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Introduction of Fermion Degrees of Freedom in IVBM for the Description of Spectra of Odd Nuclei

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Abstract. A very accurate results for the energy spectra of the even-even nuclei have been obtained in a new more physical application of the dynamical symmetry:

$$Sp(12, R) \supset U(6) \supset U(3) \otimes U(2) \supset O(3) \otimes (U(1) \otimes U(1))$$

of the Interacting Vector Boson Model (IVBM). The description of odd-mass nuclei requires the introduction of fermion degrees of freedom, that we make through the total moment $\mathbf{I} = l_n + s_n$ for a single fermion n coupled to the angular momentum L of the states of the ground and octupole bands of the even-even core $\mathbf{J} = \mathbf{L} + \mathbf{I}$. From an algebraic point of view, this can be presented as an extension of the dynamical symmetry group $Sp(12, R)$ of the IVBM to the orthosymplectic group $OSp(2\Omega/12, R)$, which contains the direct product of $SO^F(2\Omega)$ of the fermion degrees of freedom and the boson part $Sp^B(12, R)$ describing the even-even system. The approach can be extended for the description of the spectra of odd-odd nuclei as well. The results are illustrated by the comparison of the theory with the experimental spectra of the ground and octupole bands ot the even-even nuclei ^{236}U and ^{238}Pu and the long collective bands build on them in the odd ^{237}U and ^{239}Pu.

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1 Introduction

There have been a large number of theoretical investigations concerning odd nuclei but few are based on a phenomenological algebraic models, like the Interacting Boson Model (IBM) [1], which describe in their dynamical symmetries different types of collective modes. The later in a conjunction with conveniently associated Bose-Fermi symmetries of [2] have enriched our understanding of the structure of low-lying collective states in heavy even-even and odd-mass nuclei respectively.

In the early 1980s, a boson-number-preserving version of the phenomenological algebraic Interacting Vector Boson Model (IVBM) [3] was introduced and applied successfully [4] to a description of the low-lying collective rotational spectra of the even-even medium and heavy mass nuclei. With the aim of extending these applications to incorporate new experimental data on states with higher spins and to incorporate new excited bands, we explored the symplectic extension of the IVBM [5], for which the dynamical symmetry group is $Sp(12, R)$. This extension is realized from, and has its physical interpretation over the basis states of its maximal compact subgroup $U(6) \subset Sp(12, R)$, and resulted in the description of various excited bands of both positive and negative parity of complex systems exhibiting rotation-vibrational spectra. In [6] an orthosymplectic extension of the IVBM was carried out in order to encompass the treatment of the odd-mass nuclei. With the present work we review again the boson-fermion extension [6] of IVBM with the aim to describe in a more accurate and physically meaningful way the spectra of odd mass nuclei. This is achieved by introducing fermion degrees of freedom explicitly into the symplectic IVBM, where we choose the set of the basis states that correspond to the experimentally observed ones in the even-even core by redefining the parity in a more appropriate way. We approach the problem by considering the simplest physical picture in which a single particle with the moment $I = 1/2$ is coupled to the even-even nucleus whose states belong to an $Sp(12, R)$ irrep. In this paper we demonstrate that the results for the energy spectra obtained in this simplified version of the model agree rather well with the experimental data for some heavy nuclei

2 The Even-Even Core Nuclei

The algebraic structure of the IVBM is realized in terms of creation (annihilation) operators $u_m^\dagger(\alpha)(u_m(\alpha))$, in a 3-dimensional oscillator potential $m = 0, \pm 1$ of *two* types of bosons, differing by the value of the projection $\alpha = 1/2(p)$ or $\alpha = -1/2(n)$ of an additional quantum number, called "pseudo-spin" T. The pseudo-spin has the properties of the F-spin in IBM-2 [1]. These operators satisfy the boson commutation relations

$$\left[u^m(\alpha), u_n^\dagger(\beta)\right] = \delta(\alpha, \beta)\, \delta_{m,\, n}$$

and Hermitian conjugation rules $\left[u_m^\dagger(\alpha)\right]^\dagger = u^m(\alpha)$, $\left[u^m(\alpha)\right]^\dagger = u_m^\dagger(\alpha)$.

The operators $u_m^\dagger(\alpha)(u_m(\alpha))$ are by definition three-dimensional vectors ($l = 1, m = 0, \pm 1$) with respect to the group $O(3)$) belonging to two independent representations $(1, 0)$ of the group $SU(3)$ (the annihilation operators belong to the representation $(0, 1)$ [7]). The transformation properties of the vector bosons used as building blocks of the algebraic structure of the model, define in analogy with the Elliott's oscillator quarks [8] and the excitations of the p-shell of the microscopic shell model.

Unlike our previous assumptions [9], in this paper we will further use this definition for the construction of the algebraic structure and basis states.

The bilinear combinations of the vector bosons close under commutation the algebra of the symmetry group $Sp(12, R)$ and the number preserving part of them generates the maximal compact subgroup of $U(6) \subset Sp(12, R)$. In this work we aim to review again the application of one of the dynamical symmetries [9] of the symplectic extension of the Interacting Vector Boson Model /IVBM/ [3], which was used for the description of the behavior of the ground and octupole bands in the heavy even-even nuclei. We revise the assignment of sequences of states classified in the boson representation of the $Sp(12, R)$ algebra to the experimentally observed low laying even and odd parity states clarifying in particular the definition of their parity from the physical point of view. This new approach does not simplify the description of these bands, but increases its accuracy up to rather high spins. It includes the consideration of some additional excited 0^+ bands, which as is empirically observed strongly influence the behavior of the ground and octupole bands [10]. We show that the energies of the considered collective bands are strongly dependant on the number of bosons that build their band head configurations [11]. This once again outlines the advantage of the symplectic extension of the IVBM, which is based on the construction of the basis states from different number of boson excitations.

Being a noncompact group, the unitary representations of $Sp(12, R)$ are of infinite dimension, but when reduced to the group $U^B(6)$, each irrep of the group $Sp^B(12, R)$ decomposes into irreps of the subgroup characterized by the partitions $[N, 0^5]_6 \equiv [N]_6$ [5], where $N = 0, 2, 4, \ldots$ (even irrep) or $N = 1, 3, 5, \ldots$ (odd irrep).

For the description of the low lying collective bands we employ the "unitary" chain

$$Sp(12, R) \supset U(6) \supset SU(3) \otimes U(2) \supset SO(3) \otimes U(1). \tag{1}$$

$$[N] \quad (\lambda, \mu) \quad (N, T)\; K \quad L \qquad T_0 \tag{2}$$

The labels below the subgroups are the quantum numbers (2) corresponding to their irreducible representations. Hence, all the possible irreducible representations: $[N]$ of $U(6)$, (λ, μ) of $SU(3)$, K which define the multiplicity of the the angular momentum L and its projection M in the final reduction to the $SO(3)$ representations are determined uniquely through all possible sets of the eigenvalues of the Hermitian operators $\mathbf{N}$, $\mathbf{T^2}$, $\mathbf{T_0}$ and $\mathbf{L^2}$ reducing the symplectic extension $sp(12, R) \supset u(6)$ [9] to "the unitary" limit of the model [3]. Making use of the latter we can write the basis as

$$\mid [N]_6; (\lambda, \mu); K, L, M; T_0\rangle = \mid (N, T); K, L, M; T_0\rangle \tag{3}$$

The ground state of the system is the vacuum state $|0\rangle$ with $N = 0, T = 0, K = 0, L = 0, M = 0, T_0 = 0$.

The Hamiltonian, corresponding to the unitary limit of IVBM (1) expressed in terms of the first and second order invariant operators of the different subgroups in the chain is [5]:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2. \tag{4}$$

H (4) is obviously diagonal in the basis (3). Its eigenvalues are the energies of the basis states of the boson representations of $Sp(12, R)$:

$$E((N, T), L, T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2. \tag{5}$$

The most important application of the $U^B(6) \subset Sp^B(12, R)$ limit of the theory is the possibility it affords for describing both even and odd parity bands up to very high angular momentum [5]. In order to do this we first have to identify the experimentally observed bands with the sequences of basis states of the even or odd $Sp(12, R)$ irreducible representation (irreps). As we deal with the symplectic extension of the boson representations of the number preserving $U^B(6)$ symmetry we are able to consider all even or odd eigenvalues of the number of vector bosons N with the corresponding set of $T-$spins, which uniquely define the $SU^B(3)$ irreps (λ, μ), since $T = \frac{\lambda}{2}$ and $N = 2\lambda + \mu$. The multiplicity index K appearing in the final reduction to the $SO(3)$ is related to the projection of L on the body fixed frame and is used with the parity (π) to label the different bands (K^π) in the energy spectra of the nuclei. For the even-even nuclei we redefine the parity of the states as $\pi_{\text{core}} = (-1)^N$ [10]. This allowed us to describe both positive and negative bands.

The rotational spectra of some even-even nuclei in the rare earth and light actinide region exhibit, next to the ground band, a negative parity band with $K^\pi = 0^-$, which consists of the states with $L^\pi = 1^-, 3^-, 5^-, \ldots$ Such nuclei are supposed to have an asymmetric shape under reflection, associated with a static octupole deformation, which determines this new collective feature of the nuclear system. Experimentally the presence of "octupole" bands for some isotopes from the light actinide and rare earth region is firmly established.

We revise the classification of even and odd parity states in the symplectic multiplets of the $Sp(12, R)$ dynamical symmetry of the Interacting Vector Boson Model /IVBM/, taking into account the new definition of their parity. We consider first the ground band, starting with the vacuum state $|\ (0, 0); 0, 0, 0; 0\rangle$, and extending along the $SU(3)$ multiplets $(\lambda, 0)$ with $L = 0, 2, 4, \ldots$. In this case from the reduction rules follows that $N = L$ and the pseudospin eigenvalues are $T = N/2 = L/2$ with $T_0 = 0$. This assignment is rather similar to the one of the rotational $SU(3)-$limit of the Interacting Boson Model [1] and correctly reflects the increase of the quadrupole deformation through the increase of $\lambda >> \mu$ with the increase of the angular momentum L.

Thus, taking into account the reduction rules, relating N, T and L with $T_0 = 0$, the energy (5) can be rewritten only in terms of the angular momentum L in the following way:

$$E_g^{\text{core}} = aL + bL^2 + \alpha_3 \frac{L}{2}\Big(\frac{L}{2} + 1\Big) + \beta_3 L(L+1) = \beta_g L(L + \Omega_g), \qquad (6)$$

where $\beta_g = b + \alpha_3/4 + \beta_3$ gives the inertia parameter of the band, and $\Omega_g = (a + \alpha_3/2 + \beta_3)/\beta_g$ is a geometrical parameter, describing the deviation of the bands'energies from the rigid rotor behavior. It is important to note that in this assignment of basis states to the states of the ground band the values of the pseudo-spin T is maximal for the given $N = L$ and changes for each of the band states.

The new assignment of the parity, places the octupole band in the N-odd space of $Sp(12, R)$. This, as explained above, ensures the negative parity of its states. Since the gsb and the lowest 0^- band are usually considered as one and the same band with alternating parity of its states, we choose a band from N-odd space that is very similar to the ground state band, studied above. Hence for the octupole band we take the sequence of $SU(3)$ irreps $(\lambda, 0)$ with $\mu = 0$-fixed $(K = 0)$ and $\lambda = \lambda_0 + 2i$, $i = 0, 1, 2\ldots$, with $L = 2i + 1$ changing with the connection $N = \lambda = \lambda_0 + 2i = N_0 + L - 1$. In this case $T = \lambda/2$ is half-integer and also changing. Finally for the energies of the negative parity bands we get

$$\begin{aligned} E_{\text{oct}}^{\text{core}} &= a(L + N_0 - 1) + b(L + N_0 - 1)^2 \\ &+ \frac{\alpha_3}{2}\Big(\frac{1}{2}(L + N_0 - 1) + 1\Big)(L + N_0 - 1) + \beta_3 L(L+1) + \alpha_1. \end{aligned} \qquad (7)$$

After some simple transformations for the energies of the negative parity bands we get:

$$E_{\text{oct}}(L) = \beta_{gr}\, L\,(L + \Omega_{\text{oct}}) + \Big(2b + \frac{\alpha_3}{2}\Big) L N_0 + C. \qquad (8)$$

where β_{gr} is defined as in the gsb, hence it could be said that the two bands have the same inertia parameters, but they are not paralel, since

$$\Omega_{\text{oct}} = \frac{a - \alpha_3/2 - 2b + \beta_3}{\beta_g} \neq \Omega_g \qquad \text{and}$$
$$C = a(N_0 - 1) + (b + \alpha_3/4)(N_0 - 1)^2 + \alpha_1\,.$$

From the expressions (6) and (8) it is easy to see that as assigned the ground and the 0^- octupole band have similar behavior, but in (8) the value of N_0 plays an inportant role, defining the bandhead's position. In this case there is one more hamiltonion parameter α_1 that has to be fitted.

3 Fermion Degrees of Freedom

In order to incorporate the fermion degrees of freedom into the symplectic IVBM, we extend the dynamical algebra of $Sp(12, R)$ to the orthosymplectic algebra of $OSp(2\Omega/12, R)$ [6]. For this purpose we introduce a particle (quasi-particle) with moment I and consider a simple core plus particle picture. Thus, in addition to the boson collective degrees of freedom (described by dynamical symmetry group $Sp(12, R)$) we introduce creation and annihilation operators $a^{\dagger}_m$ and a_m ($m = -I, \ldots, I$), which satisfy the anticommutation relations

$$\{a^{\dagger}_m, a^{\dagger}_{m'}\} = \{a_m, a_{m'}\} = 0, \{a_m, a^{\dagger}_{m'}\} = \delta_{mm'}. \tag{9}$$

All bilinear combinations of a^{+}_m and $a_{m'}$, namely

$$f_{mm'} = a^{\dagger}_m a^{\dagger}_{m'}, \;\; m \neq m'; \;\; g_{mm'} = a_m a_{m'}, \;\; m \neq m'; \tag{10}$$

$$C_{mm'} = (a^{\dagger}_m a_{m'} - a_{m'} a^{\dagger}_m)/2 \tag{11}$$

generate the (Lie) fermion pair algebra of $SO^F(2\Omega)$ where $\Omega = 1/2(2I + 1)$. Their commutation relations are given in [6]. The number preserving operators (11) generate maximal compact subalgebra of $SO^F(2\Omega)$, i.e. $U^F(\Omega)$. The upper (lower) script B or F denotes the boson or fermion degrees of freedom, respectively. Further we construct a fermion dynamical symmetry, i.e. the group-subgroup chain for one particle occupying a single level I:

$$SO^F(2\Omega) \supset Sp(2I + 1) \supset SU^F(2). \tag{12}$$

The dynamical symmetry (12) remains valid and for the case of two particles occupying the same or two different levels I_i [12]. Hereafter, for simplicity we will use just the reduction $SO(2\Omega) \subset SU^F(2)$ (i.e. dropping all intermediate subgroups between $SO(2\Omega)$ and $SU^F(2)$) and keep in mind the proper content of the set of I values for one and/or two particles cases, respectively.

Once the fermion dynamical symmetry is determined further the Bose-Fermi symmetries can be constructed. If a fermion is coupled to a boson system having itself a dynamical symmetry (e.g., such as an IBM core), the full symmetry of the combined system is $G^B \otimes G^F$. The standard approach to supersymmetry in nuclei (dynamical supersymmetry) is to embed the Bose-Fermi subgroup chain of $G^B \otimes G^F$ into a larger supergroup G, i.e. $G \supset G^B \otimes G^F$.

Making use of the embedding $SU^F(2) \subset SO^F(2\Omega)$ and the above considerations, we make orthosymplectic (supersymmetric) extension of the IVBM which is defined through the chain [6]

$$
\begin{array}{ccccccc}
OSp(2\Omega/12,R) & \supset & SO^F(2\Omega) & & \otimes & Sp^B(12,R) & \\
 & & & & & \Downarrow & \\
 & & \Downarrow & & \otimes & U^B(6) & \\
 & & & & & N & \\
 & & & & & \Downarrow & \\
 & & SU^F(2) & & \otimes & SU^B(3)\otimes U^B_T(2) & \\
 & & I & & & (\lambda,\mu) \Longleftrightarrow (N,T) & \\
 & & & \searrow & & \Downarrow & \\
 & & & & \otimes & SO^B(3)\otimes U(1) & \\
 & & & & & L \qquad\quad T_0 & \\
 & & & & \Downarrow & & \\
 & & Spin^{BF}(3) & & \supset & Spin^{BF}(2), & \\
 & & J & & & J_0 &
\end{array}
\tag{13}
$$

where bellow the different subgroups the quantum numbers characterizing their irreducible representations are given. Here by $Spin^{BF}(n)$ ($n = 2, 3$) is denoted the universal covering group of $SO(n)$.

We can label the basis states according to the chain (13) as:

$$\mid [N]_6; (\lambda,\mu); KL; I; JJ_0; T_0 \,\rangle \equiv \mid [N]_6; (N,T); KL; I; JJ_0; T_0 \,\rangle, \tag{14}$$

where he $[N]_6, (\lambda,\mu), KL, T$ and T_0 are the quantum numbers characterizing the boson core excitations, $I-$ the orbital momentum of an odd particle, J, J_0 are the total (coupled boson-fermion) angular momentum and its third projection. Since the $SO(2\Omega)$ label is irrelevant for our application, we drop it in the states (14).

The Hamiltonian can be written as linear combination of the Casimir operators of the different subgroups in (13):

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2 + \eta I^2 + \gamma' J^2 + \zeta J_0^2 \tag{15}$$

and it is obviously diagonal in the basis (14) labeled by the quantum numbers of their representations. Then the eigenvalues of the Hamiltonian (15), that yield the spectrum of the odd-mass systems are:

$$\begin{aligned} E(N;T,T_0;L,I;J,J_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3' L(L+1) \\ + \alpha_1 T_0^2 + \eta I(I+1) + \gamma' J(J+1) + \zeta J_0^2. \end{aligned} \tag{16}$$

We note that only the last three terms of (15) come from the orthosymplectic extension.

The basis states (14) can be considered as a result of the coupling of the boson $\mid (N,T); KLM; T_0\rangle$ (3) and fermion $\phi_{I,m}$ wave functions. Then, if the parity of the single particle is π_{sp}, the parity of the collective states of the odd$-A$ nuclei will be $\pi = \pi_{\text{core}}\pi_{sp}$ [6]. Thus, the description of the positive and/or negative parity bands requires only the proper choice of the core band heads, on which the corresponding single particle is coupled to, generating in this way the different odd$-A$ collective bands.

Let consider an odd nucleus as an even-even core plus neutron and choose the simplest for our calculations approximation - nucleon angular momentum $l_n = 0$. Then the total nucleon moment coincides with it's spin s_n. Thus the total moment of the odd nucleus may have two values:$J_1 = L + 1/2$ and $J_2 = L - 1/2$.

$$|J, M; L, s\rangle = \sum_{m_S} |L, 0\rangle \otimes |s,\ m_S\rangle \, C^{J,\ M}_{L,\ 0,\ s,\ m_S} \tag{17}$$

If we take into account the coupling (17) where $m_S = \pm\frac{1}{2}$ and

$$C^{L+1/2,1/2}_{L\ 0,\ 1/2\ 1/2} = (-1)^{2L}\sqrt{\frac{1+L}{1+2L}}, \qquad C^{L-1/2,\ -1/2}_{L\ 0,\ 1/2\ -1/2} = \sqrt{\frac{L}{2L+1}},$$

than for the value of an arbitrary integer angular momentum L $(-1)^{2L} = 1$ we have

$$(C^{L+1/2,\ 1/2}_{L\ 0,\ 1/2\ 1/2})^2 + (C^{L-1/2,\ -1/2}_{L\ 0,\ 1/2\ -1/2})^2 = 1.$$

Taking into account the above couplings, for the considered simple case of a single particle with angular momentum $l_n = 0$ we obtain the following energies for the bands of the odd nucleus:

$$E^{\text{odd}}_{gr} = E^{\text{core}}_{gr} + \xi_{gr} \begin{pmatrix} \dfrac{\left(L-\frac{1}{2}\right)L\left(L+\frac{1}{2}\right)}{2L+1} + C_{gr}; \quad m_s = \frac{1}{2} \\ \dfrac{\left(L+\frac{1}{2}\right)(L+1)\left(L+\frac{3}{2}\right)}{2L+1} + C_{gr}; m_s = -\frac{1}{2} \end{pmatrix} \tag{18}$$

$$E^{\text{odd}}_{\text{oct}} = E^{\text{core}}_{\text{oct}} + \xi_{\text{oct}}, \begin{pmatrix} \dfrac{\left(L+\frac{1}{2}\right)(L+1)\left(L+\frac{3}{2}\right)}{2L+1} + C_{\text{oct}};\ m_s = \frac{1}{2} \\ \dfrac{\left(L-\frac{1}{2}\right)L\left(L+\frac{1}{2}\right)}{2L+1} + C_{\text{oct}}; \quad m_s = -\frac{1}{2} \end{pmatrix} \tag{19}$$

In this simple case, it is obvious, that the last three terms in (16) are replaced with the above expressions (18) and (19), which depend on only two parameters ξ_{gr} and C_{gr} for the gsb and ξ_{oct} and C_{oct} for the octupole band. Each expression is reproducing both the $L + 1/2$ and $L - 1/2$ bands build on the corresponding band of the even- even core. As a result, we obtain the parity doublets $(J_g)^+ = L+1/2$ with $(J_{oct})^- = L-1/2$ and $(J_g)^+ = L-1/2$ with $(J_oct)^- = L+1/2$ in the low lying spectra of the odd nuclei, in particular, when we suppose that the momentum of the odd particle is I=1/2 and it is coupled to the ground and octupole bands, which is observed also in the experiment for the chosen nuclei.

4 Results and Discussion

The aim of our present investigation was to explore the behavior of the collective bands build from coupling a single odd fermion to the alternating parity bands of an even even core. Such an investigation should be performed systematically for the nuclei from the rare-earth and actinides but here we choose only to illustrate our interpretation of these bands on the examples of nuclei that exhibit in experiment [13] rather long ground and octupole bands and their neighboring odd isotopes : ^{236}U,^{237}U and ^{238}Pu,^{239}Pu.

In Figures 1 and 2 respectively we present the comparison between the experimental data and the new theoretical calculations within the $U(6)$ limit of the IVBM [9], with the redefined parity of the states and respectively assignments of the basis states to the experimentally observed one, reviewed in the previous

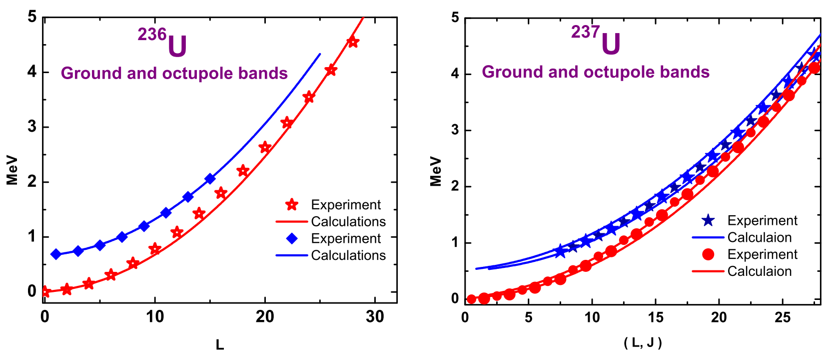

Figure 1. (color online) Theoretical results and experimental data for ground and octupole bands in ^{236}U (left) and ^{237}U (right).

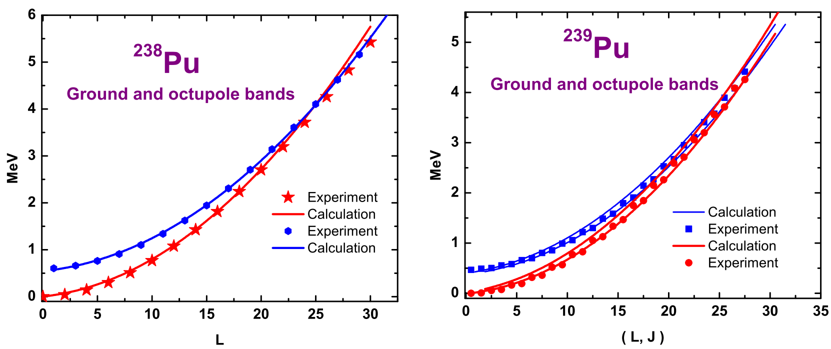

Figure 2. (color online) Theoretical results and experimental data for ground and octupole bands in ^{238}Pu (left) and ^{239}Pu (right).

sections. On left side of the figures are plotted the energies of the ground (6) and octupole bands (7) of the even-even cores and on the right side of the figures – the energies of the $L + 1/2$ and $L - 1/2$ bands of the neighboring odd nuclei build on the corresponding bands of the even-even core (18), (19). A rather good agreement between the theory and experiment is obtained in the presented examples. The parameters of IVBM obtained in the overall fitting of the energies of the assigned levels to the experiment [13] are given in Table 1.

Table 1. Parameters of the Hamiltonians for the even-even core nuclei and their odd neighbors.

Parameters	^{238}Pu and ^{239}Pu	Parameters	^{236}U and ^{237}U
a	0.01675725	a	0.00569132
b	-0.0006792	b	-0.0002316
a_1	0.544861	a_1	0.544861
β_3	0.006	β_3	0.0058
α_3	-0.0021	α_3	-0.000580
Δ_{gr}	0.0033	Δ_{gr}	0.0197
Δ_{oct}	0.0007	Δ_{oct}	0.00008
ξ_{oct}	-0.000837	ξ_{oct}	-0.00124187
ξ_{gr}	-0.0007761	ξ_{gr}	-0.000837
C_{gr}	-0.0432	C_{gr}	0.02
C_{oct}	-0.15588	C_{oct}	-0.15
Δ_{gr}	0.00395	Δ_{gr}	0.0004
Δ_{oct}	0.00139	Δ_{oct}	0.00371

For the ground state bands in both nuclei ^{236}U and ^{238}Pu in Figures 1 and 2, $N_0 = 0$ by definition. From the behavior of the energies of the ground and octupole bands in both nuclei ^{236}U and ^{238}Pu in Figures 1 and 2 it could be seen that with our new assignment of the parity the octupole bands run in parallel with the ground state bands and there they actually form the alternating parity band in the even-even core nuclei. This leads to an accurate description of the energies of their adjacent odd isotopes.

In conclusion we could summarize that in the framework of the IVBM with the present more physical assignment of the parity of the basis states and the presented new correspondence of them to the experimentally observed low laying collective 0^+ and 0^- bands, the properties for the heavy even-even nuclei from the actinides' region are very well reproduced and interpreted. Respectively the simple way of coupling them to a single fermion, which is motivated by the orthosimplectic generalization of the boson model allows for rather simple, but physically meaningful and correct description of their adjacent odd nuclei. This approach can be further generalized for the description of odd-odd nuclei as well.

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