle t-\varepsilon\rangle_0}\left(\ln R_{\langle t-\varepsilon\rangle_0}\right) = -\frac{Hu}{\sqrt{1-C(u)}}\left[-\frac{1}{u}+\frac{1}{2}\frac{1}{1+A(u)}\frac{dA(u)}{du}\right] \\ = \frac{H}{\sqrt{1-C(u)}}\left[1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right]. \tag{49}$$

Note that no assumption about perturbations is implied. Lastly, we can say that for the energy-momentum tensor of a homogeneous and isotropic fluid in singularity function variables, $T_{\mu\nu} = \mathrm{diag}\left[\rho_{\langle t-\varepsilon\rangle_0}, -p_{\langle t-\varepsilon\rangle_0}, -p_{\langle t-\varepsilon\rangle_0}, -p_{\langle t-\varepsilon\rangle_0}\right]$, the "00" component of Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{50}$$

yields to Friedmann equation in a flat spacetime in the form

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_{\langle t-\varepsilon\rangle_0}. \tag{51}$$

Now, from equation (40) we get

$$\dot{R} = \frac{1}{2}\frac{1}{Hu\sqrt{1+A(u)}}\frac{dA(u)}{du}\frac{du}{d\langle t-\varepsilon\rangle_0} - \frac{1}{Hu^2}\frac{du}{d\langle t-\varepsilon\rangle_0}\sqrt{1+A(u)}, \tag{52}$$

$$\dot{R} = \left[\frac{1}{2}\frac{1}{Hu\sqrt{1+A(u)}}\frac{dA(u)}{du} - \frac{1}{Hu^2}\sqrt{1+A(u)}\right]\frac{du}{d\langle t-\varepsilon\rangle_0}, \tag{53}$$

$$\frac{\dot{R}}{R}=\left[\frac{1}{2}\frac{1}{1+A(u)}\frac{dA(u)}{du}-\frac{1}{u}\right]\frac{du}{d\left\langle t-\varepsilon\right\rangle_0}, \tag{54}$$

$$\frac{\dot{R}}{R}=\left[-\frac{1}{2}\frac{1}{1+A(u)}\frac{dA(u)}{du}+\frac{1}{u}\right]\frac{Hu}{\sqrt{1-C(u)}}, \tag{55}$$

$$\frac{\dot{R}}{R}=\frac{H}{\sqrt{1-C(u)}}\left[1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right], \tag{56}$$

which is precisely equation (49). Therefore, in a flat spacetime, the energy density is given by

$$\left\{\frac{H\sqrt{3}}{\sqrt{8\pi G\left(1-C(u)\right)}}\left[1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right]\right\}^2=\rho_{\langle t-\varepsilon\rangle_0}. \tag{57}$$

Returning to equation (43), we conclude that, given the energy density, the expectation value of the rate at which the invariant element evolves only in time mode, within a G-closure, in a locally flat background is described by

$$\begin{aligned}\langle 0|\,g_{\mu\nu}d\left\langle x-\varepsilon\right\rangle_\mu d\left\langle x-\varepsilon\right\rangle_\nu|0\rangle &= \Omega^2\left\{-\left[1-C(u)\right]du^2\right\}\\ &= -\left[\frac{3}{8\pi Gu^2\rho_{\langle t-\varepsilon\rangle_0}}\left(1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right)^2\right]du^2,\end{aligned} \tag{58}$$

since

$$\frac{3H^2}{8\pi G\rho_{\langle t-\varepsilon_0\rangle}}\left[1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right]^2=1-C(u).$$

For the purposes of theoretical treatment, improving the semi-classical structure we may put last equation in the form of a specific relation between the expectation value of the rate and the expectation value of the energy density, so that

$$\begin{aligned}&\langle 0|\,g_{\mu\nu}d\left\langle x-\varepsilon\right\rangle_\mu d\left\langle x-\varepsilon\right\rangle_\nu|0\rangle\\ &= -\left[\frac{3}{8\pi Gu^2\left\langle\rho_{\langle t-\varepsilon\rangle_0}\right\rangle}\left(1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right)^2\right]du^2.\end{aligned} \tag{59}$$

Technical reasons given below have proven to be convenient to manipulate the right-hand side with the metric, which led us to write

$$\begin{aligned}&\langle 0|\,g_{\mu\nu}d\left\langle x-\varepsilon\right\rangle_\mu d\left\langle x-\varepsilon\right\rangle_\nu|0\rangle\\ &= -g_{\mu\nu}\left[\frac{3}{8\pi Gu^2\left\langle\rho_{\langle t-\varepsilon\rangle_0}\right\rangle g_{\mu\nu}}\left(1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right)^2\right]du^2.\end{aligned} \tag{60}$$

It is a fact that, when discussing the solution of the cosmological constant problem, sometimes one states that certain symmetry requirements in quantum field theory imply that the energy-momentum tensor in vacuum must match a constant times the metric tensor [3], the constant being equal to the expectation value of ρ, since it must have the dimension of an energy density [39–41]. Thereby, within this view, equation (60) leads to the final expression with the expectation value of the energy-momentum tensor

$$\langle 0| g_{\mu\nu} d\left\langle x-\varepsilon\right\rangle_{\mu} d\left\langle x-\varepsilon\right\rangle_{\nu} |0\rangle = -g_{\mu\nu}\left[\frac{3}{8\pi G u^2 \langle 0|\hat{T}_{\mu\nu}|0\rangle}\left(1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right)^2\right]du^2. \quad (61)$$

If we take the approach from loop quantum gravity, maintaining the conditions of isotropy and homogeneity previously assumed, the Friedmann equation becomes

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\left\langle \rho_{\langle t-\varepsilon_0\rangle}\right\rangle\left(1-\frac{\left\langle \rho_{\langle t-\varepsilon_0\rangle}\right\rangle}{\rho_{Pl}}\right), \quad (62)$$

where $\rho_{Pl} \sim 10^{96}$ kg/m^3 is the Planck density [8], [13]. Then,

$$\frac{3H^2}{8\pi G\left(1-C(u)\right)}\left[1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right]^2 = \left\langle \rho_{\langle t-\varepsilon_0\rangle}\right\rangle\left(1-\frac{\left\langle \rho_{\langle t-\varepsilon_0\rangle}\right\rangle}{\rho_{Pl}}\right). \quad (63)$$

From the last equation, we see that when the matter in the Universe has a density ρ negligible compared to the Planck density, the parenthesis is close to 1, and correction is insignificant. But near the Big Bang, density ρ becomes very high and close to ρ_{Pl}; the right term is therefore null as the derivative $\dot{R}$. There is no expansion or contraction, and the density of matter cannot grow beyond the Planck density. In this particular case,

$$1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du} = 0, \quad (64)$$

$$\frac{dA(u)}{1+A(u)} = 2\frac{du}{u}, \quad (65)$$

from which

$$\int\frac{dA(u)}{1+A(u)} = 2\int\frac{du}{u}. \quad (66)$$

As the integration entails a constant, it seems to be useful to label function $A(u)$ with some reference dimensionless parameter (still to be interpreted), in order to dispose, for possible deviations from the generalized solution, a host of scenarios. Then, function $A(u)$, near the Big Bang era, must fulfill

$$\int\frac{dA(u)}{1+A(u)} = 2\ln u + k. \quad (67)$$

[3] Applying the principle of general covariance, this is valid also in the general case where $g_{\mu\nu}$ describes a real gravitational field with non-vanishing Riemann tensor components.

The integration by parts leads to

$$\frac{A(u)}{1+A(u)}+\int\frac{A(u)}{(1+A(u))^2}dA(u)=2\ln u+k, \tag{68}$$

with the simple solution

$$A(u)=e^{2\ln u+k-1}-1. \tag{69}$$

Again, to avoid any confusion, we can observe throughout this deduction that no appeal was made to perturbative methods, and expression (49) is really a non-perturbative result, unless we want to make it perturbative; there is nothing that obligate us to assume functions $A(u)$ and $C(u)$ as evaluated perturbatively. Therefore, we compare quantum spacetime with quantum Riemannian metric to establish the conditions for further precise evaluation of $A(u)$ and $C(u)$ in present model; this is the same as to ask what functions $A(u)$ and $C(u)$ relate quantum spacetime with quantum Riemannian metric, measuring the spacetime shrinkage rate. After Wheeler, we say that the geometry of spacetime fluctuates and "saying that geometry fluctuates is the same as saying that gravity fluctuates" [22]; looking at equations (60) and (62), we complete saying that Riemannian metric fluctuates as the physical state of spacetime itself. As we have observed, Woodard rescues Green's retarded functions in their formal considerations [12], but in no way he claims an evaluation exclusively non-perturbative. The whole problem then is to manipulate $C(u)=G_C^{ret}[3a+c](u)$ and $A(u)=-4G_A^{ret}[a](u)+G_C^{ret}[3a+c](u)$ to give something useful[4], and such handing are specific to the particular geometry of our problem fixed by $\langle 0|\, g_{\mu\nu}d\langle x-\varepsilon\rangle_\mu d\langle x-\varepsilon\rangle_\nu\,|0\rangle=\Omega^2\left\{-\left[1-C(u)\right]du^2\right\}$.

6 Conclusion

From the foregoing discussion, we recognize that gravity, at least apparently, becomes more complicated at short distances and this compels us to explore all possibilities. A pleonastic look at the most recent studies, however, shows that there was not much progress in this area in the last twenty years. In addition, unfortunately many works that come to public about quantum gravity are mere mathematical exercises, with no much to add from the physical point of view. With so many open issues in this field, where no unifying methodical framework exists, this article presented and discussed possible gravitational aberrations which can behave like time machines based on two theoretical assumptions: 1) the existence of G-closures, regions where spacetime shrinks by gravitational pressure, and 2) the acceptance that in any such regions spatial displacement is counterbalanced or even overtaken by the shrinkage. These G-closures

[4] A few ongoing tries show a variety of function shapes generated by particular boundary conditions.

run as one-dimensional time machines, that is, do not require spatial displacement. Also, with the aid of singularity functions, present work discussed how can we understand the introduction of quantum concepts in gravitation, explaining a way of measuring the invariant element only in time. We conclude that, under strong gravitational compression, time expands as well as space shrinks (this conclusion is of course consistent with results from general and special relativity, including the well-known expression $l = l'\sqrt{1 - v^2/c^2}$), and in such circumstances there is no real displacement in space. Since in quantum gravity everything is very uncertain regarding the Planck scale, and being the physics of black holes and wormholes lacking of more definitive observational verifications, we see present model as phenomenological for now. However, it brings the advantage of establishing some reasonable physical predictions about the spacetime behavior under the intense gravitational compression of two supermassive bodies, and introduces an original way to adjust quantum spacetime with quantum Riemannian metric in accordance with Einstein's field equations. Lastly, the search for the appropriate functions A and C has been one of our major concerns.

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