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Isotropic Background for Interacting Two Fluid Scenario Coupled with Zero Mass Scalar Field in Modified Gravity

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Abstract. The modified theories of gravity have engrossed much attention in the last decade, especially $f(R)$gravity. In this contextual exploration, we investigate interaction between barotropic fluid and dark energy with zero-mass scalar field for the spatially homogeneous and isotropic flat FRW universe. In this universe, the field equations correspond to the particular choice of $f(R) = R + bR^m$. The exact solutions of the field equations are obtained by applying volumetric power law and exponential law of expansion. In power and exponential law of expansion, the universe shows both matter dominated and DE era for $b \leq 0$ and $b \geq 0$ and remain present in dark era respectively, but power law model is fully occupying with real matter for $b > 0$ and for $b < 0$ exponential model expands with negative pressure and remain present in matter dominated phase respectively. The physical behavior of the universe has been discussed by using some physical quantities.

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1 Introduction

Recent cosmological observations indicating that our universe experiences an accelerated expansion. The accelerating expansion of the universe is driven by mysterious energy with negative pressure known as Dark Energy (DE). The evidence of the existence of DE comes from the Supernova observations [1,2] and other observations, such as cosmic microwave background (CMB) anisotropy measured with WMAP satellite [3] and large-scale structure [4], suggest that nearly two-third of our universe consists of DE, and the remaining consists of relativistic dark matter and baryons [5]. In spite of all the observational evidence, the nature of DE is still a challenging problem in theoretical physics. A very important parameter for the DE investigation is that of the equation of state parameter (EoS). For pressure less dust, radiation and stiff-fluid dominated universe, one can assume the values 0, 1/3, and 1, respectively. DE is usually parameterized by an EoS parameter of the form $\omega = p/\rho$, where p and ρ are the pressure and the density. On/or after Friedman equation, one can see that a value of EoS parameter $\omega < -1/3$ is required for accelerated cosmic expansion. Possibilities of evolving EoS parameter were also explored in many dynamical DE models. Primary candidates in this category are scalar field models, such as Quintessence [6-9] and K-essence [10-12]. A common example to quintessence is the energy of a slowly evolving scalar field that has not yet reached the minimum of its potential $V(\phi)$, similar to the inflation field used to describe the inflationary phase of the universe. In quintessence models the range of EoS parameter is $-1 < \omega < -1/3$, and the DE density decreases by a scale factor $a(t)$ as $\rho \propto a^{-3(1+\omega)}$ [13]. A specific exotic form of DE denoted phantom energy, with $\omega < -1$, has also been proposed in [14,15].

The alternative way to explain cosmic acceleration is to anticipate that large scale dynamics of the Universe is not governed by Einstein's equations. In a classical generalization of general relativity (GR) one replaces the Ricci scalar R in the Einstein-Hilbert action by an arbitrary function of R of the well-known $f(R)$ modified gravity. These models consist of higher order curvature invariants as functions of the Ricci scalar. Carroll et al. [16] have explained the presence of a late time cosmic acceleration of the universe in $f(R)$ gravity. Viable $f(R)$ gravity models have been proposed by Nojiri and Odintsov [17] showing the unification of early-time inflation and late-time acceleration. Capozziello et al. [18] achieve both phases – the dust matter and DE phases – by deriving the exact solution from a power law $f(R)$ cosmological model. Some other useful aspects of $f(R)$gravity inspected by Nojiri and Odintsov [19]. Exact solutions of static spherically symmetric space-times in $f(R)$ gravity coupled to non-linear electrodynamics has been analyzed by Hollenstein and Lobo [20]. Using the same theory Azadi et al. [21] have studied vacuum solution in cylindrically symmetric space time. Standard cosmological evolution in a wide range of $f(R)$ gravity have been investigated by Evans et al. [22]. Miranda et al. [23] have discussed a viable singularity-free $f(R)$ gravity without a cosmological constant. Sharif and Yousaf [24] have studied the impact of DE and dark matter models on the dynamical evolution of collapsing self-gravitating systems in this gravity.

In studies of cosmological models with DE, DE and matter are usually viewed as independent substances so that the energy-momenta of the partial fluids are independently conserved. Earlier, Xin [25] has studied an interacting two-fluid scenario for quintom DE. Xin-He et al. [26] have considered Friedman cosmology with a generalized EoS and bulk viscosity to explain DE dominated universe. The tachyon cosmology in interacting and non-interacting cases in non-flat FRW Universe has been studied in [27]. Recently, an interacting and non-interacting two-fluid scenario for DE models in FRW universe has been examined by Amirhashchi et al. [28] and Pradhan et al. [29].

The study of interacting fields, one of them being zero-mass scalar field, is a fundamental challenge to look into the yet unsolved problem of the unification of the gravitational and quantum theories. In the last few decades there has been renewed concern focused on the theory of gravitation signifying zero-mass scalar field coupled with gravitational field [30,31]. Maniharsingh [32], Singh and Bhamra [33] and Singh [34,35] have studied different one-fluid models coupled with a scalar field. Singh and Deo [36] have investigated the problem of zero-mass scalar field interactions in the presence of a gravitational field for FRW space-time in general relativity and showed that the 'Big-Bang' of the universe at the initial stage can be avoided by introducing a zero-mass scalar field; along with this some authors [37,38] have investigated cosmological model with zero-mass scalar field.

Incited by above discussions, in this paper, we study interaction between barotropic fluid and DE with zero-mass scalar field for the spatially homogeneous and isotropic flat FRW universe by applying volumetric power law expansion and exponential expansion. In doing so, we consider an interacting case. This paper is organized as follows: In Section 2, we describe a brief review of the $f(R)$ theory. In Section 3, the metric and the basic equations are described. Section 4, deals with the solutions of the field equations with power law and exponential law expansion for interacting two-fluids. Finally, conclusions are summarized in the last Section 5.

2 A Brief Review of the $f(R)$ Gravity

The $f(R)$ theory of gravity is a generalization of the general relativity. The three main approaches in the $f(R)$ theory of gravity are: (i) metric approach; (ii) Palatine formalism; and (ii) affine $f(R)$ gravity'. In metric approach, the connection is the Levi-Civita connection and variation of the action is done with respect to the metric tensor. While, in Palatine formalism, the metric and the connection are independent of each other and variation is done for the two mentioned parameters, independently. In metric-affine $f(R)$ gravity, both metric tensor and connection are treated independently and it is assumed that the matter action depends on the connection as well.

The action for this theory is given by

$$S = \frac{1}{2k^2}\int d^4\sqrt{-g}f(R) + \int d^4xL_m(g_{\mu\nu}, \phi_m). \tag{1}$$

Here, $f(R)$ is a general function of the Ricci scalar, $k^2 = 8\pi G = 1g$ is the determinant of the metric $g_{\mu\nu}$, and L_m is the metric Lagrangian that depends on $g_{\mu\nu}$ and the matter field ϕ_m.

It is noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein–Hilbert action.

The corresponding field equations are found by varying the action with respect to the metric $g_{\mu\nu}$

$$F(R)\,R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_{\mu}\nabla_{\nu}F(R) + g_{\mu\nu}\Box F(R) = T^{M}_{\mu\nu}, \tag{2}$$

where $\Box \equiv \nabla^{\mu}\nabla_{\mu}$,

$$F(R) \equiv \frac{df(R)}{dR}\,, \tag{3}$$

∇_{μ} is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

3 Metric and Field Equations

We consider the spatially homogeneous and isotropic flat Friedman-Robertson-Walker (FRW) universe of the form

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2], \tag{4}$$

where a is the metric potential or the scale factor and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$.

The corresponding Ricci scalar for the universe (4) is given by

$$R = 6\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right], \tag{5}$$

where overhead dot represents derivative with respect to time t.

The energy-momentum tensor due to the barotropic fluid, DE and zero-mass scalar field is taken as

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu} + \Big(\psi_{\mu}\psi_{\nu} - \frac{1}{2}g_{\mu\nu}\psi_{,m}\psi^{,m}\Big), \tag{6}$$

together with

$$u^{\mu}u_{\mu} = -1, \tag{7}$$

where u^{μ} is the four velocity vector, p, ρ and ψ are the isotropic pressure, the energy density and the zero-mass scalar field, respectively.

$$p = (p_m + p_d), \qquad \rho = (\rho_m + \rho_d).$$

p_m and ρ_m are the pressure and the energy density of barotropic fluid, p_d and ρ_d are the pressure and the energy density of DE, respectively. Also, $p_m = w_m\rho_m$ and $p_d = w_d\rho_d$.

The scalar field ψ satisfies the equation

$$\psi^{\mu}_{;\mu} = 0. \tag{8}$$

In the co-moving co-ordinate system, we have from equation (6)

$$T_1^1 = T_2^2 = T_3^3 = p + \frac{1}{2}\dot{\psi}^2\,, \quad T_4^4 = -\rho - \frac{1}{2}\dot{\psi}^2\,, \quad T_\nu^\mu = 0\,, \quad \mu \neq \nu. \quad (9)$$

From the equation of motion (2), the Friedman equation for two fluid scenarios with zero-mass scalar field reduces to the following set of equations

$$\left(2\frac{\dot{a}^2}{a^2}\right)F - \frac{1}{2}f(R) - 2\frac{\dot{a}}{a}\dot{F} - \ddot{F} = p + \frac{1}{2}\dot{\psi}^2\,, \quad (10)$$

$$3\frac{\ddot{a}}{a}F - \frac{1}{2}f(R) - 3\frac{\dot{a}}{a}\dot{F} = -\rho - \frac{1}{2}\dot{\psi}^2, \quad (11)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = 0. \quad (12)$$

The overhead dot represents the differentiation with respect to time t.

Now we define some parameters for the universe, which are important in cosmological observations.

We define average scale factor and spatial volume, respectively as:

$$a = \sqrt[3]{V}\,, \quad V = a^3\,; \quad (13)$$

the generalized Hubble parameter,

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{\dot{a}}{a}\,; \quad (14)$$

the mean anisotropy parameter,

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \quad (15)$$

The expansion scalar and shear scalar are defined as

$$\theta = u^\mu_{;\mu} = 3H = 3\frac{\dot{a}}{a}\,, \quad (16)$$

$$\sigma^2 = \frac{3}{2}A_m H^2\,; \quad (17)$$

the deceleration parameter,

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \quad (18)$$

For any physically relevant model, the Hubble parameter H and deceleration parameter q are the most important observational quantities in cosmology. The deceleration parameter q measures the rate of expansion of the universe. The sign of q indicates the state of expanding universe. If $q < 0$ it represents inflation (expansion of the universe is accelerating); for $q > 0$ it represents deflation (expansion of the universe is decelerating); while $q = 0$ shows that universe increases with constant velocity.

4 Solution of the Field Equations

In order to solve the field equations completely, we first assume that the barotropic fluid and DE components interact minimally. Therefore, the energy momentum tensors of the two sources may be conserved separately.

The energy conservation equation $(T^{\mu\nu}_{;\nu} = 0)$ of the barotropic fluid leads to

$$(\dot{\rho}_m) + 3\frac{\dot{a}}{a}(\rho_m + p_m) = 0\,, \tag{19}$$

whereas the energy conservation equation $(T^{\mu\nu}_{;\nu} = 0)$ of the DE component yields

$$(\dot{\rho}_d) + 3\frac{\dot{a}}{a}(\rho_d + p_d) = 0. \tag{20}$$

We assume that the EoS parameter of the perfect fluid to be a constant (which is considered by Akarsu [39] and Kumar [40])

$$\omega_m = \frac{p_m}{\rho_m} = \text{const.}\,, \tag{21}$$

while w_d has been admitted to be a function of time t.

Since, the set of field equations (10)– 12) are coupled system of highly nonlinear differential equations containing six unknowns, namely a, ρ, p, F, f, ψ. Thus, we can introduce conditions to obtain unique solutions of the field equations. The solutions to the field equations are generated by using two different forms of volumetric expansion laws: (i) power law; and (ii) exponential law [41]

$$V = c_1 t^{3k}, \tag{22}$$

$$V = c_2 e^{3m_1 t}, \tag{23}$$

where c_1, c_2, k, and m are constants.

The model describes accelerating volumetric expansion with power law and exponential law for $k > 1$, and for $k < 1$ the model exhibit a decelerating volumetric expansion.

4.1 Model with power law expansion

In this section we discussed the acts of interaction between barotropic fluid and DE using power law expansion with the changing aspects of physical behavior of universe.

In this case, the energy densities of DE and matter no longer satisfy independent conservation laws, they obey instead

$$(\dot{\rho}_m) + 3\frac{\dot{a}}{a}(\rho_m + p_m) = Q\,, \tag{24}$$

$$(\dot{\rho}_d) + 3\frac{\dot{a}}{a}(\rho_d + p_d) = -Q\,. \tag{25}$$

The quantity Q ($Q > 0$) expresses the interaction term between the DE and the barotropic matter components. It should be noted that the ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction *since* we are interested to investigate the interaction between DE and matter. In our work we consider the interaction term in the form of $Q \propto H\rho_m$, which is already well-thought-out by Saha [42] and Amirhashchi [43]

$$Q = 3H\sigma\rho_m \,, \tag{26}$$

where σ is the coupling coefficient which can be considered as a constant.

Using equation (24) and (26), we obtain an energy density of barotropic fluid as

$$\rho_m = (t)^{-3k(1+w_m-\sigma)} \,. \tag{27}$$

The physical parameters of the universe, such as energy density, isotropic pressure, and the equation of state parameter of dark fluid in interacting two fluid models with power law expansion are:

(i) Energy density,

$$\rho_d = \frac{6k(2k-1)}{2t^2} - \frac{\alpha_1\left[6k(2k-1)\right]^{m-1}}{t^{2m}} - \frac{1}{2t^{6k}} - \frac{1}{t^{3k(1+\omega_m-\sigma)}}. \tag{28}$$

In power law expansion of the universe, it is observed that the energy density is always positive and decreasing function of time t. At the initial stage $t \to 0$ the universe has infinitely large energy density $\rho \to \infty$ but with the expansion of the universe it declines and at large $t \to \infty$ it is null $\rho \to 0$. Thus, our derived universe is free from big rip. This behavior is clearly shown in Figure 1, as a representative case with appropriate choice of constants and other parameters using reasonably well known situations.

(ii) Isotropic pressure,

$$p_d = \frac{\alpha_2\left[6k(2k-1)\right]^{m-1}}{t^{2m}} - \frac{[3k(2k-1)]}{t^2} - \frac{1}{2t^{6k}} - \frac{\omega_m}{t^{3k(1+\omega_m-\sigma)}}. \tag{29}$$

From equation (29), it is observed that in the power law expansion of the universe, at the initial epoch $t \to 0$, when the universe starts to expand, the fluid pressure is infinitely large $p \to \infty$ throughout the universe and decreases with the expansion of the universe and at $t \to \infty$, $p \to 0$.

(iii) EoS parameter,

$$\omega_d = -\frac{\dfrac{\alpha_2\left[6k(2k-1)\right]^{m-1}}{t^{2m}} - \dfrac{[3k(2k-1)]}{t^2} - \dfrac{1}{2t^{6k}} - \dfrac{\omega_m}{t^{3k(1+\omega_m-\sigma)}}}{\dfrac{\alpha_1\left[6k(2k-1)\right]^{m-1}}{t^{2m}} - \dfrac{[6k(2k-1)]}{2t^2} + \dfrac{1}{2t^{6k}} + \dfrac{1}{t^{3k(1+\omega_m-\sigma)}}}. \tag{30}$$

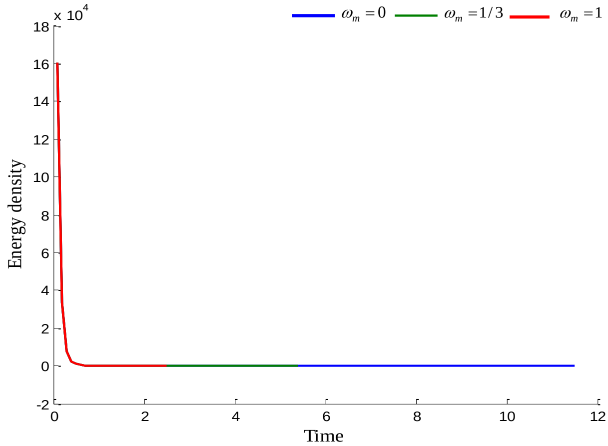

Figure 1. Energy density versus time t with the appropriate choice of constant $k = 1$, $m > 0(2)$, $b < 0(-1)$.

From equation (30), we observed that the equation of state parameter of DE (ω) is time dependent. The graphical behavior of EoS parameter versus time t is shown in Figure 2. At the initial stage when the universe started to expand, the EoS of the universe has value of $\omega > 0$, i.e. the model behaves as like matter was dominated once at early stages, while at late times it becomes $\omega < 0$. At late time the EoS parameter varies from phantom $\omega < -1$ region to quintessence $\omega > -1$ region; this is a situation in early universe, where the phantom field

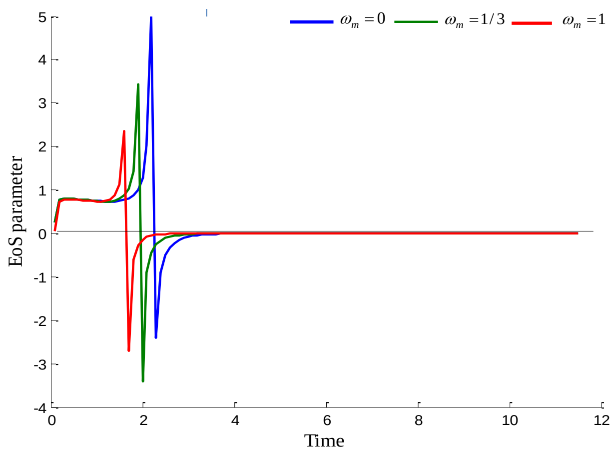

Figure 2. Equation of state parameter of DE versus time t in power law expansion with the appropriate choice of constant $k = 1$, $m > 0(2)$, $b < 0(-1)$.

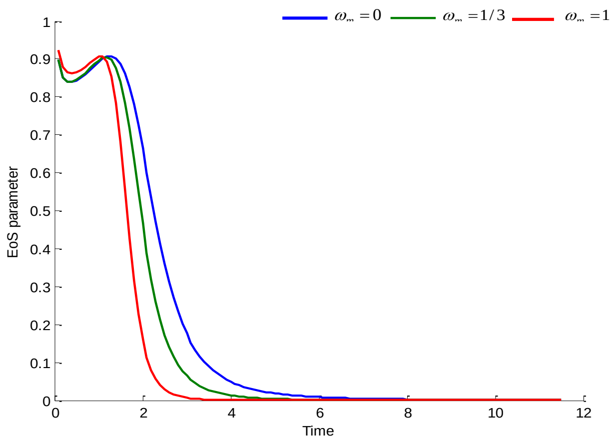

Figure 3. Equation of state parameter of DE versus time t in power law expansion of the universe with the appropriate choice of constant $k = 1$, $m > 0(2)$, $b > 0(1)$.

dominated universe may be playing an important role of the EoS parameter while for whole interval of time it remain present in quintessence region $\omega > -1$. It is interesting to note that EoS takes a negative value, which is an acceptable form found within Supernova observations.

Also, an interesting feature in power law expansion is observed that for $b \leq 0$ the universe shows both matter dominated and DE era, but for $b > 0$ it is fully occupying the real matter and there is no chance of DE. This behavior is shown in Figure 3.

Kinematical parameters of the universe: power law expansion:

The spatial volume and the average scale factor $$V = a^3 = c_1 t^{3k}.$$

The generalized Hubble parameter, $$H = \frac{k}{t}.$$

The expansion scalar, $$\theta = \frac{3k}{t}.$$

Deceleration parameter, $$q = \frac{1-k}{k}.$$

The mean anisotropy parameter, $$A_m = 0.$$

The shear scalar, $$\sigma^2 = 0.$$

We observe that the spatial volume is zero at $t \to 0$. Thus, the singularity exists at an initial stage of the evaluation of the universe, i.e. the universe starts evolving with a big-bang at $t \to 0$. The expansion scalar decreases with the expansion of the universe. Also the generalized Hubble parameter is initially large at $t \to 0$, and null at $t \to \infty$. The expansion scalar $\theta \to 0$ as $t \to \infty$

indicates that the universe is expanding and the rate of expansion decreases with increase of time. Anisotropic parameter and shear scalar comes out to be zero, hence the universe does not approach anisotropy and the universe is shear free.

Also it is observed that the deceleration parameter q comes out to be constant and it depends on the value of n. For $n > 1$ the sign of q becomes positive which correspond to the standard decelerating behavior whereas for $n < 1$ the sign of q becomes negative which correspond to the standard accelerating behavior of the universe.

4.2 Model with exponential law expansion

In this section we discussed the pieces of interaction between barotropic fluid and DE using exponential law of expansion with the changing aspects of physical behavior of universe.

Using equation (26) and (24), we obtain an energy density of barotropic fluid for interacting case

$$\rho_m = (t)^{-3k(1+w_m-\sigma)}. \tag{31}$$

The physical parameters of the universe, such as energy density, isotropic pressure and EoS parameter of DE, in interacting two fluid models with exponential expansion are

(i) Energy density,

$$\rho_d = \alpha_3 - \frac{1}{2e^{6m_1 t}} - \frac{1}{t^{3k(1+\omega_m-\sigma})}. \tag{32}$$

In an exponential expansion of the universe, it is observed that the energy density of DE is always positive and decreasing function of time t. From equation (32), it is concluded that at the initial stage of the universe the energy density is infinitely large, i.e. $\rho \to \infty$ and with the expansion of the universe it decreases and at large expansion it is null, i.e. $\rho \to 0$. Thus, our derived universe is free from big rip.

(ii) Isotropic pressure,

$$p_d = \alpha_4 - \frac{1}{2e^{6m_1 t}} - \frac{\omega_m}{t^{3k(1+\omega_m-\sigma)}}. \tag{33}$$

It is observed that, in the exponential expansion of the universe at the initial epoch $t \to 0$when universe start to expand the fluid pressure is infinitely large throughout the universe $p \to \infty$ and decreases with the expansion of the universe while at infinite expansion$t \to \infty$,$p \to 0$.

(iii) EoS parameter,

$$\omega_d = \frac{\alpha_4 - \frac{1}{2e^{6m_1 t}} - \frac{\omega_m}{t^{3k(1+\omega_m-\sigma)}}}{\alpha_3 - \frac{1}{2e^{6m_1 t}} - \frac{1}{t^{3k(1+\omega_m-\sigma})}}. \tag{34}$$

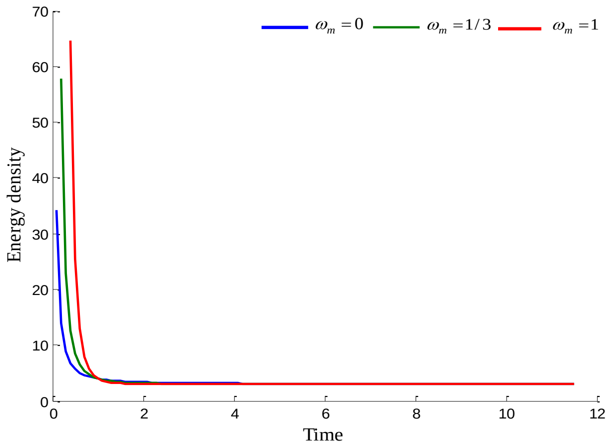

Figure 4. Energy density versus time t in an exponential expansion of the universe with the appropriate choice of constant $k = 1$, $m > 0(2)$, $b < 0(-1)$, $m_1 = 0.5$.

Equation (34) represents the EoS parameter of DE in an exponential expansion which is a function of time t. The graphical behavior of EoS parameter versus time t is shown in Figure 5. At the initial stage when the universe start to accelerate for small interval of time the EoS parameter of the universe having value $\omega < 0$ while for some interval of time it is $\omega > 0$ and for whole interval of time it is $\omega < 0$, i.e. in an exponential expansion the universe expand with quintessence $\omega > -1$ region, for some interval it behave as like matter dominated once while

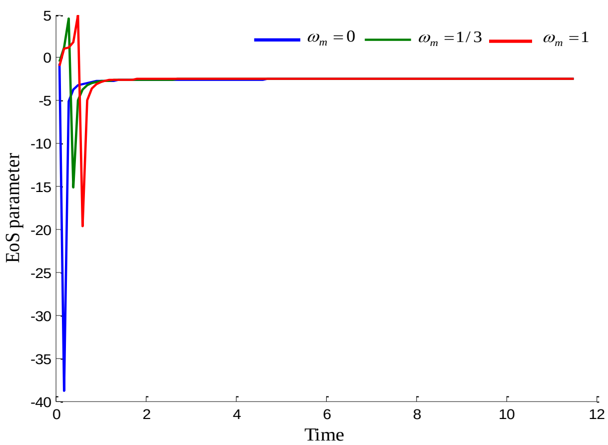

Figure 5. Equation of state parameter of DE versus time t in an exponential expansion of the universe with the appropriate choice of constant $k = 1$, $m > 0(2)$, $b \geq 0(1)$, $m_1 = 0.5$.

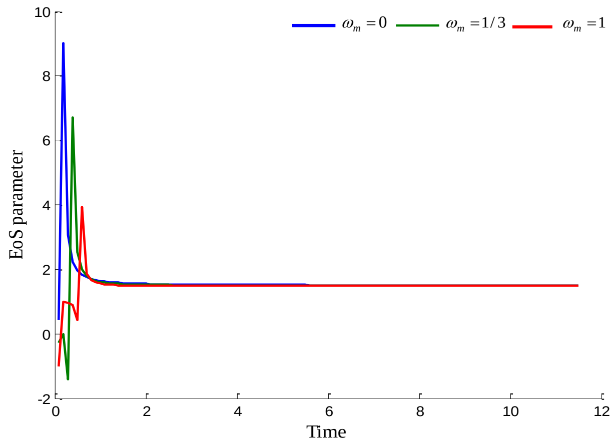

Figure 6. Equation of state parameter of DE verses time $t$in an exponential expansion of the universe with the appropriate choice of constant $k = 1$, $m > 0(2)$, $b < 0(-1)$, $m_1 = 0.5$.

at late times it remain present in Phantom $\omega < -1$ region, this is a situation in early universe where the quintessence field dominated universe may be playing an important role of the EoS parameter. It is interesting to note that EoS parameter takes a negative value which is an acceptable value observed by Supernova data.

Also, an interesting feature in our investigation is observed that for $b \geq 0$ the universe shows matter dominated and DE era and remain present in dark era but for $b < 0$ universe expands with negative pressure and remains present in matter dominated phase. The behavior is shown in Figure 6.

Kinematical parameters of the universe: exponential expansion

The spatial volume and the average scale factor as $V = a^3 = c_2 e^{3m_1 t}$.

The generalized Hubble parameter, $H = m_1$.

The expansion scalar, $\theta = 3m_1$.

The deceleration parameter, $q = -1$.

The mean anisotropy parameter, $A_m = 0$.

The shear scalar, $\sigma^2 = 0$.

We observe that the spatial volume approaches to positive small value at $t \to 0$ and with the expansion of the time the universe expands exponentially. The results of generalized Hubble's parameter yield the constant values in an exponential expansion.

The value of expansion scalar is constant and the sign of deceleration parameteris negative which shows that the accelerating expansion of the universe as we expect in this exponential law. The mean anisotropy parameter and shear scalar are zero and so the universe is isotopically same in all directions and the model is shear less.

5 Conclusions

Spatially homogeneous and isotropic two fluids model with zero-mass scalar field have been investigated in $f(R)$ theory of gravitation. The models have been studied in relation to volumetric power law and exponential law expansion. Our derived universe exhibited both accelerating as well as decelerating phase, in both phases universe is expanding and the expansion of the universe is much faster and then slows down for later time.

In power law expansion, the universe starts with zero volume, the Hubble parameter and the scalar expansion are the functions of time and initially $t \to 0$ they attain infinitely large value and decreases with expansion and approaches to zero at large expansion, the EoS parameter ω is a time dependent and for small interval, it behaves like matter dominated once while at late time it remain present in quintessence region $\omega > -1$. Hence, our investigations are supported to the observational fact that the usual matter described by known particle theory is about 4% and the DE cause the accelerating expansion of the universe and several high precision observational experiments, especially the WMAP satellite experiment (DE occupies near about 73% of the energy of the universe and dark matter is about 23% for accelerating expansion) [3].

In an exponential expansion the universe starts with constant volume for $c_2 > 0$ and for $c_2 = 0$ it starts with zero volume and expands exponentially with infinite volume. The expansion scalar and the shear scalar are constant. EoS parameter ω is time dependent, the universe expands with quintessence $\omega > -1$ region, for some interval it behaves like matter dominated once, while at late times it remain present in phantom $\omega < -1$ region.

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