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Systematics of ^{12}C Emission from Superheavy Nuclei

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Abstract. We have theoretically studied the systematics of ^{12}C emission in superheavy nuclei with $Z = 104$–130. We have also compared the ^{12}C decay half-lives with that of alpha decay and spontaneous fission. It is found that ^{12}C decay half-lives are greater than that of alpha decay and spontaneous fission. The variation of logarithmic ^{12}C decay half-lives with $Z_d^{0.6}Q^{-1/2}$ is found to be exact straight line. The superheavy nuclei which are having shorter ^{12}C half-lives are highlighted.

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1 Introduction

Study of exotic/cluster decay is important in the field of superheavy nuclei. There is an expectation that the decay of superheavy nuclei through the cluster emission of nuclei. The study of alpha decay and cluster radioactivity plays an important role in the identification and synthesis of superheavy elements. From the detail literature survey it has been observed theoretical as well as experimental works on the synthesis and decay modes of superheavy elements. The alpha and heavy particle decay in Z=116-124 was reported by Santhosh and Priyanka [1]. Manjunatha [2] studied alpha decay properties of $Z = 126$ in the range of $288 \leq A \geq 339$. Previous workers [3] extended systematics of alpha decay half-lives for exotic superheavy nuclei. Santhosh and Nithya [4] studied different decay modes of even Z superheavy isotopes. Santhosh et al. [5] reported cluster emission in $^{210-226}$Ra isotopes. Alpha decay half-lives of superheavy nuclei with $Z = 116$–118 are reported by previous workers [6]. The competition between spontaneous fission and α-decay process for superheavy element with $Z = 112$ was reported by previous researchers [7]. Alpha decay properties and structure of superheavy nuclei $Z = 102$–120 was studied by Silişteanu and Budaca [8]. A study on α-decay rates of spherical and deformed nuclei was reported by Ni and Ren [9]. Nuclear lifetimes of cluster radioactivities with $Z = 52$–122, were empirically calculated by Poenarua et al. [10]. There has been several detail reports on alpha decay half-lives of superheavy nuclei [11–24]. From the literature survey [25–29], it is also observed that there was some studies on preformation probabilities of alpha decay. There were some studies reported on cluster decay and heavy particle radioactivity [30–40]. Some of the researchers [41–46], studied both alpha and cluster emission in superheavy nuclei.

The study of cluster radioactivity is important for heavy and superheavy nuclei. Most of the researchers studied the cluster radioactivity except ^{12}C emission. A detail literature survey reveals that there is no systematic study of ^{12}C emission in superheavy nuclei. Hence in the present work, we have theoretically studied the systematics of ^{12}C emission in superheavy nuclei. This paper is organized in to three sections. First part is introduction and in the second part, we have explained the theoretical frame work used in the calculations of half-lives of ^{12}C emission. The results obtained in the present work are analyzed in the third section.

2 Theoretical Frame Work

The decay half-life of parent nuclei with the emission of ^{12}C cluster is studied by

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu P}\,, \tag{1}$$

where λ is the decay constant and ν is the assault frequency and is expressed as

$$\nu = \frac{\omega}{2\pi} = \frac{2E_\nu}{h}\,, \tag{2}$$

where E_ν is the empirical vibrational energy [47]. According to WKB approximation (Wentzel-Kramers-Brillouin) the penetration probability P through the potential barrier studied by the following equation:

$$P = \exp\left\{ -\frac{4\pi}{h}\int_{R_a}^{R_b} \sqrt{2\mu(V_T(r) - Q)}dr \right\}, \tag{3}$$

where μ is the reduced mass of fission fragments of cluster decay and alpha decay system, R_a and R_b are the inner and outer turning points and these turning points are calculated by

$$V_T(R_a) = Q = V_T(R_b)\,. \tag{4}$$

The total interacting potential is

$$V_T(r) = V_N(r) + V_C(r) + V_l(r)\,, \tag{5}$$

where $V_N(r)$ is the attractive nuclear potential, $V_C(r)$ is the repulsive Coulomb potential and $V_l(r)$ is the centrifugal potential. The Coulomb potential $V_C(r)$ for cluster decay and alpha decay is given by

$$V_C(r) = Z_\alpha Z_b e^2 \begin{cases} \dfrac{1}{R} & (R > R_C) \\ \dfrac{1}{2R_c}\Big[3 - \Big(\dfrac{R}{R_c}\Big)^2\Big] & (R < R_C) \end{cases}, \tag{6}$$

where R_c is touching radial separation between fission fragments in cluster decay and alpha decay. The nuclear potential $V_N(r)$ between cluster/alpha decay nuclei and daughter nuclei is

$$V_N(r) = 4\pi\gamma\bar{R}\Phi(\varepsilon)\,. \tag{7}$$

The nuclear potential intern depends upon geometry and shape of the nuclei and universal function $\Phi(\varepsilon)$ and surface coefficient γ is calculated as [48]

$$\gamma = \gamma_0\Big[1 - K_s\Big(\frac{N-Z}{A}\Big)^2\Big] \ \text{MeV/fm}^2\,, \tag{8}$$

where γ_0 is the surface energy constant and K_s is the surface asymmetry constant. $\gamma_0 = 1.25284$ MeV/fm^2 and $K_s = 2.345$ [49]. $\bar{R}$ is the mean curvature as

$$\bar{R} = \frac{C_1 C_2}{C_1 + C_2}\,, \tag{9}$$

where C_1 and C_2 are the sussmann's central radii of cluster/alpha nuclei and daughter nuclei, respectively. Based on droplet model [50] C_i is written as

$$C_i = c_i + \Big(\frac{N_i}{a_i}\Big)t_i\,, \qquad (i = 1, 2), \tag{10}$$

where t_i is the neutron skin and is given as

$$t_i = \frac{3}{2}r_0\Big(\frac{JI_i - \frac{1}{12}c_i Z_i A_i^{-1/3}}{Q + \frac{9}{4}JA_i^{-1/3}}\Big), \quad (i = 1, 2)\,, \tag{11}$$

r_0 is the radius constant and $r_0 = 1.14$ fm, the symmetry energy coefficient $J = 32.65$ MeV, $I = (N-Z)/A$, $c_i = 3e^2/5r_0 = 0.757895$ MeV and neutron skin stiffness coefficient $Q = 35.4$ MeV, c_i is the half-density radius of the charge distribution and it is defined as [50]

$$c_i = R_{00i}\Big(1 - \frac{7}{2}\frac{b^2}{R_{00i}^2} - \frac{49}{8}\frac{b^4}{R_{00i}^4} + ...\Big), \qquad (i = 1, 2)\,, \tag{12}$$

R_{00i} is charge radius formula and is expressed as

$$R_{00i} = 1.171A_i^{1/3} + 1.472A_i^{-1/3}\,, \quad (i = 1, 2)\,. \tag{13}$$

Universal proximity potential is given by [51]

$$\Phi(\varepsilon) = \begin{cases} -1.7817 + 0.9270\varepsilon + 0.143\varepsilon^2 - 0.09\varepsilon^3 & \text{for } \varepsilon \le 0.0, \\ -1.7817 + 0.9270\varepsilon + 0.0169\varepsilon^2 - 0.05148\varepsilon^3 & \text{for } 0 \le \varepsilon \le 1.9475, \\ -4.41\exp\left(\dfrac{-\varepsilon}{0.7176}\right) & \text{for } \varepsilon \ge 1.9475 \end{cases}, \tag{14}$$

where $\varepsilon = s/b$ is the minimum separation between fission fragments and $b \approx 0.99$ is the nuclear surface thickness and s is the distance between the near surfaces of the fragments.

Table 1. Highlighted ^{12}C emitters in the superheavy nuclei

Parent nucleus	Daughter nucleus	Q, (MeV)	$T_{1/2}$, (s)	Parent nucleus	Daughter nucleus	Q, (MeV)	$T_{1/2}$, (s)
^{257}107	^{245}101	29.63	24678.2	^{312}126	^{300}120	45.61	426.5356
^{308}123	^{296}117	37.90	397111.2	^{313}126	^{301}120	32.10	480.2658
^{309}123	^{297}118	37.87	8667.157	^{314}126	^{302}120	32.30	1232.562
^{310}123	^{298}117	37.68	41863.62	^{315}126	^{303}120	33.40	3671.034
^{311}123	^{299}117	36.89	33121.23	^{316}126	^{304}120	31.30	13081.06
^{312}123	^{300}117	37.55	43938.14	^{310}127	^{298}121	33.59	73.15606
^{314}123	^{302}117	37.92	532.7464	^{311}127	^{299}121	33.08	46.28831
^{308}124	^{296}118	39.96	55384.92	^{312}127	^{300}121	32.73	17.52527
^{312}124	^{300}118	41.44	20280.27	^{313}127	^{301}121	32.71	42.9559
^{313}124	^{301}118	41.85	44699.79	^{314}127	^{302}121	32.58	63.11274
^{306}125	^{294}119	41.32	16016.39	^{315}127	^{303}121	32.42	275.5344
^{307}125	^{295}119	41.76	11912.69	^{316}127	^{304}121	32.19	705.3964
^{309}125	^{297}119	41.72	2749.136	^{317}127	^{305}121	32.67	2993.247
^{310}125	^{298}119	42.32	683.8953	^{313}128	^{301}122	34.97	3.719666
^{311}125	^{299}119	42.98	473.7727	^{314}128	^{302}122	34.98	6.717327
^{312}125	^{300}119	43.83	1185.967	^{315}128	^{303}122	34.89	39.61285
^{313}125	^{301}119	43.66	3526.878	^{316}128	^{304}122	34.76	130.3004
^{314}125	^{302}119	43.96	8568.358	^{317}128	^{305}122	34.67	524.9683
^{315}125	^{303}119	43.66	30641.77	^{318}128	^{306}122	35.98	2318.064
^{308}126	^{296}120	44.47	2103.65	^{316}129	^{304}123	36.89	10.09031
^{309}126	^{297}120	44.60	977.4565	^{317}129	^{305}123	36.85	52.34164
^{310}126	^{298}120	44.92	646.8053	^{319}130	^{307}124	39.51	228.5082
^{311}126	^{299}120	45.74	199.4518	^{320}130	^{308}124	39.96	1499.252

3 Results and Discussion

We have studied the half-lives of ^{12}C decay from superheavy nuclei of $104 < Z < 130$. The energy released (Q) during the ^{12}C decay is plotted as a function of neutron number of the parent nuclei and it is shown in Figure 1. Figure 2 shows the variation of logarithmic half-lives with neutron number of parent nuclei for studied superheavy nuclei. From this figure it is found that half-lives of ^{12}C decay is shorter for superheavy nuclei with $Z > 120$. The highlighted ^{12}C emitters in the superheavy emitters are given in Table 1. To study the competition between the ^{12}C decay and other dominant decay modes such as alpha decay and spontaneous fission, we have also calculated the half-lives corresponds to the alpha decay and spontaneous fission. The comparison of ^{12}C decay half-lives with that of alpha decay and spontaneous fission are shown in Figure 3. From this figure it is found that ^{12}C decay half-lives are greater than that of alpha decay and spontaneous fission.

The variation of logarithmic half-lives for ^{12}C decay with mass number reveals that there is peaks correspond to the nuclei ^{286}Fl, ^{297}120, ^{301}121, ^{301}122 and ^{303}123. These nuclei are having longer ^{12}C decay half-lives than that of neighbour. This means these nuclei are having extra stability against ^{12}C decay. The variation of logarithmic half-lives for ^{12}C decay with mass number reveals that there is peaks correspond to the nuclei ^{286}Fl, ^{297}120, ^{301}121, ^{301}122 and ^{303}123. These nuclei are having longer ^{12}C decay half-lives than that of neighbour. This means these nuclei are having extra stability against ^{12}C decay.

To test the validity of Geiger–Nuttall law [52] for ^{12}C decay, we have plotted the logarithmic ^{12}C decay half-lives with inverse of square root of energy released (Q). Figure 3 shows the variation of logarithmic ^{12}C decay half-lives with inverse of square root of energy released (Q). From this figure it is found that variation of logarithmic ^{12}C decay half-lives with $Q^{-1/2}$ is found to be straight line. We have also evaluated the fitting coefficients 'a' and 'b' and these are included in the Figure 4. Experimental data of branching ratios between alpha and ^{12}C decay in superheavy nuclei are not available in the literature.

To validate the present calculations, we have evaluated branching ratios for some actinide nuclei where the experimental data is available. We have compared the calculated branching ratios between alpha and carbon cluster radioactivity with that of the experiments (Ref. [53]) and this comparison is as shown in Table 2. From the detail study of variation of logarithmic half-lives with energy released (Q) and atomic number of daughter nuclei reveals that the logarithmic half-lives can be expressed linearly using $Z_d^{0.6}Q^{-1/2}$. Figure 5 shows the variation of logarithmic ^{12}C decay half-lives with $Z_d^{0.6}Q^{-1/2}$. The variation of logarithmic ^{12}C decay half-lives with $Z_d^{0.6}Q^{-1/2}$ is found to be exact straight line. For each superheavy element, we have expressed logarithmic half-lives in terms linear equation of $Z_d^{0.6}Q^{-1/2}$. In Figure 5, to show the exactness of the fit, we have

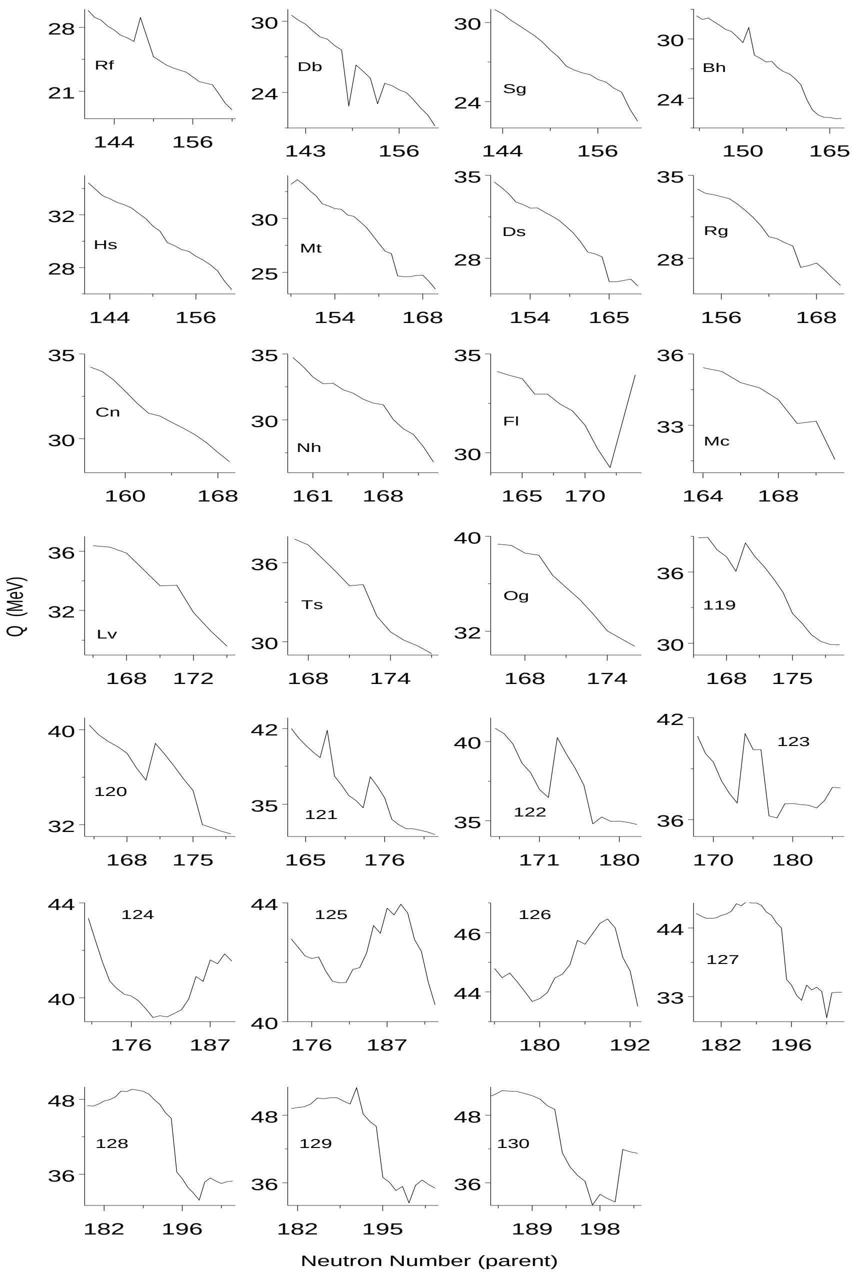

Figure 1. Variation of Q (MeV) with neutron number of parent nuclei.

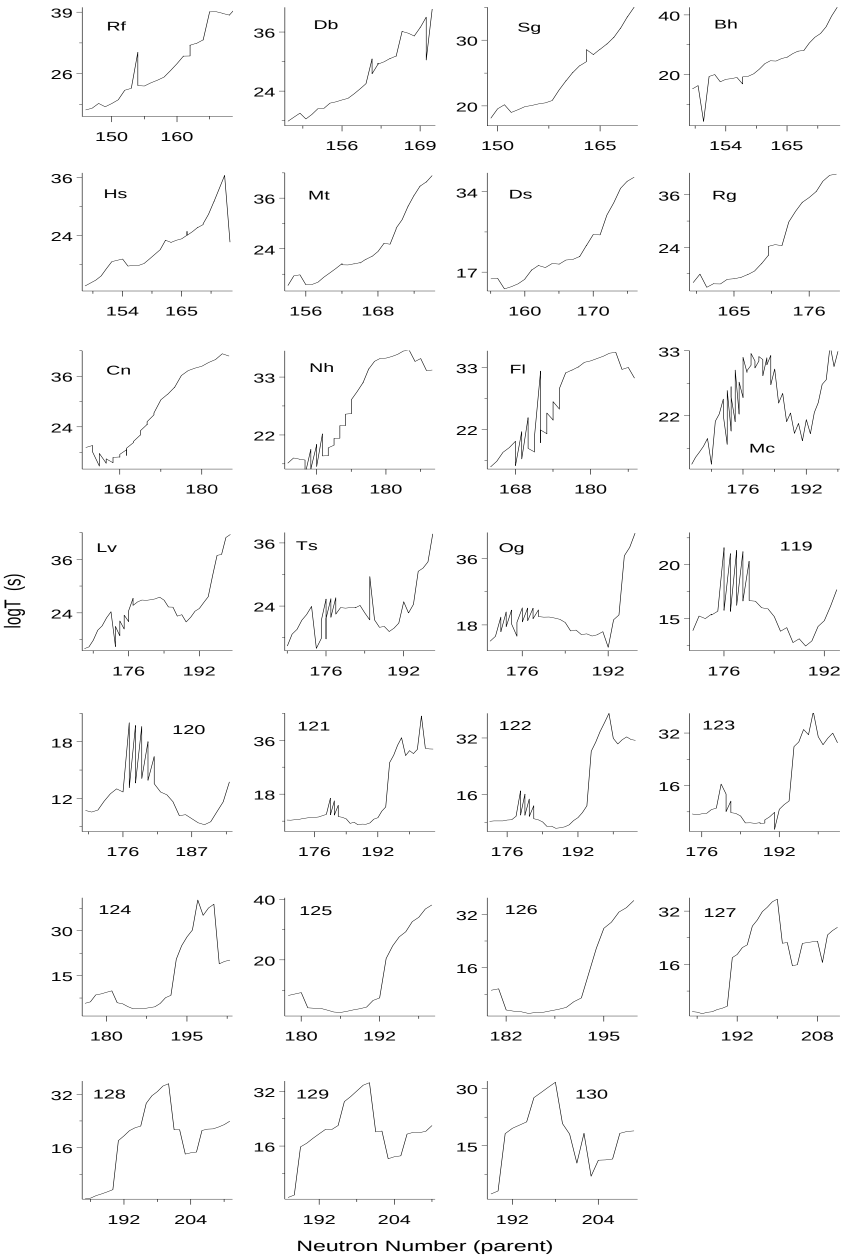

Figure 2. Variation of logarithmic half-lives with neutron number of parent nuclei.

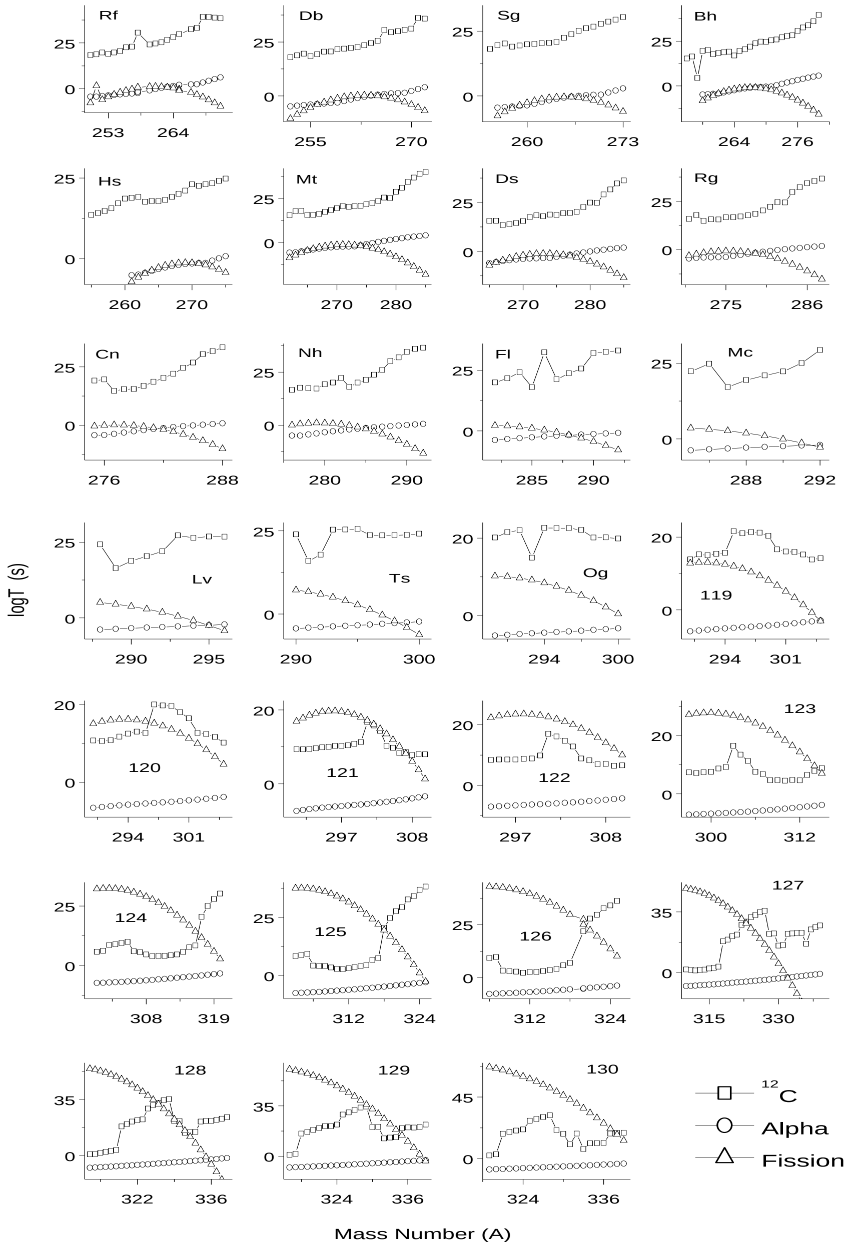

Figure 3. Comparison of logarithmic half-lives of ^{12}C decay with that of alpha decay and spontaneous fission. ^{258}Rf, ^{286}Fl, ^{297}120, ^{301}121, ^{301}122, ^{303}123.

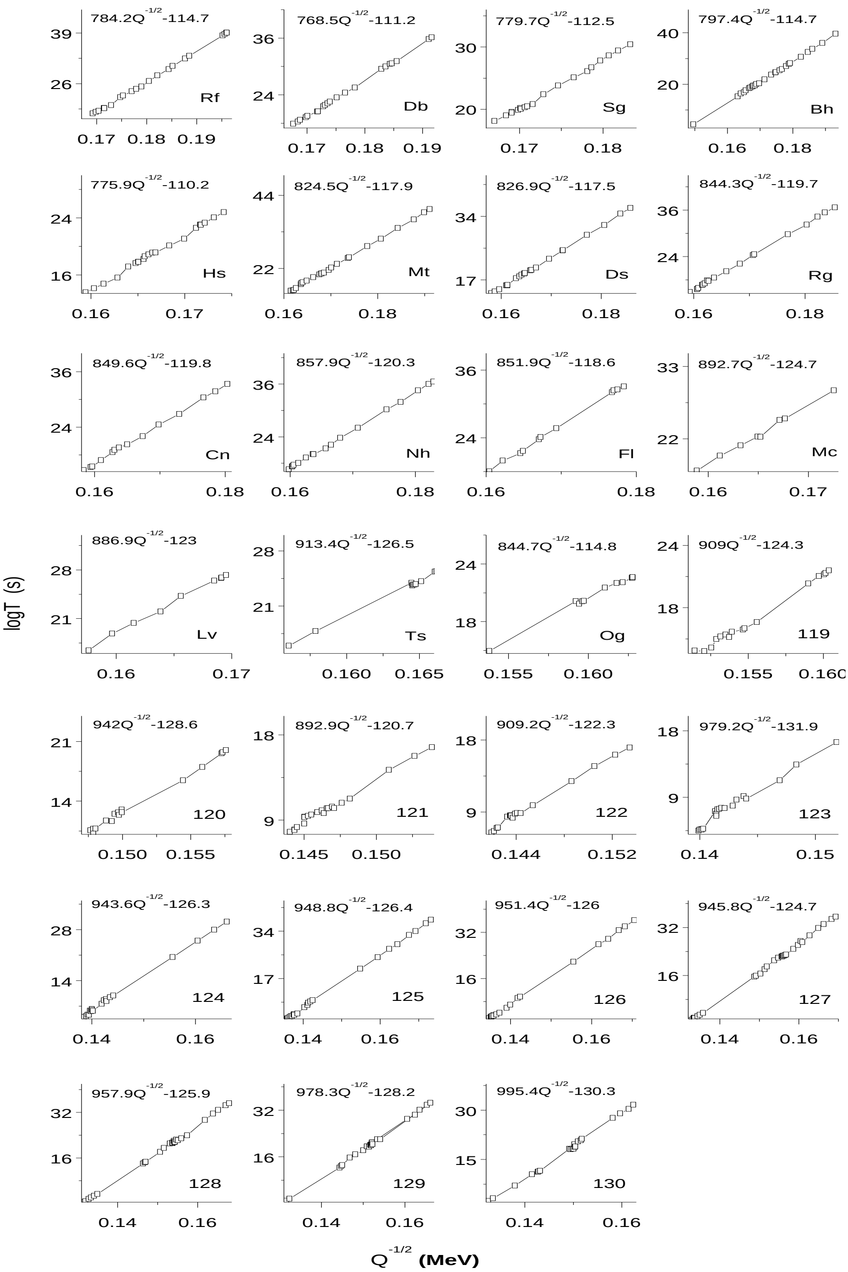

Figure 4. Variation of logarithmic ^{12}C decay half-lives with $Q^{-1/2}$ (MeV).

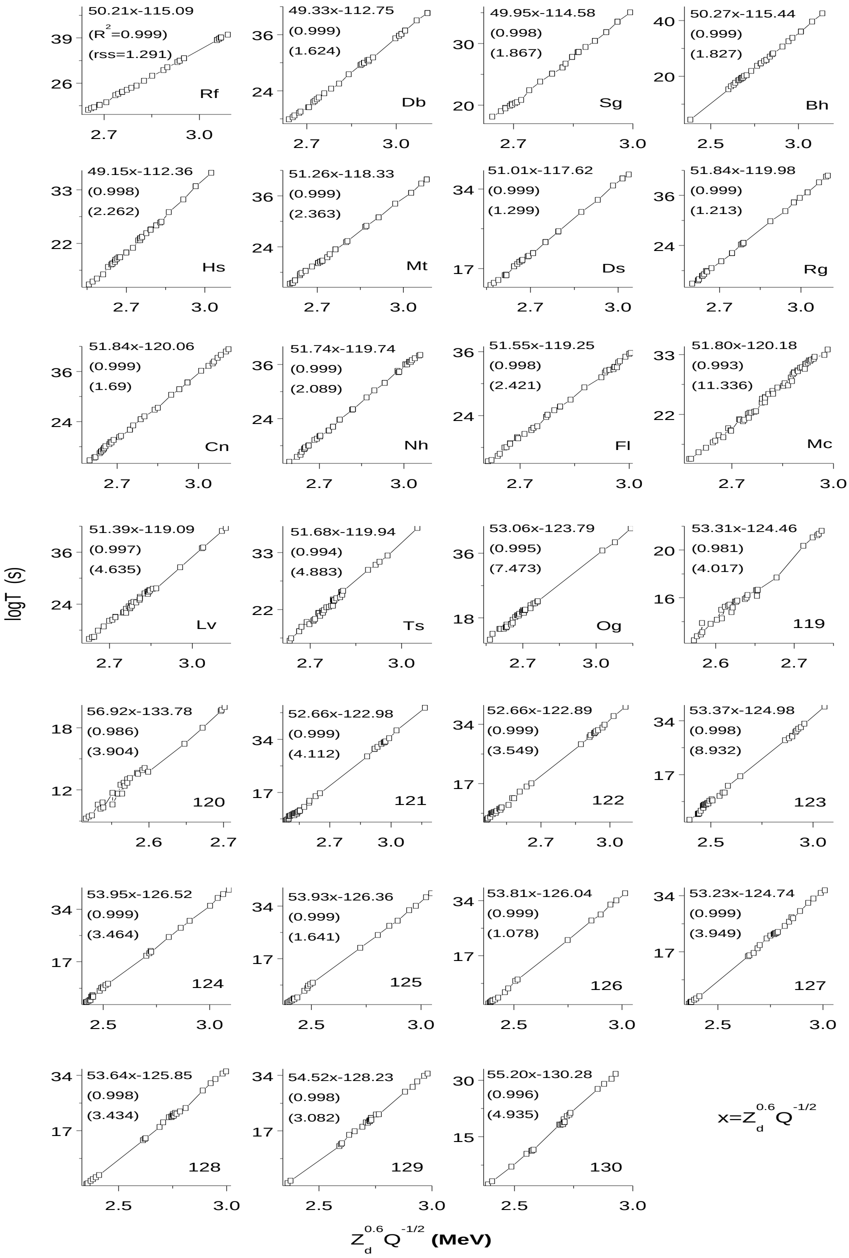

Figure 5. Variation of logarithmic of ^{12}C decay half-lives with $Z_d^{0.6}Q^{-1/2}$.

Table 2. Comparison of calculated branching ratios between alpha and cluster radioactivity with that of the experiments (Ref. [53]).

Emitter	Cluster	Detection System (Ref. [53]	Branching ratios Experiment (Ref. [53])	Present work
^{221}Fr	^{14}C	BP1 (Ref. [53])	$(8.14 \pm 1.14)10^{-13}$	9.6×10^{-13}
^{221}Ra	^{14}C	BP1 (Ref. [53])	$(1.15 \pm 0.91)10^{-12}$	1.93×10^{-12}
^{222}Ra	^{14}C	BP1 (Ref. [53])	$(3.7 \pm 0.6)10^{-10}$	3.94×10^{-10}
^{222}Ra	^{14}C	POLY (Ref. [53])	$(3.1 \pm 1.0)10^{-10}$	3.293×10^{-10}
^{222}Ra	^{14}C	SOLENO (Ref. [53])	$(2.3 \pm 0.3)10^{-10}$	2.63×10^{-10}
^{223}Ra	^{14}C	SOLENO (Ref. [53])	$(8.5 \pm 2.5)10^{-10}$	10.94×10^{-10}
223Ra	^{14}C	E XΔE (Ref. [53])	$(5.5 \pm 2.0)10^{-10}$	6.97×10^{-10}
223Ra	^{14}C	SOLENO (Ref. [53])	$(7.6 \pm 3.0)10^{-10}$	9.74×10^{-10}
223Ra	^{14}C	E XΔE (Ref. [53])	$(6.1 \pm 1.0)10^{-10}$	7.13×10^{-10}
223Ra	^{14}C	POLY (Ref. [53])	$(4.7 \pm 1.3)10^{-10}$	5.61×10^{-10}
223Ra	^{14}C	SPLIT-POLE (Ref. [53])	$(6.4 \pm 0.4)10^{-10}$	7.13×10^{-10}
223Ra	^{14}C	SOLENO (Ref. [53])	$(7.0 \pm 0.4)10^{-10}$	7.41×10^{-10}
223Ra	^{14}C	SOLENO (Ref. [53])	$(8.9 \pm 0.4)10^{-10}$	8.91×10^{-10}
224Ra	^{14}C	SOLENO (Ref. [53])	$(4.3 \pm 1.2)10^{-11}$	5.53×10^{-11}
224Ra	^{14}C	POLY (Ref. [53])	$(6.5 \pm 1.0)10^{-11}$	6.63×10^{-11}
225Ac	^{14}C	SOLENO (Ref. [53])	$(6.0 \pm 1.3)10^{-12}$	7.15×10^{-12}
225Ac	^{14}C	BP1 (Ref. [53])	$(4.5 \pm 1.4)10^{-12}$	5.16×10^{-12}
226Ra	^{14}C	BP1 (Ref. [53])	$(3.2 \pm 1.6)10^{-11}$	4.23×10^{-11}

also included the residual sum of squares (RSS) and coefficient of determination (R^2). The studied systematics of ^{12}C emission in superheavy nuclei with $Z = 104$–130 is important in the field of superheavy element.

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