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Bianchi Type VI_0 Cosmological Model with Bulk Viscosity in $f(R)$ Theory

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Abstract. The paper is devoted to study the Bianchi type VI_0 space-time filled with Bulk viscous in the framework of $f(R)$ gravity. The model initially accelerates for a certain period of time and decelerates thereafter .The physical and kinematical features of the models are studied and discussed. The function $f(R)$ of the Ricci scalar is also evaluated for the model.

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1 Introduction

Accelerated expansion of universe may be described by dark energy with positive energy and negative pressure [1-10]. There are also other interesting models to describe the dark energy such as k-essence model [11], tachyonic models [12], Chaplygin gas models [13-27] and quintessence [28-30]. The alternative way to explain cosmic acceleration is to anticipate that large scale dynamics of the Universe is not governed by Einstein's equations. In a classical generalization of general relativity, the Ricci scalar R in Einstein-Hilbert action is replaced by a more general function $f(R)$ which modifies the geometry-side of the Einstein's equation [31]. The $f(R)$ gravity provides a natural unification of the early-time inflation and late-time cosmic acceleration. Earlier interest in $f(R)$ theories was motivated by inflationary scenarios as for instance, in the Starobinsky model [32]. The constant curvature solutions in $f(R)$ gravity have been investigated by Cognola *et al.* [33]. Multamäki and Vilja [34] have shown that a large class of $f(R)$ models has the Schwarzschild–de Sitter metric as a spherically symmetric vacuum solution. The exact solutions of static spherically symmetric space times $f(R))$ modified theories of gravity have been analyzed by Lukas and Francisco [35]. Momeni, Azadi and Nouri-Zonoz [36] have explored cylindrical symmetric solutions in $f(R)$ theory. This work was extended by Momeni and Gholizade

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[37] to the general cylindrical symmetric solutions. Spherically symmetric solutions of $f(R)$ theories of gravity via the Noether symmetry approach have been explored by Capozziello *et al.* [38]. Recently, Sharif and Kausar [39] have studied non-vacuum static spherically symmetric solutions in $f(R)$ gravity. Gödel-type universes in $f(R)$ gravity have been studied by Reboucas and Santos [40]. M. Sharif and M. Shamir [41] have found exact solutions of the Bianchi types I and V spacetimes using the non-vacuum field equations. These solutions correspond to two models of the universe, i.e., a singular model and a non-singular model. M. Shamir [42] has studied the exact vacuum solutions of Bianchi type I, III and Kantowski Sachs spacetimes in the metric version of $f(R)$ gravity. M. Sharif and H. Kausar [43] have studied the exact solutions of the Bianchi type VI$_0$ universe in the metric $f(R)$ gravity and to discuss the recent cosmic acceleration. Two types of non-vacuum solutions are found corresponding to isotropic and anisotropic fluids respectively. K.S. Adhav [44] studied Bianchi type III cosmic string cosmological model in $f(R)$ gravity.

Bulk viscosity plays a significant role in cosmology in getting accelerated expansion phase of the universe popularly known as the inflationary phase. The magnitude of the viscous stress relative to the expansion is determined by the bulk viscosity coefficient. Neutrino viscosity acting in the early era might have considerably reduced the present anisotropy of the blackbody radiation during the process of evolution had been suggested by Minser [45]. Some general characteristics of anisotropic cosmological models in the presence of viscosity are investigated by Belinsky and Khalatnikov [46] and Murphy [47] derived a homogeneous isotropic model by introducing the second viscosity coefficient in the energy-momentum tensor of the fluid content. The coefficient of viscosity decreases as universe expands. The Freidmann-Robertson-Walker (FRW) models have been discussed by several authors [48-52] of getting the possibility of bulk viscosity leading to inflationary-like solutions in general relativistic. Bali and Dave [53], Bali and Pradhan [54], Tripathy *et al.* [55,56] and Katore and Shaikh [57] have studied various string cosmological models in the presence of bulk viscosity.

After noting such recent developments in $f(R)$ gravity, the authors aimed to study different cosmological models in $f(R)$ gravity with bulk viscous. The main objective of this work is to find solutions of the field equations of the Bianchi type VI$_0$ model with bulk viscous in $f(R)$ gravity.

2 $f(R)$ Gravity Formalism

The $f(R)$ theory of gravity is the generalization of General Relativity. The action for this theory is given by

$$S = \frac{1}{2k^2} \int d^4 \sqrt{-g} f(R) + \int d^4 x L_m(g_{\mu\nu}, \psi_m)\,. \tag{1}$$

Here $f(R)$ is a general function of the Ricci scalar, $k^2 = 8\pi G = 1$, g is the determinant of the metric $g_{\mu\nu}$ and L_m is the metric Lagrangian that depends on $g_{\mu\nu}$ and the matter field ψ_m.

It is noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein–Hilbert action.

The corresponding field equations are found by varying the action with respect to the metric $g_{\mu\nu}$

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = T^M_{\mu\nu}\,, \tag{2}$$

where

$$\Box \equiv \nabla^\mu\nabla_\mu\,, \qquad F(R) \equiv \frac{df(R)}{dR}\,, \tag{3}$$

∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

3 Metric and Field Equations

We consider the space time metric of the spatially homogeneous and anisotropic Bianchi type VI_0 of the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2qx}dy^2 + C^2e^{2qx}dz^2\,, \tag{4}$$

where A, B and C are functions of t only and q is a non-zero constant.

The corresponding Ricci scalar curvature for Bianchi type VI_0 model is given by

$$R = 2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{q^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right], \tag{5}$$

where dot represents derivative with respect to t.

The energy-momentum tensor T_{ij} for a bulk viscous fluid distribution is given by

$$T_{ij} = (\rho + \bar{p})u_iu_j + \bar{p}g_{ij}\,, \tag{6}$$

where

$$\bar{p} = p - \eta u^i_{;i} = p - 3\eta H \tag{7}$$

is the effective pressure, η is the coefficient of bulk viscosity, p is the isotropic pressure, ρ is the energy density, $3\eta H$ is usually known as bulk viscous pressure, H is Hubble's parameter and u^i is fluid four-velocity vector satisfying

$$u_iu^i = -1. \tag{8}$$

Therefore, the dynamics of cosmic evolution does not change fundamentally by the inclusion of viscous term in the energy momentum tensor. In the co-moving co-ordinate system, we have from the above equations

$$T_1^1 = T_2^2 = T_3^3 = \bar{p}\,, \qquad T_4^4 = -\rho\,, \qquad T_i^j = 0\;,\; i \neq j. \tag{9}$$

Using equations (6)-(9), the corresponding field equations (2) for Bulk Viscous in respect of the Bianchi type VI$_0$ space-time reduce to the following set of equations

$$\Big(\frac{\ddot{A}}{A} - 2\frac{q^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC}\Big)F + \frac{1}{2}f(R) + \ddot{F} + \Big(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big)\dot{F} = -\bar{p}\,, \tag{10}$$

$$\Big(\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC}\Big)F + \frac{1}{2}f(R) + \ddot{F} + \Big(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\Big)\dot{F} = -\bar{p}\,, \tag{11}$$

$$\Big(\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC}\Big)F + \frac{1}{2}f(R) + \ddot{F} + \Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\Big)\dot{F} = -\bar{p}\,, \tag{12}$$

$$\Big(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\Big)F + \frac{1}{2}f(R) + \Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big)\dot{F} = \rho\,, \tag{13}$$

$$q\Big(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\Big)F = 0\,. \tag{14}$$

From equation (14), we get

$$B = c_1 C,$$

where c_1 is the constant of integration.

Without loss of generality, we can consider $c_1 = 1$ for the sake of simplicity.

Hence we get

$$B = C\,. \tag{15}$$

The conservation equation for energy-momentum $T^i_{i;j} = 0$ gives

$$\dot{\rho} + (\rho + \bar{p})\Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big) = 0\,. \tag{16}$$

4 Solutions of the Field Equations

Using equation (15) in the field equations (10)–(13), we get

$$\left(\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{B}}{AB} - 2\frac{q^2}{A^2}\right)F + \frac{1}{2}f(R) + \ddot{F} + \left(2\frac{\dot{B}}{B}\right)\dot{F} = -\bar{p}, \tag{17}$$

$$\left(\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right)F + \frac{1}{2}f(R) + \ddot{F} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{F} = -\bar{p}, \tag{18}$$

$$\left(\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}\right)F+\frac{1}{2}f(R)+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)\dot{F}=\rho. \tag{19}$$

The physical quantities of observational interest in cosmology are spatial volume V, mean Hubble parameter H, the expansion scalar θ, the mean anisotropy parameter A_m, the shear scalar σ^2 and the deceleration parameter q. They are defined as

$$\text{Spatial Volume} \qquad V = AB^2\,, \tag{20}$$

$$\text{Mean Hubble Parameter} \qquad H=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right), \tag{21}$$

$$\text{Scalar Expansion} \qquad \theta = 3H\,, \tag{22}$$

$$\text{Anisotropic Parameter} \qquad A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{23}$$

where H_i $(i = 1, 2, 3)$ are the directional Hubble parameters in the directions of x, y and z axes respectively.

$$\text{Shear Scalar} \qquad \sigma^2=\frac{3}{2}A_m H^2\,, \tag{24}$$

$$\text{Deceleration Parameter} \qquad q=\frac{d}{dt}\left(\frac{1}{H}\right)-1\,. \tag{25}$$

Here the deceleration parameter q measures the rate of expansion of the universe. The sign of q indicates the state of expanding universe. If $q < s0$ or $> q0$ respectively, then it represents inflation or deflation of the universe while $q = 0$ shows expansion with constant velocity.

The average scale factor of Bianchi type VI_0 model is defined as

$$a=\left(V^{\frac{1}{3}}\right). \tag{26}$$

Here we have three independent field equations containing A, B, η, ρ, p, $\bar{p}$, F, f eight unknowns, so we shall assume extra conditions to obtain unique solutions of the filed equations. For the complete determination of the exact solutions, additional constraints relating these parameters are required.

For a Baratropic fluid the combined effect of the proper pressure and the Baratropic bulk viscous pressure can be expressed as

$$\bar{p} = p - 3\eta H = \gamma\rho\,, \tag{27}$$

where

$$p=\gamma_0\rho, \qquad 0\le\gamma_0\le 1\,. \tag{28}$$

We also assume that the result established by Sharif and Shamir [58] in $f(R)$ gravity which shows that

$$F \propto a^m\,, \tag{29}$$

where m is an arbitrary constant, $a = (V)^{1/3}$ is a scale factor.

Equation (29) leads to

$$F = la^m\,, \qquad F = l(V)^{m/3}\,, \qquad F = l(AB^2)^{m/3}, \tag{30}$$

where l is proportionality constant.

The two observable parameters q and H are related by the relation

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \tag{31}$$

This equation, on integration, gives the scale factor $a(t)$ as

$$a(t) = e^{\delta} \exp \int \frac{dt}{\int (1+q)dt + r}, \tag{32}$$

where r, δ are arbitrary constants of integration.

For the complete determination of $a(t)$, Abdusattar and Prajapati [59] proposed the following choice of q as

$$q = -\frac{\alpha}{t^2} + (\beta - 1), \tag{33}$$

where $\alpha > 0$ is a parameter having the dimension of square of time and $\beta > 1$ is a dimensionless constant. Obviously, the different values of $\alpha$and β will give rise to different models.

With q given by equation (33) and $\delta = 0$, $r = 0$, equation (32) can be integrated to give the scale factor as

$$a(t) = \left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{1}{2\beta}}. \tag{34}$$

In order to solve the system completely, we assume that $B = V^b$, where b is any constant number.

Then from equations (15), (20) and (34), we obtain the exact expression for the scale factors

$$A = \left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{3-6b}{2\beta}}, \tag{35}$$

$$B = C = \left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{3b}{2\beta}}. \tag{36}$$

Using equations (30), (34), we obtain

$$F = l\left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{m}{2\beta}}. \tag{37}$$

Using equations (16), (27), (35), (36), we get

$$\rho = c_3\left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{-3(1+\gamma)}{2\beta}}, \tag{38}$$

where c_3 is a constant of integration.
Using equations (27), (38), we have

$$\bar{p} = \gamma c_3\left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{-3(1+\gamma)}{2\beta}}. \tag{39}$$

Using equations (28), (38), we have

$$p = \gamma_0 c_3\left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{-3(1+\gamma)}{2\beta}}. \tag{40}$$

Using equations (7), (39) and (40), we have the coefficient of bulk viscosity

$$\eta = \frac{1}{3H}(\gamma_0 - \gamma)c_3\left(t^2 + \frac{\alpha}{\beta}\right)^{\frac{-3(1+\gamma)}{2\beta}}. \tag{41}$$

Ricci scalar curvature can be found out as

$$\begin{aligned} R = 2\Bigg\{&\left(\frac{3-3b}{\beta}\right)\frac{1}{\left(t^2+\frac{\alpha}{\beta}\right)}\left[\left(\frac{3-6b-2\beta}{\beta}\right)\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)}+1\right]\Bigg\} \\ &+\frac{6b}{\beta}\frac{1}{\left(t^2+\frac{\alpha}{\beta}\right)}\left[\left(\frac{3b-2\beta}{\beta}\right)\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)}+1\right] \\ &-\frac{q^2}{\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{3-6b}{2\beta}}}+\left(\frac{3b}{\beta}\right)^2\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2} \\ &+2\left(\frac{3-6b}{\beta}\right)\frac{3b}{\beta}\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}. \end{aligned} \tag{42}$$

$f(R)$ turns out to be

$$\begin{aligned}
\frac{1}{2}f(R) &= c_3\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{-3(1+\gamma)}{2\beta}} \\
&\quad -\left\{\left(\frac{3-6b}{\beta}\right)\frac{t}{\left(t^2+\frac{\alpha}{\beta}\right)}+\frac{6b}{\beta}\frac{t}{\left(t^2+\frac{\alpha}{\beta}\right)}\right\}\frac{lmt}{\beta}\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{m-2\beta}{2\beta}} \\
&\quad -\left(\left\{\left(\frac{3-6b}{\beta}\right)\frac{1}{\left(t^2+\frac{\alpha}{\beta}\right)}\left[\left(\frac{3-6b-2\beta}{\beta}\right)\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)}+1\right]\right\}\right. \\
&\quad \left.+\frac{6b}{\beta}\frac{1}{\left(t^2+\frac{\alpha}{\beta}\right)}\left[\left(\frac{3b-2\beta}{\beta}\right)\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)}+1\right]\right)l\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{m}{2\beta}} \qquad (43)
\end{aligned}$$

The mean Hubble parameter is given by

$$H=\frac{t}{\beta\left(t^2+\frac{\alpha}{\beta}\right)}. \qquad (44)$$

The spatial volume of the universe is given by

$$V=\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{3}{2\beta}}. \qquad (45)$$

The expansion scalar is given by

$$\theta=\frac{3t}{\beta\left(t^2+\frac{\alpha}{\beta}\right)}. \qquad (46)$$

The mean anisotropic parameter is

$$A_m=2(3b-1)^2. \qquad (47)$$

For $b=\frac{1}{3}A_m=0$. Hence the model is isotropic when $b=\frac{1}{3}$.
The shear scalar is given by

$$\sigma^2=3\frac{(3b-1)^2}{\beta^2}\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}. \qquad (48)$$

The deceleration parameter is given by

$$q=\frac{\beta\left(t^2-\frac{\alpha}{\beta}\right)}{t^2}-1. \qquad (49)$$

We find that $a(0) \neq 0$, $\dot{a}(0) = 0$ but $\ddot{a}(0) =$ const at initial epoch $t = 0$. This shows that the model is free from initial singularity and start expanding with finite acceleration. Equation (34) suggests that the deceleration parameter $q \to -\infty$ at $t = 0$ and reduces to zero at $t = \sqrt{\dfrac{\alpha}{\beta - 1}}$. The period of accelerated expansion also depends on the value of α and β. Afterwards, with deceleration parameter q approaching $\beta - 1$, for sufficiently large values of t, the model decelerates. For $\beta = 1$, the Universe has an accelerated expansion throughout the evolution. We see that θ, σ^2, H ultimately tend to zero as $t \to \infty$.

5 Conclusion

In this paper, we have obtained Bianchi type VI_0 cosmological model with bulk viscous as a matter in $f(R)$ gravity by using the special form of the average scale factor derived by Abdussattar and Prajapati [59] by constraining the deceleration parameter. The model starts from rest with a finite radius and finite acceleration which gradually decreases and reduces to zero after some time – this is the important aspect of the present model; thereafter the model decelerates with gradually increasing deceleration, approaching to a constant value for large values of t. It is interesting to note that the results obtained resemble with the results of Abdussattar and Prajapati [59], Shri Ram *et al.* [60] and Katore and Shaikh [61]. The model obtained may throw some light on our understanding of $f(R)$ cosmology.

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