Bulgarian Journal of Physics vol. **XX** (20XX) 1–10

An Information-Theoretic Approach to Black Hole Information Paradox

P. Kar[1], **B. Sahoo**[2], **S. Sahoo**[1]

[1]Department of Physics, National Institute of Technology Durgapur, West Bengal, India

[2]Department of Physics, T. D. B. College, Raniganj, West Bengal, India

Received *13 June 2026*

Abstract. The black hole information paradox remains one of the most fundamental challenges at the intersection of general relativity and quantum mechanics. In this work, we investigate the paradox through a combination of information-theoretic modelling, numerical analysis of entropy evolution, and the modern island framework. The results indicate that information preservation can be achieved within a framework that incorporates quantum entanglement, entropy dynamics and gravitational contributions from island regions. These findings provide additional support for the view that black hole evaporation is fundamentally compatible with quantum mechanical unitarity.

KEY WORDS: General theory of relativity, Black hole information paradox, Hawking radiation.

1 Introduction

Black holes are characterized by the presence of an event horizon, a boundary beyond which no information can escape to distant observers [1]. For many years, black holes were regarded as purely gravitational objects described by classical spacetime geometry. However, the incorporation of quantum effects established a connection between gravity, thermodynamics and quantum information theory [2].

A major development in black hole physics occurred when Hawking showed that black holes emit thermal radiation due to quantum effects near the event horizon [3]. This phenomenon, known as Hawking radiation, implies that black holes possess a finite temperature and gradually lose mass through evaporation. Together with Bekenstein's proposal that black hole entropy is proportional to the area of event horizon [4], Hawking's result established the foundations of

black hole thermodynamics. The corresponding entropy and temperature suggest the existence of microscopic degrees of freedom associated with gravitational systems.

Despite these successes, Hawking radiation leads to a fundamental problem. In the semiclassical description, the emitted thermal radiation contains no information about the matter that originally formed the black hole [3]. Consequently, if a black hole evaporates completely, an initial pure quantum state may evolve into a mixed state, violating the principle of unitarity in quantum mechanics [5]. This is known as the black hole information paradox.

Several proposals have been developed to address this problem. Black hole complementarity attempts to reconcile information preservation with the equivalence principle [6], while the firewall proposal suggests the existence of high-energy excitations near the horizon [7]. Other approaches, including the fuzzball paradigm [8], ER = EPR conjecture [9], and the ADS/CFT correspondence [10], emphasize the role of quantum structure and holography in information recovery. More recently, significant progress has been achieved through the Page curve and the island prescription, which provide a framework for understanding the entropy evolution of Hawking radiation in a manner consistent with unitary quantum mechanics [11, 12].

This paper is structured as follows. We discuss the black hole information paradox and the principle of unitarity from an information-theoretic perspective, introducing a microscopic interpretation based on degenerate vacuum states in Section 2. Section 3 presents an information-based model of black holes and provides estimates of information content, microscopic constituents and characteristic radiation frequencies for different classes of black holes. In Section 4, the evolution of entanglement entropy during black hole evaporation is investigated through numerical simulations of the Page curve. Section 5 applies the island framework to compute the generalized entropy of Hawking radiation, derive the Page time and explain the recovery of the Page curve through quantum extremal surfaces. The Page times for different classes of black holes are also estimated and compared. Finally, the main results and implications of the present study are summarized in Section 6.

2 Black Hole Information Paradox and Conservation of Unitarity

The discovery of Hawking radiation established a profound connection between gravitation, thermodynamics and quantum theory. According to quantum field theory in curved spacetime, vacuum fluctuations near the event horizon can generate entangled particle-antiparticle pairs. Under the influence of the strong gravitational field, one particle may escape to infinity while its partner falls into the black hole. The escaping particle is observed as Hawking radiation, causing the black hole to gradually lose mass and eventually evaporate [3]. The emit-

ted Hawking quanta remain quantum mechanically entangled with their partners inside the horizon. Consequently, the radiation subsystem is described by a reduced density matrix obtained after tracing over the inaccessible interior degrees of freedom. The associated von Neumann entropy [11] is

$$S_{\rm rad} = -{\rm Tr}\left(\rho_{\rm rad} \ln \rho_{\rm rad}\right) , \tag{1}$$

where $\rho_{\rm rad}$ denotes the density matrix of the radiation subsystem. As evaporation proceeds, the number of entangled pairs increases and the corresponding radiation entropy grows. In Hawking's original semiclassical treatment, the emitted radiation possesses a thermal spectrum and appears independent of the information encoded in the collapsing matter [3,5]. If the black hole evaporates completely, the final state consists solely of thermal radiation, implying a transition from an initial pure quantum state to a mixed state. This apparent loss of information constitutes the black hole information paradox.

To examine the problem from an information-theoretic perspective, we consider the microscopic structure of a black hole. According to the no-hair theorem, a black hole is completely characterized by its mass, charge and angular momentum. However, these quantities are macroscopic observables and may correspond to numerous microscopic configurations. We therefore represent the black hole state as a superposition of N degenerate ground states [13],

$$|\psi\rangle = \frac{1}{\sqrt{N}}(|0\rangle_1 + |0\rangle_2 + |0\rangle_3 + \cdots + |0\rangle_N). \tag{2}$$

The state described by Eq. (2) is a pure quantum state. In Figure 1, the different vacuum configurations may be regarded as degenerate minima of an effective potential.

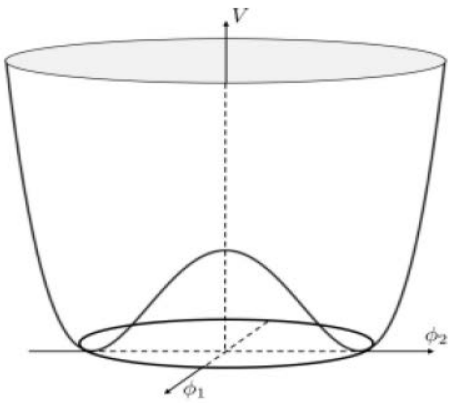


Figure 1. Schematic representation of a Mexican-hat potential illustrating degenerate vacuum states corresponding to different microscopic black hole configurations [13].

External perturbations may induce the selection of a particular vacuum configuration, resulting in spontaneous symmetry breaking. As the system settles into a specific ground state, the surrounding quantum field adjusts accordingly, leading to particle emission. The resulting radiation can be interpreted as arising from tracing over inaccessible vacuum configurations, thereby producing an effectively mixed state.

According to quantum mechanics information must be preserved under time evolution. A convenient order parameter for describing the symmetry-broken phase is the particle-number operator $\hat{n}_i$. Before symmetry breaking, the vacuum expectation value vanishes, whereas after the formation of a black hole it acquires a non-zero value. The corresponding density matrix is

$$\rho = \mid \psi\rangle\langle\psi \mid \ . \tag{3}$$

For every vacuum state characterized by n_i, a corresponding state with expectation value -n_i may also exist. These opposite configurations can be interpreted as particle and antiparticle contributions. Summing over all possible vacuum states [13] gives

$$\langle 0 \mid \hat{n}_1 \mid 0\rangle_1 + \langle 0 \mid \hat{n}_2 \mid 0\rangle_2 + \cdots + \langle 0 \mid \hat{n}_N \mid 0\rangle_N = \langle 0 \mid \hat{n}_{\text{bf}} \mid 0\rangle = 0\,. \tag{4}$$

Equation (4) suggests that when the complete set of microscopic configurations is taken into account, the net information content remains conserved. Within this interpretation, Hawking radiation does not necessarily imply information destruction but rather a redistribution of information among different microscopic degrees of freedom. This viewpoint motivates the information-based model developed in next section, where the information content, microscopic constituents and radiation characteristics of black holes are investigated quantitatively.

3 Information-Based Model of Black Holes

To investigate the microscopic origin of information storage, we develop an information-based model in which the black hole is treated as a collection of fundamental information carriers. The starting point is the Bekenstein-Hawking entropy [4],

$$S_{\text{BH}} = \frac{4\pi k_{\text{B}} G M^2}{\hbar c}\,. \tag{5}$$

According to Zeilinger's information-theoretic interpretation, one elementary quantum system can be associated with one unit of information [14]. The total information content of a black hole may therefore be expressed in terms of its entropy as

$$I_{\text{BH}} = \frac{S_{\text{BH}}}{2k_{\text{B}} \ln 2} = \frac{2\pi G M^2}{\hbar c \ln 2} \ \text{bits}\,. \tag{6}$$

Let the black hole contain n microscopic constituents, referred to as black particles. Assuming that each particle carries one bit of information and that every pairwise interaction contributes an additional bit, the total information content [14] becomes

$$I_n = \frac{n(n+1)}{2} \text{ bits.} \tag{7}$$

For $n \gg 1$, $I_n \approx n^2/2$.

The information difference between two successive configurations is

$$I_n - I_{n-1} = \frac{n(n+1)}{2} - \frac{(n-1)n}{2} = n\,. \tag{8}$$

indicating that the addition of a single constituent increases the information content by n bits [14].

Since the information content is related to the black hole mass through Eq. (6), we write

$$M_{\text{bh}}^2 = \frac{\hbar c \ln 2}{2\pi G} I_{\text{bh}}\,. \tag{9}$$

Using the approximation $I_n \approx n^2/2$ and assuming $M_{\text{bh}}^2 = n^2 m_0^2$, where m_0 denotes the mass of an individual black particle, we obtain

$$M_{\text{bh}}^2 = n^2 m_0^2 = \frac{\hbar c \ln 2}{2\pi G} I_{\text{bh}}\,, \tag{10}$$

$$m_0 = \frac{\sqrt{\ln 2}}{2\sqrt{\pi}} \sqrt{\frac{hc}{2\pi G}} = 5.09 \times 10^{-6} g\,. \tag{11}$$

The number of black particles associated with a black hole of mass M_{bh} is therefore

$$n = \frac{M_{\text{bh}}}{m_0}\,. \tag{12}$$

The black hole is considered as losing mass in black particle amount and losing one black particle is equivalent of losing n bit of information. Inorder to obey the conservation of unitarity, it can be believed as the emission of n photons of frequency ν [14].

$$nh\nu = m_0 c^2\,. \tag{13}$$

The frequency of radiation will be

$$\nu = \frac{m_0 c^2}{nh}\,. \tag{14}$$

The estimated information content, number of black particles and corresponding frequencies for different classes of black holes are presented in Table 1.

Table 1. Estimated information content, number of black particles and characteristic radiation frequency for different classes of black holes

Black hole type	**Mass (kg)**	**Information (bits)**	**Particles (n)**	**Frequency (Hz)**
Supermassive (Sgr A*)	8.56×10^{36}	1.39×10^{90}	1.68×10^{45}	0.41×10^{-3}
Intermediate	2×10^{33}	0.75×10^{83}	0.39×10^{42}	1.769
Stellar	2×10^{31}	0.75×10^{79}	0.39×10^{40}	1.769×10^{2}
Primordial	10^{12}	1.9×10^{40}	0.19×10^{21}	3.63×10^{21}

4 Entropy Evolution and the Page Curve

The black hole information paradox is closely related to the evolution of entanglement entropy during Hawking evaporation. In Hawking's original picture, the repeated creation of entangled particle pairs cause the entropy of the emitted radiation to increase continuously [3]. As a result, the final state after complete evaporation appears thermal, suggesting a loss of information.

A different perspective was proposed by Page, who analysed black hole evaporation using quantum information theory [11]. Consider the total system consisting of the black hole and its radiation,

$$H_{\text{total}} = H_{\text{BH}} \otimes H_{\text{rad}} \,. \tag{15}$$

Although the combined system remains in a pure state, the radiation subsystem is described by a reduced density matrix. The corresponding entropy is

$$S_{\text{rad}} = -\text{Tr}\left(\rho_{\text{rad}} \ln \rho_{\text{rad}}\right) \,. \tag{16}$$

According to Page's argument, the radiation entropy should initially increase as entangled Hawking quanta are emitted. However, after a characteristic time known as the Page time, information begins to emerge in the outgoing radiation, causing the entropy to decrease. Consequently, the entropy evolution follows a Page curve rather than a monotonically increasing profile. To investigate this behaviour numerically, the mass evolution of an evaporating black hole is approximated by

$$\frac{dM}{dt} \propto -\frac{1}{M^2} \,, \tag{17}$$

which leads to

$$M\left(t\right) = M_0 \left(1 - \frac{t}{t_{\text{evap}}}\right)^{1/3} \,, \tag{18}$$

where M_0 is the initial mass and t_{evap} is the total evaporation time.

The corresponding Bekenstein-Hawking entropy decreases with time, whereas the radiation entropy increases due to the continuous production of Hawking

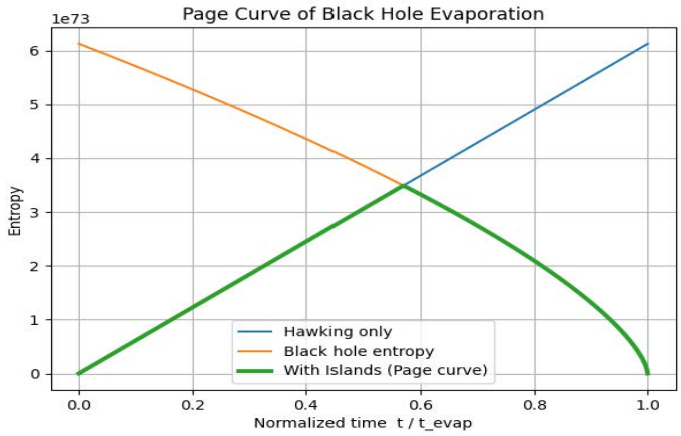


Figure 2. Numerical simulation of entropy evolution during black hole evaporation. The radiation entropy initially increases, reaches a maximum at the Page time and subsequently decreases, producing the characteristic Page curve.

quanta. Following Page's prescription, the physical entropy is obtained by selecting the smaller of the two contributions,

$$S(t) = \min\left(S_{\rm rad}(t), S_{BH}(t)\right) . \tag{19}$$

The resulting time evolution of the radiation entropy is illustrated in Figure 2.

5 Island Framework and Recovery of the Page Curve

The numerical results reproduce the expected Page-curve behaviour. At early times, the entropy growth is dominated by the increasing entanglement between the black hole and the radiation. Near the Page time, both entropy contributions become comparable, resulting in a maximum entropy. At later times, information gradually becomes encoded in the outgoing radiation, leading to a decrease in entropy and restoring consistency with unitary evolution. The appearance of the Page curve therefore provides strong evidence that information need not be lost during black hole evaporation. After reproducing the Page curve numerically, we now investigate its theoretical origin using the island prescription. Recent developments in semiclassical gravity have shown that the entropy of Hawking radiation must be computed using the generalized entropy rather than the matter entropy alone. The generalized entropy contains both the geometric contribution from the black hole horizon and the quantum contribution from matter fields [15],

$$S_{\rm gen} = \frac{A}{4G_N} + S_{\rm matter} . \tag{20}$$

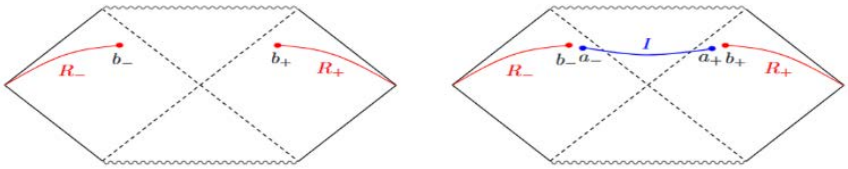


Figure 3. Penrose diagram illustrating the computation of radiation entropy with and without island contributions [16].

The physical entropy of the radiation is obtained by extremizing the generalized entropy and selecting the minimum extremum. This procedure allows a region inside the black hole, known as the island, to contribute to the entropy calculation. The corresponding no-island and island configurations are illustrated in Figure 3.

To evaluate the entropy explicitly, we consider the effective two-dimensional Schwarzschild geometry

$$ds^2 = -f\,(r)\,dt^2 + \frac{dr^2}{f(r)}, \qquad f\,(r) = 1 - \frac{r_h}{r}\,, \tag{21}$$

where r_h denotes the horizon radius. Introducing the tortoise coordinate

$$r^* = r + r_h \ln\left(\frac{r - r_h}{r_h}\right) \tag{22}$$

and the Kruskal coordinates

$$U = -e^{-(t-r^*)/2r_h}\,, \quad V = e^{(t+r^*)/2r_h} \tag{23}$$

allows the near-horizon geometry to be treated in a regular coordinate system.

Using the conformal field theory expression for entanglement entropy, the radiation entropy is found to increase with time. In the late-time regime $(t_b \,|\, r_h)$, the dominant contribution simplifies [16] to

$$S_{\mathrm{no-island}} \simeq \frac{ct_b}{6r_h}\,, \tag{24}$$

demonstrating the linear growth predicted by Hawking's semiclassical calculation [16].

To incorporate quantum gravitational effects, an island region bounded by the quantum extremal surface is introduced. The corresponding matter entropy can be evaluated from a four-point correlation function. After retaining the dominant late-time contributions, the entropy [16] becomes

$$S_{\mathrm{matter}} \simeq \frac{c}{3} \ln\left(\frac{b}{r_h}\frac{a - r_h}{r_h}\right) + \frac{ct_b}{6r_h}\,. \tag{25}$$

Adding the geometric contribution yields the generalized entropy as a function of the island boundary a. The location of the island is then determined by extremizing the generalized entropy,

$$\frac{dS_{\rm gen}}{da} = 0\,. \tag{26}$$

Assuming that the island lies close to the horizon, $a = r_h + \delta$ with $\delta \ll r_h$, the extremization condition gives

$$\delta = \frac{cG_N}{6\pi r_h}\,, \tag{27}$$

indicating that the quantum extremal surface is located only a small distance outside the horizon [16]. Substitution of this result into the entropy expression shows that the island contribution is approximately equal to the Bekenstein-Hawking entropy. Consequently, the physical entropy is determined by comparing the island and no-island configurations. At early times, the no-island entropy is smaller and therefore dominates the entropy calculation. At late times, the island configuration becomes favourable, preventing the indefinite growth of radiation entropy.

The transition between these two regimes occurs when the entropies become equal, yielding [16] the Page time

$$t_{\rm Page} = \frac{6r_h^3}{cG_N}\,. \tag{28}$$

As a result, the entropy initially increases linearly, reaches a maximum near the Page time and subsequently saturates, reproducing the Page curve obtained in the numerical simulation. The inclusion of the island region therefore restores unitary evolution and provides a consistent resolution of the black hole information paradox.

The expression for Page time for a Schwarzschild black hole is

$$t_{\rm Page} = \frac{6}{cG}\left(\frac{2GM}{c^2}\right)^3 = \frac{48G^2M^3}{c^7}\,. \tag{29}$$

The estimated Page time for different classes of black holes are summarized in Table 2.

Table 2. Estimated Page time for different classes of black holes

Black hole type	**Mass (kg)**	**Page time (s)**	**Page time (years)**
Stellar-Mass Black Hole	$10M_\odot \approx 2\times10^{31}$	7.8×10^{15}	2.5×10^{8}
Supermassive Black Hole	$10^6M_\odot \approx 2\times10^{36}$	7.8×10^{30}	2.5×10^{23}
Primordial Black Hole	10^{12}	9.6×10^{-44}	3.0×10^{-51}

6 Conclusion

In this work, the black hole information paradox is investigated using an information-theoretic model, numerical Page-curve analysis and the island framework. The proposed model is used to estimate the information content, microscopic constituents and radiation characteristics of black holes with different masses. Numerical simulations reproduced the Page curve, demonstrating the expected transition from entropy growth to information recovery. Furthermore, the island prescription was shown to modify the late-time entropy evolution, yielding a finite Page time and restoring consistency with unitary quantum evolution. The combined results suggest that the apparent information loss in Hawking's semiclassical description arises from an incomplete treatment of quantum gravitational effects rather than a fundamental violation of information conservation.

References

[1] K. Schwarzschild (1916) *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften*, Berlin, pp. 189-196.
[2] R.M. Wald (1984) *General Relativity*. University of Chicago Press, Chicago.
[3] W. Hawking (1975) *Commun. Math. Phys.* **43** 199-220.
[4] J.D. Bekenstein (1973) *Phys. Rev. D* **7** 2333-2346.
[5] S.W. Hawking (1976) *Phys. Rev. D* **14** 2460-2473.
[6] L. Susskind, L. Thorlacius, J. Uglum (1993) *Phys. Rev. D* **48** 3743-3761.
[7] A. Almheiri, D. Marolf, J. Polchinski, J. Sully (2013) *J. High Energy Phys.* **2013**(2) 62.
[8] S.D. Mathur (2005) *Fortschr. Phys.* **53** 793-827.
[9] J. Maldacena, L. Susskind (2013) *Fortschr. Phys.* **61** 781-811.
[10] J.M. Maldacena (1998) *Adv. Theor. Math. Phys.* **2** 231-252.
[11] D.N. Page (1993) *Phys. Rev. Lett.* **71** 3743-3746.
[12] A. Almheiri, N. Engelhardt, D. Marolf, H. Maxfield (2019) *J. High Energy Phys.* **2019** Article No. 63.
[13] I. Arraut (2014) *Appl. Math.* **5**(4) 736-746.
[14] I. Gurevich (2013) *Open Journal of Microphycis* **3** 128-140.
[15] G. Penington (2020) *J. High Energy Phys.* **2020** Article No. 2.
[16] K. Hashimoto, N. Iizuka, Y. Matsuo (2020) *J. High Energy Phys.* Article No. 085.