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Shapes, Quartets and Clusters of Atomic Nuclei in a Semimicroscopic Framework

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Abstract. A symmetry-based semimicroscopic framework is presented, which enables us to find the exotic shapes of light nuclei as well as to relate them to cluster configurations and reaction channels. The connection is realised via the shell model in terms of the multichannel dynamical symmetry. This symmetry can predict a complete high-lying cluster spectrum, based on the description of the low-lying quartet (shell) spectrum. The relation to the symmetry-adapted no-core shell model, as well as to the reaction studies is discussed.

1 Introduction

The basic concepts of our understanding of nuclear structure are related to different physical pictures. The shape or deformation e.g. on which we have rich experimental information is an ingredient of the collective i.e. liquid drop model. This description, however, does not take into account the requirement of the antisymmetrization of the many-nucleon system. On the other hand it is treated properly in shell and mean-field models. Furthermore, for handling the decay properties or nuclear reactions, the cluster models are also needed.

Having this diverse nature of the fundamental structure models, it is essential to understand their interrelation, and to find their common intersection. In this respect the symmetry-considerations proved to be extremely helpful.

Elliott [1] showed that the quadrupole deformation and the collective rotation can be understood in terms of the spherical shell model based on the SU(3) symmetry. Soon after the formulation of the cluster model [2] Bayman and Bohr pointed out [3] that the SU(3) symmetry is a common intersection of the cluster, shell and collective models. At this early stage the investigations and the conclusions concerned a single major shell. Later studies incorporated many major shell excitations, and it turned out that the intersection of the fundamental structure models is given by the SU(3) $\otimes$ SU(3) $\supset$ SU(3) dynamical symmetry [4].

In this contribution we discuss exotic nuclear shapes with special attention on the role of symmetries in finding them, and relating them to different structure models and nuclear reactions. For this purpose we apply a semimicroscopic approach, in which the model space is constructed microscopically, and it is combined with phenomenological interactions. The description is fully algebraic, i.e. not only the basis states, but also the physical operators carry group symmetries.

It turns out that the composite dynamical symmetry, found as the common intersection of the fundamental structure models, can be very helpful from the "practical" viewpoint as well. For instance, it enables us to predict a complete high-lying cluster spectrum from the description of the low-lying shell (quartet) spectrum.

We discuss here, how the symmetry-adapted semimicroscopic framework offers a unified approach to different physical phenomena, from exotic shapes via quartet and cluster excitations up to reactions. It builds up a bridge between ab initio no-core shell model approach and an easy-to-apply phenomenological description of high-lying molecular resonances.

2 Shapes

The deformed harmonic oscillator shows large degeneracies, i.e. well-developed shell structure at some special ratios of the major axes, like e.g. for the superdeformed (2:1:1), hyperdeformed (3:1:1), etc. shapes [5]. At these deformations the SU(3) symmetry is found to be valid [6], just like in case of the spherical oscillator.

It is also a joint conclusion of many *realistic* studies of nuclear structure that the superdeformed, hyperdeformed, etc. shapes are especially stable [7]. A typical way of seeing this feature is looking at the potential energy surface as a function of the quadrupole deformation. A schematic illustration is shown in Figure 1. The ground state of the nucleus sits in the valley of the absolute minimum, which is usually located at very small deformation. As the deformation grows, the energy increases, however, at large deformation a second and third minima show up, corresponding to the superdeformed and hyperdeformed shape.

Recently, we have found an alternative way of observing the stability of the nuclear deformation [8]. It is based on the investigation of the stability and self-consistency of the quadrupole deformation. In particular, Nilsson-model calculations are carried out systematically for changing β and γ parameters in small steps, and from the distribution of the nucleons on the single-particle orbitals

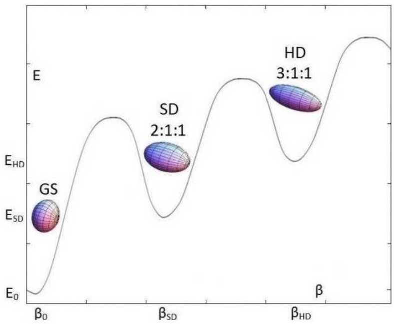

Figure 1. (color online) Schematic illustration of the appearance of the superdeformed and (SD) hyperdeformed (HD) shapes in the energy-plot.

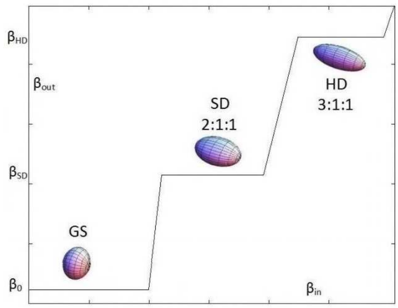

Figure 2. (color online) Schematic illustration of the appearance of the SD and HD shapes in the deformation self-consistency calculation.

(according to the energy-minimum and Pauli-principle) the resulting deformation is obtained. These functions show regions of stability, where the outcome is stable with respect to small changes of input, i.e. horizontal plateaus appear, as shown in Figure 2 (for $\gamma = 0$). They represent stable shapes of the nucleus. At these values the self-consistency of the quadrupole deformation is also fulfilled to a good approximation. The stable shapes obtained from the deformation self-consistency calculations are either in complete coincidence with the local energy minima of the traditional method, or they are very close to them [9].

The actual calculation is carried out based on the quasi-dynamical SU(3) symmetry [10]. The quasi-dynamical symmetry is a very general symmetry-concept of the quantum mechanics, it refers to a situation, when neither the operator is symmetric (scalar), nor are its eigenvectors (i.e. they do not transform according to a single irreducible representation) [11], yet the symmetry is valid for a subset of the Hilbert space, either exactly, or approximately. The distribution of nucleons on the asymptotic Nilsson-orbitals defines the quasi-dynamical SU(3) symmetry of the many-nucleon system [12], and this symmetry determines the parameters of the quadrupole deformation [13]:

$$\beta^2 = \frac{16\pi}{5N_0^2}(\lambda^2 + \mu^2 + \lambda\mu), \qquad \gamma = \arctan\left(\frac{\sqrt{3}\mu}{2\lambda + \mu}\right). \tag{1}$$

For light nuclei, where the real SU(3) is a good approximate symmetry, the quasi-dynamical one coincides with it [14].

Having the (real or quasidynamical) SU(3) symmetry of the shape isomers one can apply a selection rule [15] in order to select the allowed binary cluster configurations:

$$[n_1^{C_1}, n_2^{C_1}, n_3^{C_1}] \otimes [n_1^{C_2}, n_2^{C_2}, n_3^{C_2}] \otimes [n_R, 0, 0] = [n_1, n_2, n_3] \oplus ..., \tag{2}$$

where $[n_1, n_2, n_3]$ is the set of U(3) quantum numbers, C_i refers to cluster i, and R stands for the relative motion. The $[n_1, n_2, n_3]$ representation labels define uniquely the quadrupole shape of the nucleus via $\lambda = n_1 - n_2$, $\mu = n_2 - n_3$, and Eq. (1). From the physics viewpoint the selection rule we apply indicates the similarity of the mass distributions in the shell model and in the cluster model descriptions (taking into account deformed clusters, and arbitrary relative orientations).

A cluster configuration is obviously related to a reaction channel (actually it is defined by a reaction channel), and thus the cluster configurations provide us with useful information on the possible nuclear reactions to populate the shape isomers. This is the most valuable contribution of our new method, based on the stability and self-consistency of deformation (and symmetry).

For the determination of the shape isomers several models are applied: large-scale shell model, mean-field, Nilsson-model, alpha-cluster-model, anti-symmetrized molecular dynamics, etc. The results are usually in good agreement with each other, giving a strong support to the theoretical prediction. As for the relation to the allowed cluster configurations and preferred reaction channels, we have much less information, that is why our stable-symmetry-method can be helpful.

The SU(3) symmetry of light nuclei is an approximate one. It is very remarkable, that just as in case of the deformed harmonic oscillator [6] also for the much more realistic Nilsson-interactions, the SU(3) recovers for the commensurable major axes [8]. The validity of the quasi-dynamical SU(3) symmetry is even more approximate. Nevertheless, it is uniquely related to the quadrupole deformation, and a shell-model configuration can be associated to it, thus it represents a well-defined approximation.

3 Quarteting

Although the shell model reduces considerably the many-nucleon-problem by freezing the nucleons of the core, yet the computational task can still be very large. Therefore, further models were introduced in order to achieve more simplification. For example, the even-even nuclei can be described successfully by the interacting boson model [16], in which the basic building blocks are nucleon-pairs, i.e. pairs of the valence shell.

The quartet models arise from a similar idea. In these models the basic building block is formed by four nucleons: two protons and two neutrons. The quartet models have a long history, their origin goes back to Wigner's supermultiplet theory [17]. The effective interaction between the nucleons is short-range and attractive. This force tend to arrange the nucleons in such a configuration which provide large spatial overlap between their wavefunction. The best arrangement is obtained by putting the nucleons on the same single-particle orbital, and the exclusion principle allows two protons and two neutrons to occupy a single-particle orbit. These four nucleons have a very strong interaction among each other, they form a quartet. The interaction between different quartets is weaker.

For a long time the quartet models were applied only for the description of the ground state properties of the atomic nuclei, especially for the explanation of the binding energy. In the 70th's, however, it was proposed that the four nucleons may jump to another, higher-lying orbital, giving in this way the excitation spectrum of the quartets [18]. An important generalization of the concept of quartets was introduced by Harvey [19], defining them by their symmetry properties: two protons and two neutrons form a quartet if their SU(4) symmetry is {1,1,1,1} i.e. they form a Wigner-scalar. This is a much wider definition of he quartets, due to the fact that four nucleons may have such a symmetry, even if they sit on different major shells. Therefore, the models of this generalized quartet concept can incorporate 0, 1, 2, etc major shell excitations, as compared to the much more limited 0, 4, 8,... excitations of the original concept [18].

In the 70th's the excitation spectra of the quartets were described by models, which used empirical matrix elements. They did not have the elegant and efficient algebraic framework, like the shell [1] or interacting boson models [16]. The quartet models were equipped with a similar group-theoretical framework only recently [20]. (In the 80th's algebraic quartet models were also introduced [21, 22], but they were not based on the shell-like quartet concepts, rather they applied bosons as basic building blocks.)

For the description of the shell-like quartets Elliott's SU(3) formalism could be imported [20]. When it is combined with the simpler quartet-concpet of [18], then we end up with a phenomenologic algebraic quartet model, while when the symmetry-based quartet definition of [19] is applied, the result is a semimicroscopic algebraic quartet model. (A model is called phenomenologic if it has phenomenologic model space, i.e. the Pauli-principle is not, or not fully included, and the interactions are also phenomenologic ones, with parameters to fit. A semimicroscopic model takes into account the Pauli-principle, i.e. its model space is constructed microscopically, but the interactions are phenomenologic ones. The fully microscopic model combines a microscopic model space with nucleon-nucleon forces.)

The algebraic structure and the set of the relevant quantum numbers of the semimicroscopic algebraic quartet model (SAQM), concerning the space part is:

$$\begin{array}{c} U(3) \supset SU(3) \supset \ SO(3) \supset SO(2) \\ |[n_1, n_2, n_3], (\lambda, \mu) \ \ K \ \ , \ \ L \ \ , \ \ \ M > \ . \end{array} \tag{3}$$

The shell model connection of our quartet description is transparent. The model space is a symmetry-governed truncation of that of the no-core shell model (NCSM). It can be realised most easily by applying the symmetry adapted no-core shell model (SA-NCSM) [23, 24] which also uses L-S coupling and SU(3) basis. The SA-NCSM applies proton-neutron formalism, i.e. its basis states are:

$$|S_p S_n S N_p(\lambda_p, \mu_p) N_n(\lambda_n, \mu_n) N(\lambda, \mu) KLJM >,$$

where S_p, S_n and S are the proton, neutron and coupled spins, N is the number of oscillator quanta, (λ, μ) gives the spatial SU(3) symmetry, L is the orbital, J is the total angular momentum with projection M, and K distinguishes different L's within an (λ, μ) representation. In the quartet model the spin-isospin formulation is used: $|[f_1 f_2 f_3 f_3] STT_z N(\lambda, \mu) KLJM >$, where $[f_1 f_2 f_3 f_3]$, is the set of quantum numbers of Wigner's U(4) spin-isospin symmetry, and T and T_z are the isospin and its z component. The correspondence between the two model spaces is straightforward, as illustrated in Table 1, for the lowest-lying two major shells of the ^{16}O nucleus (before removing the centre of mass excitation).

The realistic SA-NCSM calculations incorporate many major shell excitations, while the quartet model calculations in the limit of the dynamical symmetry represent an effective model in the sense of [25]. It means that the bands of different quadrupole shapes are described by their lowest-grade U(3) irreducible representations (irreps) without taking into account the giant-resonance excitations built upon them, and the model parameters are renormalised for the subspace of the lowest U(3) irreps. Work is in progress in order to reveal the exact microscopic content of the effective quartet model description. In particular, the results will be compared with those of the SA-NCSM for the description of the shape-coexistence in the ^{16}O nucleus.

The quartet model connects the shell model to the cluster model. This is natural, if we recall that the best know cluster is an alpha-particle, which consists of two protons and two neutrons, just like a qurtet. The exact relation between the qaurtets and the clusters is provided by some symmetry-considerations.

Table 1. List of the SU(3) irreducible representations spanning the the lowest-lying major shells of the ^{16}O nucleus in the spin-isospin and proton-neutron formalisms. The angular momentum content of the SU(3) representations, and their coupling to the spin are the same in both methods, therefore, they are not indicated. When a quantum number is missing, it is identical with the one in the previous line.

$\hbar\omega$	$[f_1 f_2 f_3 f_4]$	STT_z	$N(\lambda\mu)$	$S_p S_n$S	$N_p(\lambda_p \mu_p)$	$N_n(\lambda_n \mu_n)$	$N(\lambda\mu)$
0	[4,4,4,4]	0 0 0	0(0,0)	0 0 0	0(0,0)	0(0,0)	0(0,0)
1	[4,4,4,4]	0 0 0	1(2,1)	0 0 0	1(2,1)	0(0,0)	1(2,1)
			(1,0)		(1,0)		(1,0)
	[5,4,4,3]	0 1 0	1(2,1)		0(0,0)	1(2,1)	1(2,1)
			(1,0)			(1,0)	(1,0)
		1 0 0	1(2,1)	1 0 1	1(2,1)	0(0,0)	1(2,1)
			(1,0)		(1,0)		(1,0)
		1 1 0	1(2,1)	0 1 1	0(0,0)	1(2,1)	1(2,1)
			(1,0)			(1,0)	(1,0)

4 Clustering

A cluster configuration is similar to that of a molecule; the nucleus is thought to be built up from smaller nuclei (clusters) which are especially stable, e.g. alpha particles. There are many cluster models available, the oldest one (the alpha-model by George Gamow [26]) was actually invented before the discovery of the neutron, i.e. before the nucleon model of the nucleus.

In what follows we consider here the semimicroscopic algebraic cluster model (SACM) [27]. The reason for selecting this model is the following. For studying the interrelations of the basic structure models, microscopic model spaces are needed, i.e. fully or semimicroscopic models should be considered. (The overlap of the seemingly different model wavefunctions energies from the effect of the antisymmetrization.) Furthermore, the intersections of different models usually revealed by their symmetry properties, therefore, fully algebraic description is preferred. The SACM fulfills these requirements.

In this framework the internal structure of the clusters is described by the Elliott model, while the relative motion is taken care by the vibron model [28], which is an algebraic ($\mathrm{U}_R(4)$) model of the dipole motion. Therefore, the algebraic structure of the model for a two-cluster-configuration is: $\mathrm{U}^{ST}_{C_1}(4) \otimes \mathrm{U}_{C_1}(3) \otimes \mathrm{U}^{ST}_{C_2}(4) \otimes \mathrm{U}_{C_2}(3) \otimes \mathrm{U}_R(4)$.

In the construction of the model space the role of the spin-isospin symmetry is essential, of course. When, however, the investigation is restricted to a single $\mathrm{U}^{ST}(4)$ symmetry, which is typical in cluster studies, the relevant group-chain simplifies to:

$$U_{C_1}(3) \otimes U_{C_2}(3) \otimes U_R(4) \supset U_C(3) \otimes U_R(3) \supset U(3) \supset SU(3) \supset SO(3) \quad (4)$$

5 Connection

As mentioned in he Introduction, the common intersection of the shell, collective, and cluster models is an SU(3) dynamical symmetry for the single shell problem, and an SU(3)⊗SU(3) symmetry for the multi-major-shell excitations [29]. In particular, the relevant quantum numbers are provided by the group-chain:

$$\begin{array}{c} U_x(3) \;\otimes\; U_s(3) \;\;\supset U(3) \;\supset SU(3) \;\supset SO(3) \supset SO(2) \\ |[n_1^x, n_2^x, n_3^x], [n_1^s, n_2^s, n_3^s], [n_1, n_2, n_3], (\lambda, \mu) \;\; K \;\; , \;\; L \;\; , \;\; M > \; . \end{array} \quad (5)$$

Here x refers to the excitation, and s stands for the valence shell, or for the internal shell structure of the clusters. The same symmetry connects different cluster configurations as well [30, 31], and it is called multichannel dynamical symmetry (MUSY). (It holds, when both cluster configurations can be described by an U(3) dynamical symmetry and, in addition, a further symmetry connects them to each other. This latter symmetry acts in the pseudo space of the particle indices.)

The MUSY has two interesting aspects. In addition to connecting the fundamental structure models to each other, that can be considered as a principal interest, it has a practical interest as well. This latter feature comes from its predictive power, due to the fact that it unifies the multiplet structure of a shell (or quartet) and a cluster model (or those of two different cluster configurations), and applies the same physical operators. As a result it may be able to predict a high-lying cluster spectrum from the description of a low-lying quartet spectrum. In what follows we derive the spectrum of the ^{12}C+^{16}O clusterization from the quartet spectrum of the ^{28}Si nucleus.

6 An Application

Figure 3 (lower left part) shows the experimental bands of the ^{28}Si nucleus, as established in [32] together with the recently found superdeformed (SD) band [33–35]. An especially favourable circumstance is that SU(3) quantum numbers are associated to several experimental bands, without any reference to the quartet or cluster studies.

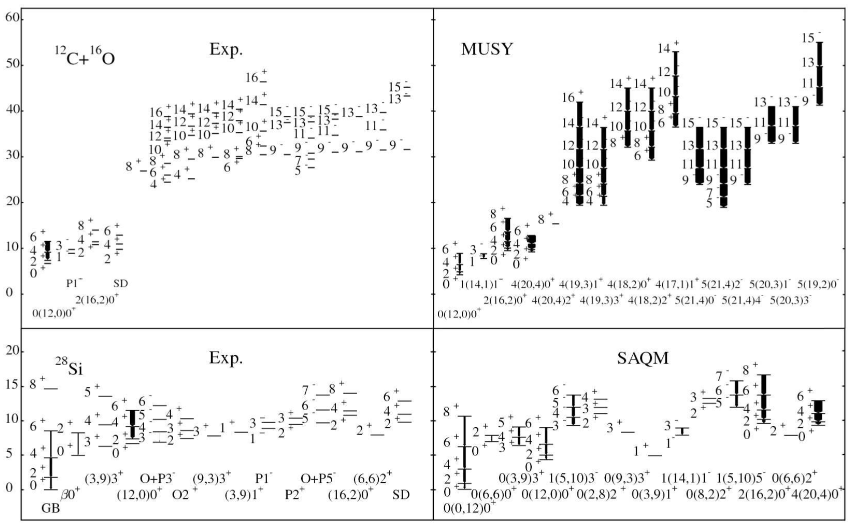

Figure 3. Unified description of the low-lying quartet and high-lying cluster spectra of the ^{28}Si nucleus in terms of the multichannel dynamical symmetry. The model parameters have been fitted to the low-energy part (lower-panel), and the cluster spectrum (upper panel) is obtained as a pure prediction, due to the unified multiplet-stucture and identical physical operators.

The theoretical spectrum (lower right part) is calculated within the SAQM approach [36]. We have applied a U(3) dynamically symmetric Hamiltonian, which is invariant with respect to the transformation between the quartet and cluster models [36]. In particular, it is expressed in terms of the invariant operators of the group-chain: U(3) ⊃ SU(3) ⊃ SO(3):

$$\hat{H} = (\hbar\omega)\hat{n} + a\hat{C}^{(2)}_{SU3} + b\hat{C}^{(3)}_{SU3} + d\frac{1}{2\theta}\hat{L}^2. \tag{6}$$

$\hbar\omega$ is calculated from the systematics [37], while a, b, d are fitted to the experimental data: $a = -0.133$ MeV, $b = 0.000444$ MeV, $d = 1.003$. θ is the moment of inertia calculated classically for the rigid shape determined by the $[n_1, n_2, n_3]$ U(3) quantum numbers [20]. In particular, the ratio of the semi-major axes z, x, and y is obtained from a self-consistency argument [38]:

$$\frac{z}{y} = \frac{n_1 + \frac{A}{2}}{n_3 + \frac{A}{2}}, \qquad \frac{x}{y} = \frac{n_2 + \frac{A}{2}}{n_3 + \frac{A}{2}}. \tag{7}$$

Their lengths are determined by the volume conservation:

$$y = R_0 \sqrt[3]{A\frac{(n_3 + \frac{A}{2})^2}{(n_1 + \frac{A}{2})(n_2 + \frac{A}{2})}}\ . \tag{8}$$

(Here we applied $R_0 = 1.2fm$.) The moment of inertia (in units of $\frac{\hbar^2}{MeV}$) for a rotor with axial symmetry ($x = y$) is given by

$$\theta = \frac{1}{5}m(z^2 + x^2). \tag{9}$$

The high-lying ^{12}C+^{16}O cluster states are those in the second, superdeformed valley (where the SD state corresponds to the "ground"-band) of the ^{28}Si nucleus. The upper left panel shows the resonance spectrum from the heavy ion experiments, according to the compilation of [39]. These states are organised into bands based on their energy-differences.

A similarly good agreement was found between the prediction of the MUSY and the experimental results on the high-lying 0^+ spectrum of alpha-scattering [40].

7 Reactions

By revealing the allowed cluster configurations of a shell model state we get useful information on the possible reaction channels which can populate the state, or in which it can decay. Therefore, the structure studies provide us with some hint on the reactions that can produce the shape isomer states. This is, however, only qualitative "yes-or-no" information of a selection rule. In order to obtain quantitative information one needs to calculate cross sections from a reaction theory. For this purpose the statistical pre-equilibrium (exciton) model [41] was generalized recently. This reaction model incorporates both the (equilibrated) compound nucleus and the fast direct reactions as limiting cases, as well as the situations in between. In [42, 43] the model was extended in order to describe cluster emission from heavy ion collisions. In particular, the coalescence treatment of the clusters [44, 45] was applied, first without taking into account the angular momentum degree of freedom [42], then by incorporating also the spin-dependence [43].

8 Conclusion

We have discussed different physical phenomena and their relations within a symmetry-based semimicroscopic framework, as shown in Table 2.

Table 2. Comparison of different physical phenomena, models, methods and symmetries. The last column shows their experimental observations.

Phenomenon	Model	Method	Symmetry	Experiment
shape isomers	Nilsson	q-self-consistency	quasi-dynamical	superdef., hyperdef.
quarteting	SAQM	effective model	dynamical	ground-region spectrum
clustering	SACM	various configur.	multichannel	cluster spectra (predict.)

The method seems to be easily applicable for the description of experimental data, and due to its transparent symmetry-properties it sheds some light on the connection of different phenomena and models (including the ab inito approach). We find especially remarkable the predictive power of the multichannel dynamical symmetry. In case of the ^{28}Si nucleus e.g. it was able to predict a complete high-lying spectrum of the ^{12}C+^{16}O clusterization from the description of the low-lying quartet spectrum.

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