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Six-Dimensional Bulk Viscous Fluid Cosmological Model in $f(R,T)$ Gravity Theory

B.B. Chaturvedi, B.K. Gupta
Department of Pure and Applied Mathematics, Guru Ghasidas Vishwavidyalaya Bilaspur (C.G.), India

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Abstract. The present paper contains five sections. First section is introductory and in section second we considered a six-dimensional homogeneous anisotropic model and calculated the scalar curvature with the help of Einstein field equation in $f(R,T)$ gravity theory. In Section 3, we have discussed the solution of Einstein field equation. In Section 4, we have described the physical and geometrical features of the expansion scalar θ, shear scalar σ, average Hubble parameter H, mean anisotropc A_m and deceleration parameter q. At last, we have drawn graphs and discussed the variation of pressure p, Hubble parameter H, expansion scalar θ and deceleration parameter q with time t.

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1 Introduction

With the help of FRW model which is described by Friedman–Robertson–Walker, many mathematicians and physicists suggested that the present universe is almost homogeneous and isotropic. We also know that the cosmological models with bulk viscosity play a very important role in the discussion of early stage of evolution of the universe. Johri and Sudarshan [1] observed that the presence of bulk viscosity leads to an inflationary universe in Brans–Dicke theory. Pavon and Zimdaml [2], Hobill, Burd and Coley [3], Bali and Dave [4], Tripathy et al. [5], Mohanty and Pradhan [6] and some other physicists and mathematicians have studied and constructed various viscous fluid cosmological models. In 2012, Bali and Singh [7] investigated Bianchi type-I bulk viscous barotropic fluid cosmological models and explained the physical and geometrical aspects of the model related with astronomical observation. Das and Ali [8] have studied axially symmetric Bianchi type-I bulk viscous cosmological model with time varying gravitational and cosmological constant. Chaubey and Shukla [9] have found a new class of Bianchi cosmological models in $f(R,T)$ gravity by using a special law of variation for the average scale factor when the deceleration parameter is linear with negative slope. Samanta and Dhal [10] have constructed higher dimensional cosmological models with a perfect fluid source in $f(R,T)$ gravity. In consequences of the above studies, this paper deals the metric and the field equation in Section 2. In Section 3 an exact solution of the field equation is obtained. In Section 4 the physical and geometrical features are described. Finally, in Section 5 we draw graphs and describe their physical and geometrical aspects.

2 Metric and Field Equation

We consider a six-dimensional homogeneous anisotropic model, in which the metric is given by

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 - D^2d\mu^2 - E^2d\nu^2\,, \tag{2.1}$$

where A, B, C, D, and E are functions of time t only.

The energy momentum tensor for imperfect bulk viscous fluid is difined by

$$T_{i\,j} = (\rho + \bar{p})u_i u_j - \bar{p}\, g_{j\,i}\,, \tag{2.2}$$

where $u^i = (0,0,0,0,0,1)$ is the six velocity in co-moving coordinates which satisfies

$$u_i\, u^i = 1\,, \qquad \text{and} \qquad u^i\Delta_j u_i = 0\,, \tag{2.3}$$

here $\bar{p}$ and ρ are effective pressure and energy density of the fluid.

The effective pressure is given by

$$\bar{p} = p - \zeta u^i_{;\,i}, \tag{2.4}$$

where p satisfies the linear equation of state, given by

$$p = \epsilon\rho\,, \qquad 0 \le \epsilon \le 1\,, \tag{2.5}$$

here p and ζ are the pressure and the coefficient of bulk viscosity.

The field equations in $f(R,T)$ gravity theory with the particular choice of the function $f(R,T)$ is given by (K.L. Mahanta [11])

$$f(R,T) = R + 2f(T). \tag{2.6}$$

The Einstein field equation, when the matter source is a bulk viscous fluid, is given by (Reddy et al. [12])

$$R_{i\,j} - \frac{1}{2}Rg_{i\,j} = 8\pi T_{i\,j} + 2f'(T)T_{i\,j} + [2\bar{p}f'(T) + f(T)]g_{i\,j}\,, \tag{2.7}$$

where R, $R_{i\,j}$ and T are the scalar curvature, the Ricci tensor, and the energy momentum tensor respectively, and $f(T) = \lambda T$, where λ is a constant. Equation

(2.1) can be written as in matrix form

$$K = \begin{pmatrix} -A^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -B^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -D^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -E^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The determinant of above matrix is given by

$$|K| = -A^2\, B^2\, C^2\, D^2\, E^2\,. \tag{2.8}$$

The non-vanishing components of Christoffel's symbols corresponding to the metric (2.1) are given by

$$\begin{gathered} \Gamma^6_{1\,1} = A_6 A\,, \quad \Gamma^6_{2\,2} = B_6 B\,, \quad \Gamma^6_{3\,3} = C_6 C\,, \\ \Gamma^6_{4\,4} = D_6 D\,, \quad \Gamma^6_{5\,5} = E_6 E \\ \Gamma^1_{6\,1} = \frac{A_6}{A}\,, \quad \Gamma^2_{6\,2} = \frac{B_6}{B}\,, \quad \Gamma^3_{6\,3} = \frac{C_6}{C}\,, \\ \Gamma^4_{6\,4} = \frac{D_6}{D}\,, \quad \Gamma^5_{6\,5} = \frac{E_6}{E}\,. \end{gathered} \tag{2.9}$$

We know that the Ricci tensor of type (0,2) is defined as

$$R_{i\,j} = \frac{\partial \Gamma^k_{k\,i}}{\partial x^j} - \frac{\partial \Gamma^k_{i\,j}}{\partial x^k} + \Gamma^q_{k\,i}\Gamma^k_{q\,j} - \Gamma^q_{i\,j}\Gamma^k_{q\,k}\,. \tag{2.10}$$

Using (2.9) in (2.10), we get

$$R_{1\,1} = -A\,A_{6\,6} - \frac{A\,A_6\,B_6}{A} - \frac{A\,A_6\,C_6}{C} - \frac{A\,A_6\,D_6}{D} - \frac{A\,A_6\,E_6}{E}\,. \tag{2.11}$$

Similarly, we can calculated other non-zero components of Ricci tensor, which are given by

$$R_{2\,2} = -B\,B_{6\,6} - \frac{B\,A_6\,B_6}{A} - \frac{B\,B_6\,C_6}{C} - \frac{B\,B_6\,D_6}{D} - \frac{B\,B_6\,E_6}{E}\,, \tag{2.12}$$

$$R_{3\,3} = -C\,C_{6\,6} - \frac{C\,C_6\,A_6}{A} - \frac{C\,D_6\,C_6}{D} - \frac{C\,C_6\,B_6}{B} - \frac{C\,C_6\,E_6}{E}\,, \tag{2.13}$$

$$R_{4\,4} = -D\,D_{6\,6} - \frac{D\,A_6\,D_6}{A} - \frac{D\,B_6\,D_6}{B} - \frac{D\,C_6\,D_6}{C} - \frac{D\,D_6\,E_6}{E}\,, \tag{2.14}$$

$$R_{5\,5} = -E\,E_{6\,6} - \frac{E\,A_6\,E_6}{A} - \frac{E\,E_6\,B_6}{B} - \frac{E\,E_6\,D_6}{D} - \frac{E\,C_6\,E_6}{C}\,, \tag{2.15}$$

and

$$R_{6\,6} = \frac{A_{6\,6}}{A} + \frac{B_{6\,6}}{B} + \frac{C_{6\,6}}{C} + \frac{D_{6\,6}}{D} + \frac{E_{6\,6}}{E}\,.. \tag{2.16}$$

Now, the scalar curvature tensor is given by

$$R = R_{1\,1}\, g^{1\,1} + R_{2\,2}\, g^{2\,2} + R_{3\,3}\, g^{3\,3} + R_{4\,4}\, g^{4\,4} + R_{5\,5}\, g^{5\,5} + R_{6\,6}\, g^{6\,6}, \quad (2.17)$$

using (2.11)–(2.16) in (2.17), we get

$$\begin{aligned} R = 2\Big(&\frac{A_{6\,6}}{A} + \frac{B_{6\,6}}{B} + \frac{C_{6\,6}}{C} + \frac{D_{6\,6}}{D} + \frac{E_{6\,6}}{E}\Big) \\ &+ 2\Big(\frac{A_6\,B_6}{A\,B} + \frac{A_6\,C_6}{A\,C} + \frac{A_6\,D_6}{A\,D} + \frac{A_6\,E_6}{A\,E} + \frac{C_6\,B_6}{C\,B} \\ &\quad + \frac{D_6\,B_6}{D\,B} + \frac{E_6\,B_6}{E\,B} + \frac{C_6\,D_6}{C\,D} + \frac{C_6\,E_6}{C\,E} + \frac{D_6\,E_6}{D\,E}\Big), \end{aligned} \quad (2.18)$$

which is the required scalar curvature for this model.

3 Solution of Field Equations

Using (2.2), (2.11) and (2.18) in (2.7), we get

$$\begin{aligned} &\frac{B_{6\,6}}{B} + \frac{C_{6\,6}}{C} + \frac{D_{6\,6}}{D} + \frac{E_{6\,6}}{E} \\ &\quad + \Big(\frac{C_6\,B_6}{C\,B} + \frac{D_6\,B_6}{D\,B} + \frac{E_6\,B_6}{E\,B} + \frac{C_6\,D_6}{C\,D} + \frac{C_6\,E_6}{C\,E} + \frac{D_6\,E_6}{D\,E}\Big) \\ &\qquad = (8\pi + 5\lambda)\bar{p} - \rho\lambda, . \end{aligned} \quad (3.1)$$

Similarly, we can find remaining field equations which are given by

$$\begin{aligned} &\frac{A_{6\,6}}{A} + \frac{C_{6\,6}}{C} + \frac{D_{6\,6}}{D} + \frac{E_{6\,6}}{E} \\ &\quad + \Big(\frac{C_6\,A_6}{C\,A} + \frac{D_6\,A_6}{D\,A} + \frac{E_6\,A_6}{E\,A} + \frac{C_6\,D_6}{C\,D} + \frac{C_6\,E_6}{C\,E} + \frac{D_6\,E_6}{D\,E}\Big) \\ &\qquad = (8\pi + 5\lambda)\bar{p} - \rho\lambda, \end{aligned} \quad (3.2)$$

$$\begin{aligned} &\frac{A_{6\,6}}{A} + \frac{B_{6\,6}}{B} + \frac{D_{6\,6}}{D} + \frac{E_{6\,6}}{E} \\ &\quad + \Big(\frac{B_6\,A_6}{B\,A} + \frac{D_6\,A_6}{D\,A} + \frac{E_6\,A_6}{E\,A} + \frac{B_6\,D_6}{B\,D} + \frac{B_6\,E_6}{B\,E} + \frac{D_6\,E_6}{D\,E}\Big) \\ &\qquad = (8\pi + 5\lambda)\bar{p} - \rho\lambda, \end{aligned} \quad (3.3)$$

$$\begin{aligned} &\frac{A_{6\,6}}{A} + \frac{B_{6\,6}}{B} + \frac{C_{6\,6}}{C} + \frac{E_{6\,6}}{E}) \\ &\quad + \Big(\frac{C_6\,E_6}{C\,E} + \frac{D_6\,B_6}{D\,B} + \frac{E_6\,A_6}{E\,A} + \frac{C_6\,A_6}{C\,A} + \frac{C_6\,B_6}{C\,B} + \frac{B_6\,E_6}{B\,E}\Big) \\ &\qquad = (8\pi + 5\lambda)\bar{p} - \rho\lambda, \end{aligned} \quad (3.4)$$

$$\frac{A_{66}}{A}+\frac{B_{66}}{B}+\frac{C_{66}}{C}+\frac{D_{66}}{D} + \Big(\frac{A_6\,B_6}{A\,B}+\frac{D_6\,B_6}{D\,B}+\frac{C_6\,B_6}{C\,B}+\frac{C_6\,A_6}{C\,A}+\frac{C_6\,E_6}{C\,E}+\frac{D_6\,A_6}{D\,A}\Big) = (8\pi+5\lambda)\bar{p}-\rho\lambda, \quad (3.5)$$

and

$$(8\pi+3\lambda)\rho-3\lambda\bar{p} = -\Big(\frac{A_6\,B_6}{A\,B}+\frac{A_6\,C_6}{A\,C}+\frac{A_6\,D_6}{A\,D}+\frac{A_6\,E_6}{A\,E}+\frac{C_6\,B_6}{C\,B} + \frac{D_6\,B_6}{D\,B}+\frac{E_6\,B_6}{E\,B}+\frac{C_6\,D_6}{C\,D}+\frac{C_6\,E_6}{C\,E}+\frac{D_6\,E_6}{D\,E}\Big). \quad (3.6)$$

Subtracting (3.1) from (3.2), (3.2) from (3.3), (3.3) from (3.4), (3.4) from (3.5) and (3.5) from (3.1), we get

$$\frac{A_{66}}{A}-\frac{B_{66}}{B}+\Big(\frac{A_6}{A}-\frac{B_6}{B}\Big)\Big(\frac{C_6}{C}+\frac{D_6}{D}+\frac{E_6}{E}\Big)=0\,, \quad (3.7)$$

$$\frac{B_{66}}{B}-\frac{C_{66}}{C}+\Big(\frac{B_6}{B}-\frac{C_6}{C}\Big)\Big(\frac{A_6}{A}+\frac{D_6}{D}+\frac{E_6}{E}\Big)=0\,, \quad (3.8)$$

$$\frac{C_{66}}{C}-\frac{D_{66}}{D}+\Big(\frac{C_6}{C}-\frac{D_6}{D}\Big)\Big(\frac{A_6}{A}+\frac{B_6}{B}+\frac{E_6}{E}\Big)=0\,, \quad (3.9)$$

$$\frac{D_{66}}{D}-\frac{E_{66}}{E}+\Big(\frac{D_6}{D}-\frac{E_6}{E}\Big)\Big(\frac{C_6}{C}+\frac{B_6}{B}+\frac{A_6}{A}\Big)=0, \quad (3.10)$$

and

$$\frac{E_{66}}{E}-\frac{A_{66}}{A}+\Big(\frac{E_6}{E}-\frac{A_6}{A}\Big)\Big(\frac{C_6}{C}+\frac{D_6}{D}+\frac{B_6}{B}\Big)=0\,. \quad (3.11)$$

Equation (3.7) can be written as

$$\frac{\partial}{\partial t}\Big(\frac{A_6}{A}-\frac{B_6}{B}\Big)+\Big(\frac{A_6}{A}-\frac{B_6}{B}\Big)\Big(\frac{A_6}{A}+\frac{B_6}{B}+\frac{C_6}{C}+\frac{D_6}{D}+\frac{E_6}{E}\Big)=0. \quad (3.12)$$

Intergrating (3.12) with respect to t, we get

$$\frac{A}{B}=d_1\,\exp\Big[k_1\Big(\int\frac{1}{(ABCDE)}\,dt\Big)\Big]. \quad (3.13)$$

Similarly, we can obtain

$$\frac{B}{C}=d_2\,\exp\Big[k_2\Big(\int\frac{1}{(ABCDE)}\,dt\Big)\Big], \quad (3.14)$$

$$\frac{C}{D}=d_3\,\exp\Big[k_3\Big(\int\frac{1}{(ABCDE)}\,dt\Big)\Big], \quad (3.15)$$

$$\frac{D}{E} = d_4 \exp\Big[k_4\Big(\int \frac{1}{(ABCDE)}\,dt\Big)\Big], \tag{3.16}$$

and

$$\frac{E}{A} = d_5 \exp\Big[k_5\Big(\int \frac{1}{(ABCDE)}\,dt\Big)\Big]. \tag{3.17}$$

We define the average scale factor V by

$$V = a^5 = ABCDE, \tag{3.18}$$

solving (3.13), (3.14), (3.15), (3.16), (3.17) and (3.18), we get

$$B = ap_2 \exp\Big[q_2\Big(\int \frac{1}{(a^5)}\,dt\Big)\Big], \tag{3.19}$$

similary, we can obtain other scale factors which are given by

$$A = ap_1 \exp\Big[q_1\Big(\int \frac{1}{a^5}\,dt\Big)\Big], \tag{3.20}$$

$$C = ap_3 \exp\Big[q_3\Big(\int \frac{1}{a^5}\,dt\Big)\Big], \tag{3.21}$$

$$D = ap_4 \exp\Big[q_4\Big(\int \frac{1}{a^5}\,dt\Big)\Big], \tag{3.22}$$

$$E = ap_5 \exp\Big[q_5\Big(\int \frac{1}{a^5}\,dt\Big)\Big], \tag{3.23}$$

where

$$\begin{aligned}
p_1 &= (d_2^{-2}d_3^{-1}d_1^{2}d_5)^{\frac{-1}{5}}, & q_1 &= \frac{-(-3k_1 - 2k_2 - k_3 + k_5)}{5},\\
p_2 &= (d_2^{-2}d_3^{-1}d_1^{2}d_5)^{\frac{-1}{5}}, & q_2 &= \frac{-(-3k_1 - 2k_2 - k_3 + k_5)}{5},\\
p_3 &= (d_2^{3}d_3^{-1}d_1^{2}d_5)^{\frac{-1}{5}}, & q_3 &= \frac{-(2k_1 + 3k_2 - k_3 + k_5)}{5},\\
p_4 &= (d_2^{3}d_3^{4}d_1^{-3}d_5)^{\frac{-1}{5}}, & q_4 &= \frac{-(2k_1 + 3k_2 + 4k_3 + k_5)}{5},\\
p_5 &= (d_2^{-2}d_3^{-1}d_1^{-3}d_5^{-4})^{\frac{-1}{5}}, & q_5 &= \frac{-(-3k_1 - 2k_2 + k_3 - 4k_5)}{5},
\end{aligned} \tag{3.24}$$

p_1, p_2, p_3, p_4, p_5 and q_1, q_2, q_3, q_4, q_5 are intergration constants which satisfy the following relations

$$p_1p_2p_3p_4p_5 = 1 \quad \text{and} \quad q_1 + q_2 + q_3 + q_4 + q_5 = 0. \tag{3.25}$$

4 Physical and Geometrical Features

The expansion scalar θ, the shear scalar σ, the average Hubble parameter H, and the anisotropy parameter A_m for the metric (2.1) are given by

$$H = \frac{V_6}{5V} = \frac{1}{5}\Big(\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E}\Big), \tag{4.1}$$

$$\theta = v^i_{;i} = \Big(\frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E}\Big), \tag{4.2}$$

$$\sigma^2 = \frac{1}{2}\Big[\Big(\frac{A_6}{A}\Big)^2 + \Big(\frac{B_6}{B}\Big)^2 + \Big(\frac{C_6}{C}\Big)^2 + \Big(\frac{D_6}{D}\Big)^2 + \Big(\frac{E_6}{E}\Big)^2\Big] - \frac{1}{10}\theta^2, \tag{4.3}$$

and

$$A_m = \frac{1}{5}\sum_{i=1}^{5}\Big(\frac{\Delta H_i}{H}\Big)^2, \tag{4.4}$$

where

$$\Delta H_i = H_i - H\,, \qquad (i = 1,2,3,4,5) \qquad \text{and}$$

$$H_1 = \frac{A_6}{A}\,, \quad H_2 = \frac{B_6}{B}\,, \quad H_3 = \frac{C_6}{C}\,, \quad H_4 = \frac{D_6}{D}\,, \quad H_5 = \frac{E_6}{E}$$

are the directional Hubble parameters. For construction of cosmological model the hubble parameter and deceleration parameter (DP) play a very important role. In 1983, Berman proposed a law of Hubble parameter in FRW model and observed that a constant deceleration parameter, which leads to power-law and exponential forms of the average scale factors. Some mathematicians and physicists proposed time-dependent forms of DP and obtained a differential form of the average scale factors of the model.

The deceleration parameter q is given by

$$q = -1 + \frac{\partial}{\partial t}\Big(\frac{1}{H}\Big)\,. \tag{4.5}$$

Using (3,18) and (4.1) in (4.5), we get

$$q = -\frac{a a_{6\,6}}{a_6^2}\,, \tag{4.6}$$

after intergrating (4.6), we get

$$a(t) = e^{\delta}\,\exp\Big(\int \frac{dt}{\int(1+q)dt + \lambda}\Big). \tag{4.7}$$

In 2011, Abdussattar and Prajapati [13] have obtained a solution for the time dependent form of q, which is given by

$$q = -\frac{\alpha}{t^2} + (\beta - 1)\,. \tag{4.8}$$

In 2011, Abdussattar and Prajapati [13] using (4.8), (4.7) have derived three different forms of $a(t)$, the simplest form of which is given by

$$a(t) = e^{\delta}\Big(t^2 + \frac{\alpha}{\beta}\Big)^{\frac{1}{2\beta}}\,, \tag{4.9}$$

if we take $\delta = 0$ and $\alpha = \beta = 5/2$ in (4.9), we get

$$a(t) = (t^2+1)^{\frac{1}{5}}\,, \tag{4.10}$$

substituting (4.10) in (3.19)–(3.23) and integrating, we have

$$A(t) = p_1\,(t^2+1)^{\frac{1}{5}}\exp[q_1\tan^{-1}t], \tag{4.11}$$

$$B(t) = p_2\,(t^2+1)^{\frac{1}{5}}\exp[q_2\tan^{-1}t], \tag{4.12}$$

$$C(t) = p_3\,(t^2+1)^{\frac{1}{5}}\exp[q_3\tan^{-1}t], \tag{4.13}$$

$$D(t) = p_4\,(t^2+1)^{\frac{1}{5}}\exp[q_4\tan^{-1}t], \tag{4.14}$$

and

$$E(t) = p_5\,(t^2+1)^{\frac{1}{5}}\exp[q_5\tan^{-1}t], \tag{4.15}$$

Taking covariant derivative of (4.11)–(4.15) once and twice with respect to t, and using these in (3.2) and (3.7), we calculate effective pressure $\bar{p}$, energy density ρ, and pressure p, which are given by

$$\begin{aligned}\bar{p} = \frac{1}{3\lambda(t^2+1)^2}\Big[\frac{1}{25}\Big(\frac{[720\pi+250\lambda]}{[(8\pi+3\lambda)(8\pi+5\lambda)-3\lambda^2]} - 90\Big)t^2 \\ + \Big(14 - \frac{(500\pi+182\lambda+6q_1)}{5[(8\pi+3\lambda)(8\pi+5\lambda)-3\lambda^2]}\Big)t \\ + \frac{(L_1(40\pi+49\lambda)+3\lambda\,L_2)}{[(8\pi+3\lambda)(8\pi+5\lambda)-3\lambda^2]} - L_1\Big],\end{aligned} \tag{4.16}$$

$$\begin{aligned}\rho = \frac{1}{(8\pi+3\lambda)(8\pi+5\lambda-3\lambda^2)(t^2+1)^2}\Big[-\frac{(720\pi+258\lambda)}{25}t^2 \\ + \frac{1}{5}(560\pi+182\lambda+6q_1)t - L_1(40\pi+49\lambda)+3\lambda L_2\Big],\end{aligned} \tag{4.17}$$

and

$$\begin{aligned}p = \frac{\epsilon}{(8\pi+3\lambda)(8\pi+5\lambda-3\lambda^2)(t^2+1)^2}\Big[-\frac{(720\pi+258\lambda)}{25}t^2 \\ + \frac{1}{5}(560\pi+182\lambda+6q_1)t - L_1(40\pi+49\lambda)+3\lambda L_2\Big],\end{aligned} \tag{4.18}$$

where

$$L_2 = q_1 + q_2\, q_3 + q_2\, q_4 + q_2\, q_5 + q_3\, q_4 + q_3\, q_5 + q_4\, q_5 \text{ and}$$
$$L_1 = 2 - q_1^2 + q_1\, q_2 + q_3\, q_4 + q_3\, q_5 + q_4\, q_5\, .$$

Using (2.4), (2.5), (4.16) and (4.18), we obtain the coefficient of bulk viscosity ζ, given by

$$\begin{aligned}\zeta = \Big[- \frac{(720\pi + 258\lambda)(3\lambda\epsilon + 1)}{75\lambda[(8\pi + 3\lambda)(8\pi + 5\lambda) - 3\lambda^2]} + \frac{90}{3\lambda}\Big] 2t^3 \\ + \Big[\frac{(560\pi + 182\lambda + 6q_1)(3\lambda\epsilon + 1)}{15\lambda[(8\pi + 3\lambda)(8\pi + 5\lambda) - 3\lambda^2]} - \frac{14}{3\lambda}\Big] 2t^2 \\ - \Big[\frac{(L_1(40\pi + 49\lambda) + 3\lambda L_2)(3\lambda\epsilon + 1)}{3\lambda[(8\pi + 3\lambda)(8\pi + 5\lambda) - 3\lambda^2]} + \frac{L_1}{3\lambda}\Big] 2t\, . \end{aligned} \tag{4.19}$$

The Hubble parameters and average Hubble parameter are given by

$$\begin{aligned} H_1 &= \Big(\frac{2t}{5} + q_1\Big)(t^2 + 1)^{-1}\, , \qquad H_2 = \Big(\frac{2t}{5} + q_2\Big)(t^2 + 1)^{-1}\, , \\ H_3 &= \Big(\frac{2t}{5} + q_3\Big)(t^2 + 1)^{-1}\, , \qquad H_4 = \Big(\frac{2t}{5} + q_4\Big)(t^2 + 1)^{-1}\, , \\ H_5 &= \Big(\frac{2t}{5} + q_5\Big)(t^2 + 1)^{-1} \text{ and } H = \frac{2t}{5}(t^2 + 1)^{-1}. \end{aligned} \tag{4.20}$$

The expansion scalar, shear scalar and mean anisotropic parameters are given by

$$\theta = V^i_{;i} = \frac{A_6}{A} + \frac{B_6}{B} + \frac{C_6}{C} + \frac{D_6}{D} + \frac{E_6}{E} = \frac{2t}{(t^2 + 1)}\, , \tag{4.21}$$

$$A_m = 0\, , \tag{4.22}$$

and

$$\sigma^2 = \frac{1}{2}(\sum_{i=1}^{5} H_i^2 - \frac{1}{5}\theta^2) = 0\, , \tag{4.23}$$

we have also caculated deceleration parameter which is given by

$$q = \frac{\partial}{\partial t}\Big(\frac{1}{H}\Big) - 1 = \frac{3t^2 - 5}{2t^2}\, . \tag{4.24}$$

We observed that the model has no initial singularity at $t = 0$ and the Hubble parameter H, the expansion scalar θ, the shear scalar σ, the effective pressure $\overline{p}$, the energy density ρ, and the coefficient of the bulk viscosity ζ are finite at $t = 0$. Also, we can see that the ratio $\lim_{t\to\infty} \Big(\frac{\sigma}{\theta}\Big)^2 = 0$, therefore, this model will be anisotropic through the evolution of the universe.

5 Graphs and Conclusion

From Figure 1 and Figure 2 it is clear that the expansion scalar θ and the Hubble parameter H both are decreasing functions of time t and both will reach maximum value at the initial point at $t = 0$.

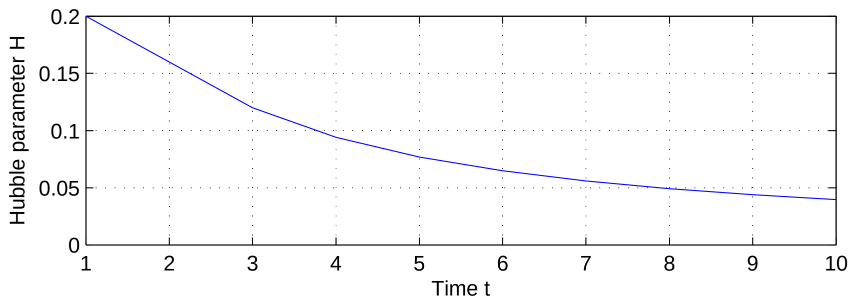

Figure 1. Hubble parameter H vs time t.

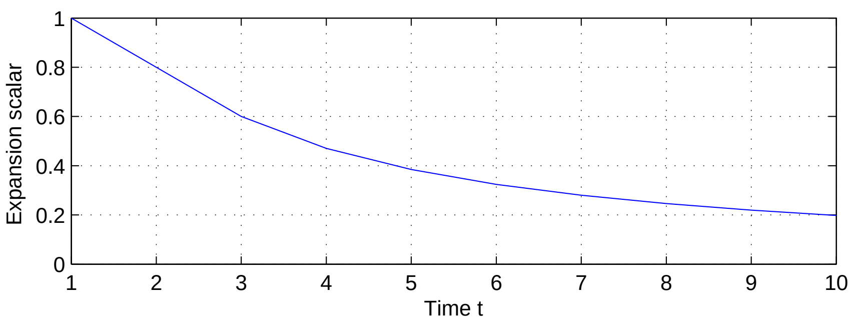

Figure 2. Expansion scalar vs time t.

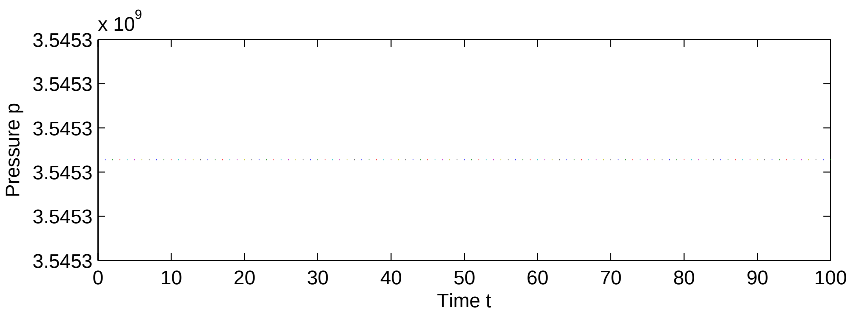

Figure 3. Pressure p vs time t.

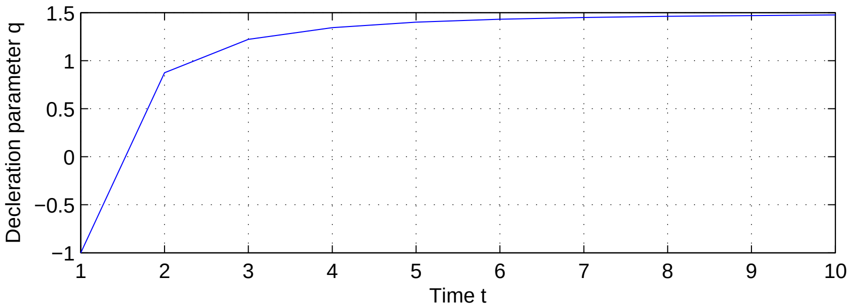

Figure 4. Deceleration parameter q vs time t.

Figure 3 shows that the pressure is independent on time t. From Figure 4 it is identified that the deceleration parameter is increasing function of time t. Initially the rate of increment is large, after some time the rate of increment decreases, and for large value of time t deceleration parameter looks like constant.

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