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Anisotropic and Homogeneous Cosmological Models with Polytropic Equation of State in General Relativity

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Abstract. In this paper, we have studied the anisotropic and homogeneous Bianchi type models filled with perfect fluid obeying the polytropic equation of state (EoS). We have considered the Hybrid expansion law for obtaining the exact solutions of the Einstein's field equations. The different physical and geometrical properties of the models have been obtained and discussed.

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1 Introduction

The recent observations of supernovae Ia (SNe Ia) experiments [1,2], large scale structures [3], cosmic microwave background [4,5], WMAP [6], weak lensing [7], have verified that the universe is undergoing a phase of accelerated expansion. This phenomenon is considered due to the presence of the dark energy component occupying 68.3% of the universe ruling the other parts of the universe [i.e. dark matter component (26.8%) and baryonic matter (4.9%)] as shown by PLANCK 2013 [8].

The equation of state is a barotropic relation, i.e. a relation expressing pressure of the model in terms of the density of the model, i.e. $p = f(\rho)$. EoS parameters are considered quite useful in obtaining a better understanding about the dark energy and its physical aspects [9]. Different EoS have attracted the attention of the researchers over a span of the last 2-3 decades. Nojiri and Odintsov [10] have studied different EoS in the presence of dark energy. Linder and Scherrer [11], Ananda and Bruni [12,13], Bamba *et al.* [14], Reddy *et al.* [15], Singh and Bishi [16] have included linear and quadratic EoS in their cosmological studies.

Mukhopadhyay and Ray [17] have studied polytropic equation of state as an alternative for describing the accelerated expansion of the universe, i.e. a model describing the universe with all its aspects in the absence of dark energy. The

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general form of the polytropic EoS can be written as $p = \alpha\rho + K\rho^n$ which is the sum of a linear EoS $p = \alpha\rho$ and a polytropic component $p = K\rho^n$. For the positive indices $(n > 0)$, in the early universe when the energy density is remarkably high, the polytropic term dominates the linear term and decides the dynamics of the universe, whereas in the late universe when the energy density of the universe is quite low, the linear term prevails the polytropic term and the evolution of the universe is determined by the linear term [18]. For the negative indices $(n < 0)$, the cases are exactly crosswise i.e. for the high density early universe, the linear term is in the ruling position, while for the low density late universe, the polytropic term surpasses the linear term. Also, the positive pressure $(K > 0)$ generates past or future singularities while negative pressure $(K < 0)$ precipitates to exponential expansion in the past or future. A more detailed study of the polytropic EoS with all the possible cases has been done by Chavanis [19-22].

From the observational data, it is well known that our universe is homogeneous and isotropic on a large scale, however, no physical evidence denies the chances of an anisotropic universe. In fact, theoretical arguments are present promoting the existence of an anisotropic phase of the universe that approaches the isotropic phase [23-26]. The anisotropy plays a vital role in the early phase of evolution of the universe and so studying homogeneous and anisotropic cosmological models is considered important. The Bianchi type models are spatially homogeneous and are in general anisotropic. The simplicity of the field equations made Bianchi type space-time useful in constructing models which were spatially homogeneous and anisotropic. Rahman and Ansari [27] have investigated interacting holographic polytropic gas model of dark energy with hybrid expansion law in Bianchi type VI_0 space-time. Vijaya Santhi *et al.* [28] have recently studied Bianchi type III, V and VI_0 models with generalized ghost pilgrim dark energy. Also, Aditya *et al.* [29] and Adhav *et al.* [30-33] have investigated Bianchi type models in different contexts.

In the following paper, we have investigated Bianchi type I, III, V & VI_0 models with polytropic EoS and hybrid expansion law and deduced various physical and geometrical properties describing the models.

2 Metric and Field Equations

We consider spatially homogeneous Bianchi type metrics in the general form

$$ds^2 = dt^2 - a_1^2 dx^2 - e^{2Ax} a_2^2 dy^2 - e^{2Bx} a_3^2 dz^2 \quad , \tag{1}$$

where a_1, a_2, a_3 are functions of time t only.

It represents

Bianchi-type I if $A = B = 0$,

Bianchi-type III if $A = -\alpha$ and $B = 0$,

Bianchi-type V if $A = B = -\alpha$,
Bianchi-type VI_0 if $A = -\alpha^2$ and $B = \alpha^2$,
where $\alpha \neq 0$ is constant.

The Einstein's field equations in natural limits ($8\pi G = 1$ & $c = 1$) are

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \,, \tag{2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar and T_{ij} is the energy momentum tensor.

The energy momentum tensor T_{ij} for the perfect fluid is given by

$$T_{ij} = (p + \rho)\, u_i u_j - p g_{ij} \,, \tag{3}$$

where ρ is the energy density, p is the pressure and u^i is the four velocity vector satisfying $g_{ij} u^i u^j = 1$.

The above perfect fluid obeys the polytropic equation of state given by

$$p = K \rho^n - \rho \,, \tag{4}$$

where K and n are constants known as polytropic constant and polytropic index respectively.

The energy conservation equation $T^{ij}_{;j} = 0$ leads to the following simpler expression:

$$\dot{\rho} + \frac{\dot{V}}{V}(p + \rho) = 0 \,, \tag{5}$$

where $V = a_1 a_2 a_3$ is the spatial volume of the universe.

Integrating above equation, we get

$$\rho = \gamma (\ln V)^{1/(1-n)} \,, \tag{6}$$

where $\gamma = [K(n-1)]^{1/(1-n)}$.

The directional Hubble parameters are given by

$$H_x = \frac{\dot{a}_1}{a_1} \,, \qquad H_y = \frac{\dot{a}_2}{a_2} \,, \qquad H_z = \frac{\dot{a}_3}{a_3} \,. \tag{7}$$

The mean Hubble parameter H and expansion scalar θ are given by

$$H = \frac{1}{3}\theta = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_x + H_y + H_z) \,. \tag{8}$$

The anisotropy parameter Δ of the expansion and shear scalar σ are defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^{3} \Big(\frac{H_i - H}{H} \Big)^2 , \tag{9}$$

$$\sigma^2 = \frac{1}{2} \Big(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2 \Big) . \tag{10}$$

In a co-moving coordinate system, the Einstein's field equations (2) for the metric (1) with the help of equations (3) reduce to the following set of equations:

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_1\dot{a}_3}{a_1a_3}-\frac{(A^2+AB+B^2)}{a_1^2}=\rho\,, \tag{11}$$

$$\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}-\frac{AB}{a_1^2}=-p\,, \tag{12}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_1\dot{a}_3}{a_1a_3}-\frac{B^2}{a_1^2}=-p\,, \tag{13}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}-\frac{A^2}{a_1^2}=-p\,, \tag{14}$$

$$A\Big(\frac{\dot{a}_2}{a_2}\Big)+B\Big(\frac{\dot{a}_3}{a_3}\Big)=(A+B)\Big(\frac{\dot{a}_1}{a_1}\Big), \tag{15}$$

where overhead dot represents differentiation with respect to time t.

As the above system of equations is highly non-linear in nature, so to obtain the exact solutions of the field equations we consider an Ansatz for mean scale factor a as

$$a=(t^{\beta}e^{t})^{1/\mu}\,, \tag{16}$$

where $\beta>0$ and $\mu\geq 0$ are constants.

This is the Hybrid expansion law which is a combination of exponential and power law. The beauty of this relation is that the obtained deceleration parameter exhibits time dependence which shows the transition of the universe from the initial decelerating phase to the present accelerating phase. So, such a choice of the average scale factor is physically acceptable. Above considerations have also been used by Pradhan and Amirhashchi [34], Yadav [35-36], Pradhan *et al.* [37], Das and Sultana [38] and Vijaya Santhi *et al.* [28].

3 Bianchi type-I Model $[A=B=0]$

For this model, the field equations (11)–(15) reduce to the following set of equations:

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_1\dot{a}_3}{a_1a_3}=\rho\,, \tag{17}$$

$$\frac{\ddot{a}_i}{a_i}+\frac{\ddot{a}_j}{a_j}+\frac{\dot{a}_i\dot{a}_j}{a_ia_j}=-p\,, \tag{18}$$

where $i,\ j=1,\ 2,\ 3$ and $i\neq j$.

Solving the set of field equations and using equation (16), we obtain

$$a_i=D_i(t^{\beta}e^{t})^{1/\mu}\exp[X_iF(t)]\,, \tag{19}$$

where $i = 1,\ 2,\ 3$; D_1, D_2, D_3, X_1, X_2, X_3 are constants of integration satisfying the relations

$$D_1 D_2 D_3 = 1; \qquad X_1 + X_2 + X_3 = 0;$$
$$F(t) = \int t^{-3\beta/\mu} e^{-3t/\mu} dt = -t^{1-3\beta/\mu} \mathrm{E}_{3\beta/\mu}(3t/\mu),$$

where $\mathrm{E}_n(z)$ is the exponential integral function.

The directional Hubble parameters become

$$\begin{aligned} H_x &= \frac{\beta + t}{\mu t} + \frac{X_1}{(t^\beta e^t)^{3/\mu}}\,, \\ H_y &= \frac{\beta + t}{\mu t} + \frac{X_2}{(t^\beta e^t)^{3/\mu}}\,, \\ H_z &= \frac{\beta + t}{\mu t} + \frac{X_3}{(t^\beta e^t)^{3/\mu}}\,. \end{aligned} \tag{20}$$

Thus, the mean Hubble parameter becomes

$$H = \frac{\beta + t}{\mu t}\,. \tag{21}$$

Also, the anisotropy parameter and shear scalar defined by equations (9)–(10) are given by

$$\Delta = \frac{Z \mu^2 t^2}{3(\beta + t)^2 (t^\beta e^t)^{6/\mu}}\,, \tag{22}$$

$$\sigma^2 = \frac{Z}{2(t^\beta e^t)^{6/\mu}}\,, \tag{23}$$

where $Z = X_1^2 + X_2^2 + X_3^2$.

4 Bianchi type-III Model $[A = -\alpha\ ,\ B = 0]$

For this model, the field equations (11)–(15) reduce to the following set of equations:

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{\alpha^2}{a_1^2} = \rho\,, \tag{24}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} = -p\,, \tag{25}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} = -p\,, \tag{26}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}-\frac{\alpha^2}{a_1^2}=-p\,, \tag{27}$$

$$\alpha\Big(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\Big)=0\,. \tag{28}$$

Integrating equation (28), we get $a_1 = c_1a_2$, where the constant of integration c_1 can be ignored finally giving

$$a_1 = a_2\,. \tag{29}$$

As we know that shear σ is proportional to expansion θ, we can write

$$a_3 = a_1^m\,, \tag{30}$$

where $m \neq 1$ is an arbitrary constant.

Using equations (16), (29) and (30), we have

$$a_1 = (t^\beta e^t)^{\frac{3}{\mu(m+2)}}\,, \tag{31}$$

$$a_2 = (t^\beta e^t)^{\frac{3}{\mu(m+2)}}\,, \tag{32}$$

$$a_3 = (t^\beta e^t)^{\frac{3m}{\mu(m+2)}}\,. \tag{33}$$

Thus, the directional Hubble parameters defined by equation (7) take the form

$$H_x=\frac{3(\beta+t)}{(m+2)\mu t}\,,\quad H_y=\frac{3(\beta+t)}{(m+2)\mu t}\,,\quad H_z=\frac{3m(\beta+t)}{(m+2)\mu t}\,. \tag{34}$$

Thus, the mean Hubble parameter is given by

$$H=\frac{(\beta+t)}{\mu t}\,. \tag{35}$$

Also, the anisotropy parameter and shear scalar are given by

$$\Delta=\frac{2(m-1)^2}{(m+2)^2}\,, \tag{36}$$

$$\sigma^2=\frac{3(m-1)^2(\beta+t)^2}{(m+2)^2\mu^2t^2}\,. \tag{37}$$

5 Bianchi type-V Model [$A = B = -\alpha$]

For this model, the field equations (11)-(15) reduce to the following set of equations:

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_1\dot{a}_3}{a_1a_3}-\frac{3\alpha^2}{a_1^2}=\rho\,, \tag{38}$$

$$\frac{\ddot{a}_i}{a_i} + \frac{\ddot{a}_j}{a_j} + \frac{\dot{a}_i \dot{a}_j}{a_i a_j} - \frac{\alpha^2}{a_1^2} = -p \,, \tag{39}$$

$$\frac{2\dot{a}_1}{a_1} = \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \,, \tag{40}$$

where $i,\ j = 1,\ 2,\ 3$ and $i \neq j$.

Integrating equation (40), we obtain $a_1^2 = c_2 a_2 a_3$, where the constant of integration c_2 can be ignored, and we finally get

$$a_1^2 = a_2 a_3 \,. \tag{41}$$

Solving the field equations, using equation (41) and the relation $V = a_1 a_2 a_3$, the directional scale factors are

$$a_1 = (t^\beta e^t)^{1/\mu} \,, \tag{42}$$

$$a_2 = D (t^\beta e^t)^{1/\mu} \exp[X\, F(t)] \,, \tag{43}$$

$$a_3 = \frac{(t^\beta e^t)^{1/\mu}}{D} \exp[-X\, F(t)] \,, \tag{44}$$

where D and X are constants of integration and

$$F(t) = \int t^{-3\beta/\mu} e^{-3t/\mu} dt = -t^{1-(3\beta)/\mu} \mathrm{E}_{3\beta/\mu}(3t/\mu) \,.$$

Thus, the directional Hubble parameters are

$$H_x = \frac{\beta + t}{\mu t} \,, \quad H_y = \frac{\beta + t}{\mu t} + \frac{X}{(t^\beta e^t)^{3/\mu}} \,, \quad H_z = \frac{\beta + t}{\mu t} - \frac{X}{(t^\beta e^t)^{3/\mu}} \,, \tag{45}$$

which gives the mean Hubble parameter as

$$H = \frac{\beta + t}{\mu t} \,, \tag{46}$$

giving the anisotropy parameter and shear scalar as

$$\Delta = \frac{2X^2 \mu^2 t^2}{(\beta + t)^2 (t^\beta e^t)^{6/\mu}} \,, \tag{47}$$

$$\sigma^2 = \frac{X^2}{(t^\beta e^t)^{6/\mu}} \,. \tag{48}$$

6 Bianchi type-VI$_0$ Model $[A = -\alpha^2,\ B = \alpha^2]$

For this model, the field equations (11)–(15) reduce to the following set of equations:

$$\frac{\dot a_1\dot a_2}{a_1a_2}+\frac{\dot a_2\dot a_3}{a_2a_3}+\frac{\dot a_1\dot a_3}{a_1a_3}-\frac{\alpha^4}{a_1^2}=\rho\,, \tag{49}$$

$$\frac{\ddot a_2}{a_2}+\frac{\ddot a_3}{a_3}+\frac{\dot a_2\dot a_3}{a_2a_3}+\frac{\alpha^4}{a_1^2}=-p\,, \tag{50}$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_3}{a_3}+\frac{\dot a_1\dot a_3}{a_1a_3}-\frac{\alpha^4}{a_1^2}=-p\,, \tag{51}$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_2}{a_2}+\frac{\dot a_1\dot a_2}{a_1a_2}-\frac{\alpha^4}{a_1^2}=-p\,, \tag{52}$$

$$\alpha^2\Big(\frac{\dot a_3}{a_3}-\frac{\dot a_2}{a_2}\Big)=0\,. \tag{53}$$

Integrating equation (53), we get $a_3 = c_3a_2$, where the constant of integration c_3 can be ignored finally giving

$$a_3 = a_2\,. \tag{54}$$

As we know that shear σ is proportional to expansion θ, we can write

$$a_1 = a_2^m\,, \tag{55}$$

where $m \neq 1$ is an arbitrary constant.

Using equations (16), (54) and (55), we have

$$a_1 = (t^\beta e^t)^{\frac{3m}{\mu(m+2)}}\,, \tag{56}$$

$$a_2 = (t^\beta e^t)^{\frac{3}{\mu(m+2)}}\,, \tag{57}$$

$$a_3 = (t^\beta e^t)^{\frac{3}{\mu(m+2)}}\,. \tag{58}$$

Thus, the directional Hubble parameters are

$$H_x=\frac{3m(\beta+t)}{(m+2)\mu t}\,,\quad H_y=\frac{3(\beta+t)}{(m+2)\mu t}\,,\quad H_z=\frac{3(\beta+t)}{(m+2)\mu t}\,. \tag{59}$$

Thus, the mean Hubble parameter is given by

$$H=\frac{(\beta+t)}{\mu t}\,. \tag{60}$$

Also, the anisotropy parameter and shear scalar are given by

$$\Delta=\frac{2(m-1)^2}{(m+2)^2}\,, \tag{61}$$

$$\sigma^2=\frac{3(m-1)^2(\beta+t)^2}{(m+2)^2\mu^2t^2}\,. \tag{62}$$

7 Common Physical Properties of the Models

We now discuss common physical properties for all the four models. The spatial volume, the scalar of expansion and the deceleration parameter of all models are given by

$$V = a^3 = (t^\beta e^t)^{3/\mu}\,, \tag{63}$$

$$\theta = \frac{3(\beta + t)}{\mu t}\,, \tag{64}$$

$$q = \frac{\mu\beta}{(\beta + t)^2} - 1\,. \tag{65}$$

Using equations (6) and (63), the energy density becomes

$$\rho = \xi(\beta \ln t + t)^{1/(1-n)}\,, \tag{66}$$

where $\xi = \left(\frac{3K\,(n-1)}{\mu}\right)^{1/(1-n)}$.

Using the polytropic EoS and equation (66), the pressure has the value

$$p = \xi(\beta \ln t + t)^{1/(1-n)} \left(\frac{K\xi^{n-1}}{\beta \ln t + t} - 1\right), \tag{67}$$

giving the EoS parameter as

$$\omega = \frac{p}{\rho} = \frac{K\xi^{n-1}}{\beta \ln t + t} - 1\,. \tag{68}$$

Also, the energy density parameter is

$$\Omega = \frac{\rho}{3H^2} = \frac{\xi\mu^2 t^2(\beta \ln t + t)^{1/(1-n)}}{3(\beta + t)^2}\,. \tag{69}$$

8 Discussion and Conclusion

It can be seen from equation (63) and Figure 1 that the models under discussion evolve with a zero volume at $t = 0$. As time increases, the volume goes on increasing and becomes infinitely large as $t \to \infty$.

One can observe from equation (65) and Figure 2 that at $t \to \infty$, $q = -1$ and $\frac{dH}{dt} = 0$, which gives the greatest value of the Hubble parameter and fastest rate of expansion of the universe. Further, it exhibits the rapid decrease of the deceleration parameter with time and it approaches towards -1 as $t \to \infty$. This can be noted as de-Sitter like expansion at late time. Also, the transition of the universe

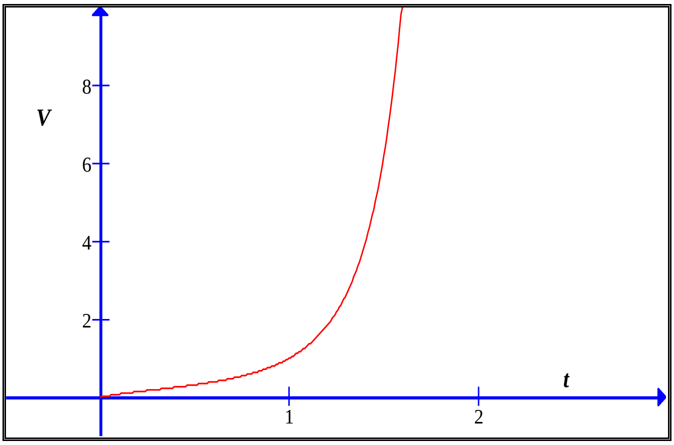

Figure 1. The variation of V vs. t for $\beta = 3$, $\mu = 9$.

from the early decelerated expansion phase to the current accelerated expansion phase can be observed. Thus, our considered models can be used for describing the late time structure of the universe as the value of the deceleration parameter lies in the range $-1 < q < 0$. These results of the deceleration parameter are harmonious with the observations made by Perlmutter *et al.* [2,39], Riess *et al.* [1] and Planck results [8].

Figure 3 and equation (69) show that the energy density parameter decreases with time t from infinite value and becomes very close to 1 i.e. the universe approaches flatness. But, it achieves flatness for a few moments only. After that,

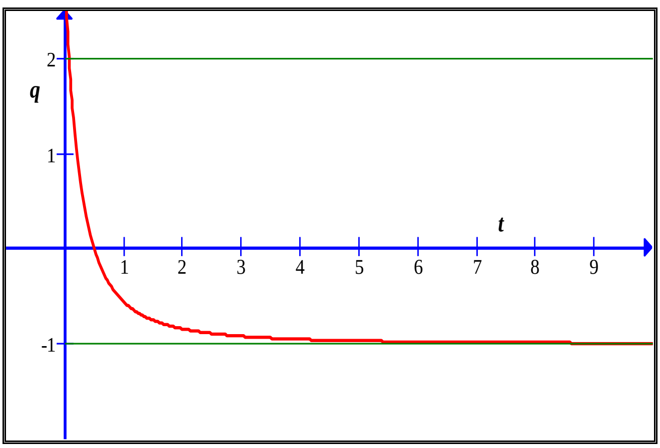

Figure 2. The variation of q vs. t for $m = 1$.

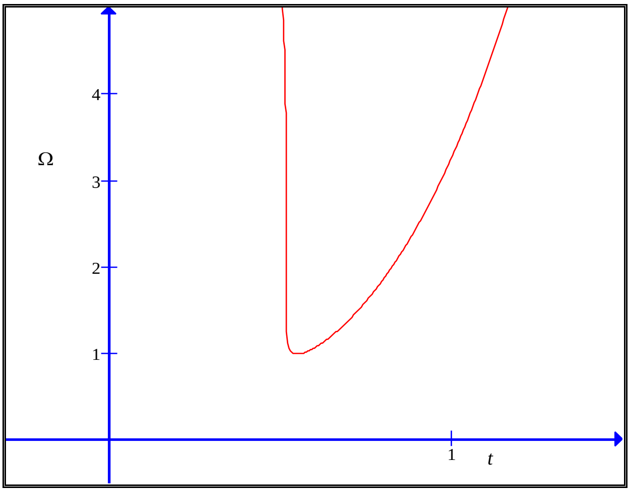

Figure 3. The variation of Ω vs. t for $\xi = 5$, $\mu = {}^1\!/_2$, $\beta = 1.8$, $n = -6$.

it again starts increasing with time and becomes infinite as $t \to \infty$. Similar results are obtained by Sarkar [40].

The variation of the EoS parameter with increasing time shown in Figure 4 and equation (68) suggest that the EoS parameter increases rapidly and it approaches towards -1. Here, it can be noted that $\omega = -1$ is the EoS of the cosmological constant Λ. So, for the future evolution of the universe, our models reduce to ΛCDM model. The ΛCDM model or the models that reduce to ΛCDM model

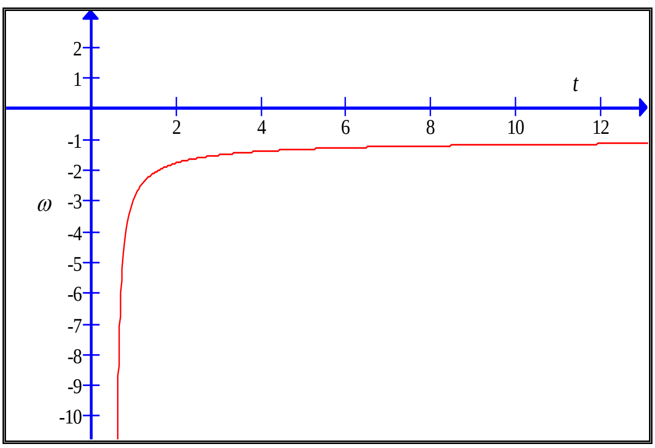

Figure 4. The variation of ω vs. t for $K = -2$, $\xi = 1$, $n = 3$, $\beta = 2$.

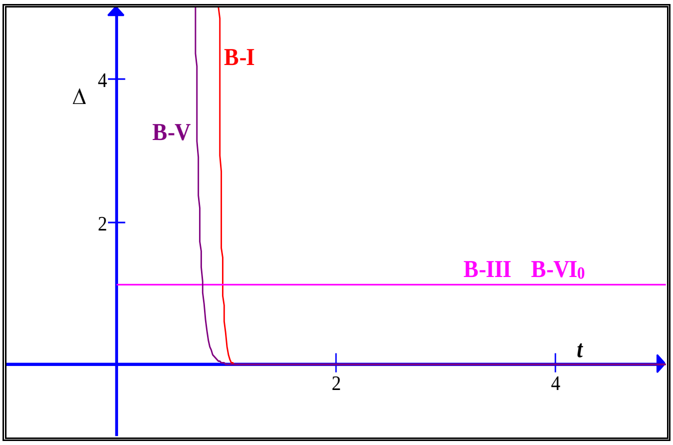

Figure 5. The variation of Δ vs. t, where for B-I, $Z = 2$, $\mu = 2$, $\beta = 6$; for B-V, $X = 2$, $\mu = 2$, $\beta = 6$ and for B-III and B-VI$_0$, $m = 10$.

are the best candidates for describing the cosmological evolution of the universe. This predicament has been pinpointed by Spergel *et al.* [4], Riess *et al.* [41,42], Eisenstein *et al.* [43], Astier *et al.* [44] and Bamba *et al.* [14].

For the Bianchi type I and type V cosmological models, the corresponding anisotropy parameters are time dependent. As time increases, for a suitable choice of the scalars, the universe which was initially anisotropic starts to isotropize and finally attains isotropy after some large cosmic time. Whereas for the Bianchi type III and type VI$_0$ models, the corresponding anisotropy parameters are independent of time. These models remain anisotropic at all times except at $m = 1$. This variation of isotropy of the models can be verified by equations (22), (36), (47), (61) and Figure 5. Thus, our results match with the present day observational data.

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