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Coherently Driven Two-Level Atom in Open Space and Interacting with Vacuum Modes

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Abstract. In this paper, we consider a coherently driven two-level atom in open space and interacting with vacuum reservoir. Using the master equation and the interaction Hamiltonian, we obtain the time evolution of the expectation values of the atomic operators. Applying the large time approximation scheme on these equation, we determined the solution of the expectation values of the atomic operators. With the aid of the resulting solution, we study the power spectrum, the atomic inversion, the mean photon number, and the normalised second-order correlation function of the fluorescent light in the weak and strong driving light limits. Moreover, we have found that the probability for the atom to be in the upper level.

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1 Introduction

The properties of the fluorescent light emitted by a two-level atom in a cavity driven by coherent light and coupled to a vacuum reservoir have been studied by several authors [1–12]. A single two-level atom has been the center of attention for the last several decades. The interaction of two-level atom with vacuum reservoir has bee studied by several groups. This kind of interaction is involved in various physical processes of interest such as resonance fluorescence and laser dynamics. Gardiner [1] studies a single two-level atom embedded in a broadband squeezed vacuum and showed that the two quadratures of the atomic polarization decay at two distict decay rates that are sensitive to the phase correlations of the squeezed vacuum. Moreover, Carmichael et al. [2] have studied the fluorescent spectrum of an atom immersed in a broadband squeezed vacuum and the atom is driven by coherent field. They predicted that for weak driving fields the incoherent spectrum would narrow as the amount of squeezing is increased. In this limit the spectrum is insensitive to the relative phase between the driving field and the squeezed vacuum. For strong fields, on the other hand, the central peak of Mollow spectrum can broaden or narrow, dependng on the relative phase between the squeezed vacuum and the driving field. Furthermore, Eyob [3] has studied a coherently driven two-level atom inside a parametric oscillator operating below threshold. He found that the presence of the parametric amplifier leads to an increase in the width of the power spectrum of the fluorescent light in the weak and strong driving light limits and the effect of the presence of the parametric amplifier on the second-order correlation function is to enhance its decay rate.

On the other hand, Abebe and Gemechu [4] has considered a single two-level atom inside a degenerate parametric oscillator coupled to a vacuum reservoir. They analyzed that the effect of the squeezed light from a parametric oscillator on the quantum properties of the fluorescent light emitted by the two-level atom. They also showed that the half-width of the power spectrum for the fluorescent light in the presence of a parametric amplifier increases, while it decreases for the cavity-mode light.

In this study, we analyze the quantum properties of the light emitted by a two-level atom, in open space, driven by coherent light and interacting with vacuum modes. Applying the master equation, we obtain the expectation values of the atomic operators. These results are then used to determine the the power spectrum, the atomic inversion, the mean photon number, and the normalised second-order correlation function of the fluorescent light in the weak and strong driving light limits with the aid of the phenomenon of photon antibunching. Moreover, we determine the atomic inversion in the weak and strong driving light limits.

2 Atomic Expectation Values

A two-level atom in open space, driven by coherent light and interacting with vacuum modes is considered as shown in Figure 1. As clearly indicated, upper and lower levels of the atom are represented by $|a\rangle$ and $|b\rangle$. It so turns out that the signal light is in a squeezed state. With the pump mode treated classically, the interaction of a two-level atom with a classical coherent light is described at

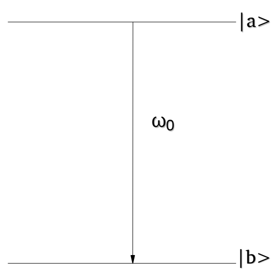

Figure 1. A two level atom.

resonance by the Hamiltonian

$$\hat{H} = \frac{i\Omega}{2}\big[\hat{\sigma}_+ - \hat{\sigma}_-\big], \tag{1}$$

where

$$\Omega = 2\varepsilon g. \tag{2}$$

Here ε is the amplitude of the driving light, g is the atom-cavity mode coupling constant, and $\hat{\sigma}_+ = |a\rangle\langle b|$ and $\hat{\sigma}_- = |b\rangle\langle a|$ are atomic operators satisfying the commutation relations $[\hat{\sigma}_\pm, \hat{\sigma}_\mp] = \pm\hat{\sigma}_z$, $[\hat{\sigma}_\pm, \hat{\sigma}_z] = \mp 2\hat{\sigma}_\pm$, $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$ with $\hat{\sigma}_z = |a\rangle\langle a| - |b\rangle\langle b|$.

In studying the nonclassical features and the quantum properties of the generated radiation, deriving various correlations from the master equation is essential. For the system under consideration, the master equation is expressible as [1]

$$\frac{d}{dt}\hat{\rho}(t) = -i\big[\hat{H}_S(t), \hat{\rho}\big] + \frac{\gamma}{2}\big[2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-\big]. \tag{3}$$

Substituting Eq. (1) into (3) results in

$$\begin{aligned}\frac{d}{dt}\hat{\rho}(t) &= \frac{\Omega}{2}\big[\hat{\sigma}_+\hat{\rho} - \hat{\rho}\hat{\sigma}_+ + \hat{\rho}\hat{\sigma}_- - \hat{\sigma}_-\hat{\rho}\big] \\ &\quad + \frac{\gamma}{2}\big[2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-\big],\end{aligned} \tag{4}$$

where γ is the atomic decay rate. Now using this equation and the cyclic property of the trace operation, one can readily obtain

$$\frac{d}{dt}\langle\hat{\sigma}_-\rangle = Tr\left[\frac{\Omega}{2}(\hat{\rho}\hat{\sigma}_-\hat{\sigma}_+ - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-) + \frac{\gamma}{2}(\hat{\rho}\hat{\sigma}_+\hat{\sigma}_-^2 - \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+\hat{\sigma}_-)\right], \tag{5}$$

so that in view of the fact that $\hat{\sigma}_-\hat{\sigma}_+\hat{\sigma}_- = \hat{\sigma}_-$, $\hat{\sigma}_-^2 = \hat{\sigma}_+^2 = 0$, $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$, and with the aid of the identities $\hat{\sigma}_z\hat{\sigma}_\pm = \pm\hat{\sigma}_\pm$, $\hat{\sigma}_\pm\hat{\sigma}_z = \mp\hat{\sigma}_\pm$, there follows

$$\frac{d}{dt}\langle\hat{\sigma}_-\rangle = -\frac{\gamma}{2}\langle\hat{\sigma}_-\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_z\rangle. \tag{6}$$

In view of the fact that $\langle\hat{\sigma}_-\rangle^* = \langle\hat{\sigma}_+\rangle$ and $\langle\hat{\sigma}_z\rangle^* = \langle\hat{\sigma}_z\rangle$ together with the relation given by[1]

$$\rho_{aa} = \langle\hat{\sigma}_+\hat{\sigma}_-\rangle = (\langle\hat{\sigma}_z\rangle + 1)/2, \tag{7}$$

we see that

$$\frac{d}{dt}\langle\hat{\sigma}_+\rangle = -\frac{\gamma}{2}\langle\hat{\sigma}_+\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_z\rangle, \tag{8}$$

$$\frac{d}{dt}\langle\hat{\sigma}_z\rangle = \gamma\langle\hat{\sigma}_z\rangle + \Omega(\langle\hat{\sigma}_-\rangle + \langle\hat{\sigma}_+\rangle) - \gamma. \tag{9}$$

We now proceed to obtain a decoupled differential equation for $\langle\hat{\sigma}_z(t)\rangle$. To this end, from (6) and (8), we get

$$\frac{d}{dt}\langle\hat{\sigma}_-\rangle + \frac{d}{dt}\langle\hat{\sigma}_+\rangle = -\frac{\gamma}{2}(\langle\hat{\sigma}_-\rangle + \langle\hat{\sigma}_+\rangle) - \Omega\langle\hat{\sigma}_z\rangle, \tag{10}$$

so that with aid of (9), we easily find

$$\Omega\left(\frac{d}{dt}\langle\hat{\sigma}_-\rangle + \frac{d}{dt}\langle\hat{\sigma}_+\rangle\right) = -\frac{\gamma}{2}\frac{d}{dt}\langle\hat{\sigma}_z\rangle - (\frac{\gamma^2}{2} + \Omega^2)\langle\hat{\sigma}_z\rangle - \frac{\gamma^2}{2}. \tag{11}$$

Thus on differentiating (9) and taking into account (11), there follows

$$\frac{d^2}{dt^2}\langle\hat{\sigma}_z\rangle + \frac{3\gamma}{2}\frac{d}{dt}\langle\hat{\sigma}_z\rangle + \left(\frac{\gamma^2}{2} + \Omega^2\right)\langle\hat{\sigma}_z\rangle = -\frac{\gamma^2}{2}. \tag{12}$$

This equation can be put in the form

$$\left(\frac{d}{dt} + \lambda\right)\left(\frac{d}{dt} + \eta\right)\langle\hat{\sigma}_z\rangle = -\frac{\gamma^2}{2}, \tag{13}$$

where $\lambda = \frac{\gamma}{4} + \mu$, $\eta = \frac{\gamma}{4} - \mu$ with $\mu = \sqrt{(\frac{\gamma}{4})^2 - \Omega^2}$. Now upon setting

$$\frac{d}{dt}\langle\hat{\sigma}_z\rangle + \eta\langle\hat{\sigma}_z\rangle = z, \tag{14}$$

we see that

$$\frac{dz}{dt} + \lambda z = -\frac{\gamma^2}{2}. \tag{15}$$

The solution of equation (15) is found to be

$$z(t+\tau) = z(t)e^{-\lambda\tau} - \frac{\gamma^2}{2\lambda}[1 - e^{-\lambda\tau}]. \tag{16}$$

On the other hand, in view of this result, the solution of Eq. (14) is expressible as

$$\begin{aligned}\langle\hat{\sigma}_z(t+\tau)\rangle = \langle\hat{\sigma}_z(t)\rangle e^{-\eta\tau} + \frac{z(t)}{\lambda-\eta}[e^{-\eta\tau} - e^{-\lambda\tau}] \\ - \frac{\gamma^2}{2\lambda\eta}[1 - e^{-\eta\tau}] + \frac{\gamma^2}{2\lambda(\lambda-\eta)}[e^{-\eta\tau} - e^{-\lambda\tau}].\end{aligned} \tag{17}$$

From (9) and (14), we note that

$$z(t) = (\eta - \gamma)\langle\hat{\sigma}_z(t)\rangle + \Omega(\langle\hat{\sigma}_-(t)\rangle + \langle\hat{\sigma}_+(t)\rangle) - \gamma. \tag{18}$$

Hence with the aid of (17) and (18), we arrive at

$$
\begin{aligned}
(\langle\hat{\sigma}_z(t+\tau)\rangle+1)/2 = {} & \left(e^{-\eta\tau}+\frac{\eta-\gamma}{\lambda-\eta}\big[e^{-\eta\tau}-e^{-\lambda\tau}\big]\right)\langle\hat{\sigma}_z(t)\rangle/2 \\
& +\frac{\Omega}{2(\lambda-\eta)}\big[e^{-\eta\tau}-e^{-\lambda\tau}\big](\langle\hat{\sigma}_-(t)\rangle+\langle\hat{\sigma}_+(t)\rangle) \\
& +\frac{1}{2}-\frac{\gamma^2}{4\lambda\eta}+\frac{\gamma^2}{4(\lambda-\eta)}\big[e^{-\eta\tau}-e^{-\lambda\tau}\big] \\
& +\frac{\gamma^2}{4\lambda\eta}e^{-\eta\tau}-\frac{\gamma}{2(\lambda-\eta)}\big[e^{-\eta\tau}-e^{-\lambda\tau}\big].
\end{aligned} \tag{19}
$$

It can be readily established that

$$
e^{-\eta\tau}+\frac{\eta-\gamma}{\lambda-\eta}\big[e^{-\eta\tau}-e^{-\lambda\tau}\big]=e^{-3\gamma\tau/4}(\cosh\mu\tau-\frac{\gamma}{4\mu}\sinh\mu\tau), \tag{20}
$$

$$
\frac{\Omega}{2(\lambda-\eta)}\big[e^{-\eta\tau}-e^{-\lambda\tau}\big]=\frac{\Omega}{2\mu}e^{-3\gamma\tau/4}\sinh\mu\tau, \tag{21}
$$

and

$$
\begin{aligned}
& \frac{1}{2}-\frac{\gamma^2}{4\lambda\eta}+\frac{\gamma^2}{4(\lambda-\eta)}\big[e^{-\eta\tau}-e^{-\lambda\tau}\big] \\
& +\frac{\gamma^2}{4\lambda\eta}e^{-\eta\tau}-\frac{\gamma}{2(\lambda-\eta)}\big[e^{-\eta\tau}-e^{-\lambda\tau}\big] \\
& =\frac{\Omega^2}{\gamma^2+2\Omega^2}\left[1-e^{-3\gamma\tau/4}(\cosh\mu\tau+\frac{3\gamma}{4\mu}\sinh\mu\tau)\right] \\
& +\frac{1}{2}e^{-3\gamma\tau/4}(\cosh\mu\tau-\frac{\gamma}{4\mu}\sinh\mu\tau).
\end{aligned} \tag{22}
$$

Now combination of (19), (20), (21), and (22) leads to

$$
\begin{aligned}
(\langle\hat{\sigma}_z(t+\tau)\rangle+1)/2 = \Gamma_1(\tau)(\langle\hat{\sigma}_z(t)\rangle+1)/2+\Gamma_2(\tau)(\langle\hat{\sigma}_-(t)\rangle \\
+\langle\hat{\sigma}_+(t)\rangle)+\Gamma_3(\tau),
\end{aligned} \tag{23}
$$

where

$$
\Gamma_1(\tau)=e^{-3\gamma\tau/4}\big(\cosh\mu\tau-\frac{\gamma}{4\mu}\sinh\mu\tau\big), \tag{24}
$$

$$
\Gamma_2(\tau)=\frac{\Omega}{2\mu}e^{-3\gamma\tau/4}\sinh\mu\tau, \tag{25}
$$

$$
\Gamma_3(\tau)=\frac{\Omega^2}{\gamma^2+2\Omega^2}\left[1-e^{-3\gamma\tau/4}\big(\cosh\mu\tau+\frac{3\gamma}{4\mu}\sinh\mu\tau\big)\right]. \tag{26}
$$

We realize that $\langle\hat{\sigma}_z(t)\rangle$ can be obtained from the expression for $\langle\hat{\sigma}_z(t+\tau)\rangle$ by setting $t=0$ and $\tau=t$. We then see that

$$
\begin{aligned}
(\langle\hat{\sigma}_z(t)\rangle+1)/2 = \Gamma_1(t)(\langle\hat{\sigma}_z(0)\rangle+1)/2+\Gamma_2(t)(\langle\hat{\sigma}_-(0)\rangle \\
+\langle\hat{\sigma}_+(0)\rangle)+\Gamma_3(t)
\end{aligned} \tag{27}
$$

and at steady state

$$(\langle\hat{\sigma}_z(t)\rangle_{ss} + 1)/2 = \frac{\Omega^2}{\gamma^2 + 2\Omega^2}. \tag{28}$$

On the other hand, employing (23) the solution of Eq. (6) can be put in the form

$$\begin{aligned}\langle\hat{\sigma}_-(t+\tau)\rangle = {}& \langle\hat{\sigma}_-(t)\rangle e^{-\gamma\tau/2} \\ & - \frac{\Omega}{2}(\langle\hat{\sigma}_z(t)\rangle + 1)e^{-\gamma\tau/2}\int_0^\tau e^{\gamma\tau'/2}\Gamma_1(\tau')d\tau' \\ & - \Omega(\langle\hat{\sigma}_-(t)\rangle + \langle\hat{\sigma}_+(t)\rangle)e^{-\gamma\tau/2}\int_0^\tau e^{\gamma\tau'/2}\Gamma_2(\tau')d\tau' \\ & - \Omega e^{-\gamma\tau/2}\int_0^\tau e^{\gamma\tau'/2}(\Gamma_3(\tau') - 1/2)d\tau'.\end{aligned} \tag{29}$$

With the help of (24), (25), and (26), one can readily verify that

$$\begin{aligned}\langle\hat{\sigma}_-(t+\tau)\rangle = {}& \chi_1(\tau)\langle\hat{\sigma}_-(t)\rangle + \chi_2(\tau)\langle\hat{\sigma}_+(t)\rangle \\ & + \chi_3(\tau)(\langle\hat{\sigma}_z(t)\rangle + 1)/2 + \chi_4(\tau),\end{aligned} \tag{30}$$

in which

$$\chi_1(\tau) = \frac{1}{2}e^{-3\gamma\tau/4}(\cosh\mu\tau + \frac{\gamma}{4\mu}\sinh\mu\tau) + \frac{1}{2}e^{-\gamma\tau/2}, \tag{31}$$

$$\chi_2(\tau) = \frac{1}{2}e^{-3\gamma\tau/4}(\cosh\mu\tau + \frac{\gamma}{4\mu}\sinh\mu\tau) - \frac{1}{2}e^{-\gamma\tau/2}, \tag{32}$$

$$\chi_3(\tau) = -\frac{\Omega}{\mu}e^{-3\gamma\tau/4}\sinh\mu\tau, \tag{33}$$

$$\chi_4(\tau) = \frac{\gamma\Omega}{\gamma^2 + 2\Omega^2}\left[1 - e^{-3\gamma\tau/4}\left(\cosh\mu\tau + \left(\frac{\gamma^2 - 4\Omega^2}{4\gamma\mu}\right)\sinh\mu\tau\right)\right]. \tag{34}$$

On account of these results, Eq. (30) takes the form

$$\begin{aligned}\langle\hat{\sigma}_-(t)\rangle = {}& \chi_1(t)\langle\hat{\sigma}_-(0)\rangle + \chi_2(t)\langle\hat{\sigma}_+(0)\rangle \\ & + \chi_3(t)(\langle\hat{\sigma}_z(0)\rangle + 1)/2 + \chi_4(t)\end{aligned} \tag{35}$$

and at steady state

$$\langle\hat{\sigma}_-(t)\rangle_{ss} = \frac{\gamma\Omega}{\gamma^2 + 2\Omega^2}. \tag{36}$$

3 Power spectrum

In terms of the atomic operators, the power spectrum of the fluorescent light can be expressed as

$$S(\omega) = 2Re\int_0^\infty d\tau e^{i(\omega-\omega_0)\tau}\langle\hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle_{ss}, \tag{37}$$

where ω_0 is the transition frequency of the atom, Re denotes the real part, and ss stands for steady state.

We next proceed to evaluate the two-time correlation function involved in (37). To this end, applying the quantum regression theorem to (35) and taking into account (7), one finds

$$\langle\hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle = \chi_1(\tau)(\langle\hat{\sigma}_z(t)\rangle + 1)/2 + \chi_4\langle\hat{\sigma}_+(t)\rangle, \tag{38}$$

so that in view of (28), (31), (34), and (36), there follows

$$\begin{aligned}\langle\hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle_{ss} = \frac{\Omega^2}{\gamma^2+2\Omega^2}\Bigg[&\frac{\gamma^2}{\gamma^2+2\Omega^2} + \frac{1}{2}e^{-\gamma\tau/2} \\ &+ \frac{1}{4(\gamma^2+2\Omega^2)}\Bigg[\left(2\Omega^2-\gamma^2+\frac{\gamma}{4\mu}(10\Omega^2-\gamma^2)\right)e^{(\mu-\gamma/4)\tau} \\ &+ \left(2\Omega^2-\gamma^2-\frac{\gamma}{4\mu}(10\Omega^2-\gamma^2)\right)e^{-(\mu+\gamma/4)\tau}\Bigg]\Bigg].\end{aligned} \tag{39}$$

Applying (39) in (37), one can easily obtain an explicit expression for the power spectrum. However, the power spectrum shows remarkably different features in the weak and strong deriving light limits. We thus wish to treat these two limiting cases separately. We note that for $\Omega \ll \frac{\gamma}{4}$ (weak driving light), $\mu = \frac{\gamma}{4}$. One then readily finds

$$\frac{1}{4(\gamma^2+2\Omega^2)}\left[2\Omega^2-\gamma^2+\frac{\gamma}{4\mu}(10\Omega^2-\gamma^2)\right]e^{(\mu-\gamma/4)\tau} = -\frac{1}{2}e^{-\gamma\tau/2}, \tag{40}$$

$$\frac{1}{4(\gamma^2+2\Omega^2)}\left[2\Omega^2-\gamma^2-\frac{\gamma}{4\mu}(10\Omega^2-\gamma^2)\right]e^{-(\mu+\gamma/4)\tau} = 0. \tag{41}$$

Hence for weak driving light ($\Omega \ll \frac{\gamma}{4}$), the two-time correlation function (39) turns out to be

$$\langle\hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle_{ss} = \left(\frac{\Omega}{\gamma}\right)^2, \tag{42}$$

so that employing this result in (37), one readily obtains the normalized power spectrum

$$S(\omega) = \delta(\omega-\omega_0) \tag{43}$$

where ω_0 is the atomic transition frequency or for that matter the frequency of the resonant driving light. According to Eq. (43) the frequency of the fluorescent light is fixed at $\omega = \omega_0$. This result is to be expected since, for a resonant deriving light, the atom absorbs a photon whose frequency equal to the transition frequency of the atom and energy conservation requires that the emitted photon have the same frequency.

On the other hand, for $\Omega \gg \frac{\gamma}{4}$ (strong driving light), we see that $\mu = i\Omega$. One thus easily obtains

$$\frac{1}{4(\gamma^2+2\Omega^2)}\left[2\Omega^2-\gamma^2+\frac{\gamma}{4\mu}(10\Omega^2-\gamma^2)\right]e^{(\mu-\gamma/4)\tau}=\frac{1}{4}e^{(i\Omega-3\gamma/4)\tau}, \tag{44}$$

$$\frac{1}{4(\gamma^2+2\Omega^2)}\left[2\Omega^2-\gamma^2-\frac{\gamma}{4\mu}(10\Omega^2-\gamma^2)\right]e^{-(\mu+\gamma/4)\tau}=\frac{1}{4}e^{-(i\Omega+3\gamma/4)\tau}. \tag{45}$$

Consequently, we get

$$\langle\hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle_{ss} = \frac{1}{4}e^{-\gamma\tau/4} + \frac{1}{8}e^{(i\Omega-3\gamma/4)\tau} + \frac{1}{8}e^{-(i\Omega+3\gamma/4)\tau}. \tag{46}$$

Now taking into account this result, the power spectrum in the strong driving light limit is found to be

$$\begin{aligned} S(\omega) = {} & \frac{3\gamma/16\pi}{(\omega-\omega_0-\Omega)^2+(\frac{3\gamma}{4})^2} + \frac{\gamma/4\pi}{(\omega-\omega_0)^2+(\frac{\gamma}{2})^2} \\ & + \frac{3\gamma/16\pi}{(\omega-\omega_0+\Omega)^2+(\frac{3\gamma}{4})^2}. \end{aligned} \tag{47}$$

This power spectrum has three peaks similar to the Mollow spectrum [13] with a central peak centered at $\omega = \omega_0$ with a half width of $\gamma/2$, and two side peaks centered at $\omega = \omega_0 \pm \Omega$ each having a half width of $3\gamma/4$. We also see from Figure (2) that the power spectrum has three peaks for the case of $\frac{\Omega}{\gamma} \neq 0$ and when $\frac{\Omega}{\gamma} = 0$ has a single peak. Moreover, it is not hard to see that the side peaks disappear as the value of $\frac{\Omega}{\gamma} = 0$.

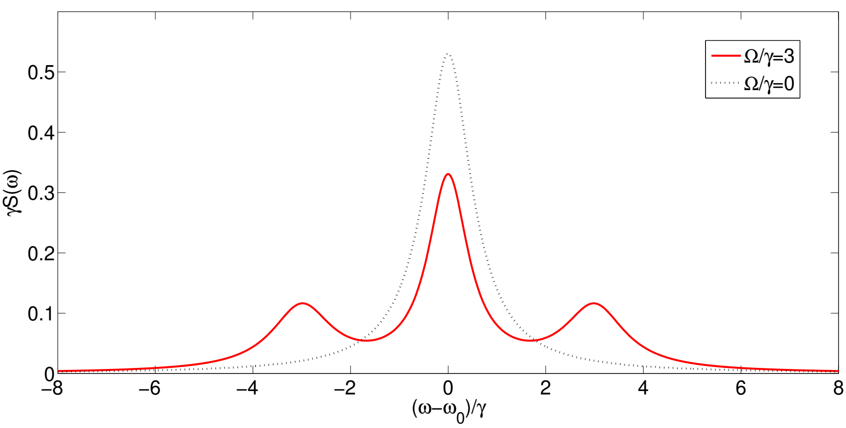

Figure 2. Plots of the normalized power spectrum ($S(\omega)$) [Eq. (47)] vs. $(\omega-\omega_0)/\gamma$.

4 Photon Antibunching

Resonance fluorescence may be defined as the process of light emission by a two-level atom undergoing some interaction. Among other things, the nonclassical feature of photon antibunching exhibited by the fluorescent light makes the phenomenon of resonance fluorescence to be quite interesting. The second-order correlation function for the light emitted by a two-level atom in open space is expressible as

$$g^{(2)}(\tau) = \frac{\langle \hat{\sigma}_+(t)\hat{\sigma}_+(t+\tau)\hat{\sigma}_-(t+\tau)\hat{\sigma}_-(t)\rangle}{\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t)\rangle^2}. \tag{48}$$

We recall that

$$\langle \hat{\sigma}_+(t+\tau)\hat{\sigma}_-(t+\tau)\rangle = (\langle \hat{\sigma}_z(t+\tau)\rangle + 1)/2, \tag{49}$$

so that taking into account (27) and applying the quantum regression theorem, we have

$$g^{(2)}(\tau) = \frac{\Gamma_3(\tau)}{(\langle \hat{\sigma}_+(t)\rangle + 1)/2}. \tag{50}$$

Finally, in view of (26) and (28), the second order correlation function takes at steady state the form

$$g^{(2)}(\tau) = 1 - e^{-\gamma\eta\tau/4}\left(\cosh \mu\tau + \frac{3\gamma}{4\mu}\sinh \mu\tau\right). \tag{51}$$

Once more it is interesting to analyse the second-order correlation function in the weak and strong driving light limits as it exhibits different features. In the

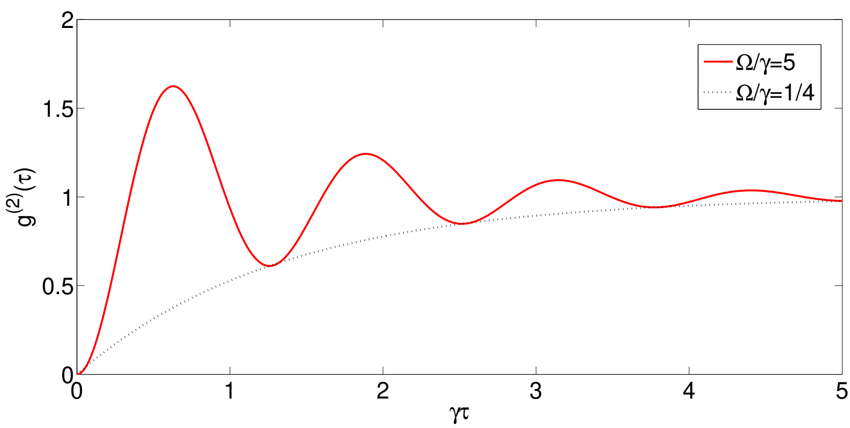

Figure 3. Plots of the the second order correlation function ($g^{(2)}(\tau)$)[Eq. (51)] vs. $\gamma\tau$.

weak driving light limit, i.e., $\Omega \ll \frac{\gamma}{4}$, the value of μ is real ($\mu = \frac{\gamma}{4}$). One can observe from the above equation, $g^{(2)}(0) = 0$ and for $\tau > 0$, $g^{(2)}(\tau) > 0$. Therefore, for $\tau > 0$, the second-order correlation function $g^{(2)}(\tau) > g^{(2)}(0)$. This shows that the fluorescent light thus exhibits the phenomenon of photon antibunching. This is due to the fact that a two-level atom can not emit two or more photons simultaneously. After each emission the atom returns to the lower level and it must absorb a photon before another emission can take place. That is the photons have a tendency to arrive at a detector separately rather than in pair. The experimental verification [14] of photon antibunching could thus be taken as a direct evidence for the quantum nature of light.

On the other hand, in the strong driving light limit ($\Omega \gg \frac{\gamma}{4}$) the value of μ is complex ($\mu = i\Omega$). This leads to an oscillatory behaviour of $g^{(2)}(\tau)$. We easily see from this figure that the behaviour of $g^{(2)}(\tau)$ is oscillatory which ultimately tends to unity. This oscillatory nature of $g^{(2)}(\tau)$ is due to the Rabi oscillation of the two-level atom.

5 Atomic Inversion

The atom inversion is defined as

$$W(t) = \rho_{aa}(t) - \rho_{bb}(t) = \langle \hat{\sigma}_z(t) \rangle. \tag{52}$$

where $\rho_{aa}(t) = \langle \hat{\sigma}_+(t)\hat{\sigma}_-(t) \rangle$ and $\rho_{bb}(t) = \langle \hat{\sigma}_-(t)\hat{\sigma}_+(t) \rangle$ are the probability for the atom to be in the upper and lower levels, respectively.

Now in view of (27), we see that

$$\begin{aligned} W(t) = 2[&\Gamma_1(t)(\langle \hat{\sigma}_z(0) \rangle + 1)/2 \\ &+ \Gamma_2(t)(\langle \hat{\sigma}_-(0) \rangle + \langle \hat{\sigma}_+(0) \rangle) + \Gamma_3(t)] - 1. \end{aligned} \tag{53}$$

Now replacing $t + \tau$ by t and t by 0 in Eq. (23), we get

$$\langle \hat{\sigma}_z(t) \rangle = 2\Gamma_1(t)\langle \hat{\sigma}_+(0)\hat{\sigma}_-(0) \rangle + 2\Gamma_3(t) - 1. \tag{54}$$

Employing the relation $(\langle \hat{\sigma}_z(0) \rangle + 1)/2 = \langle \hat{\sigma}_+(0)\hat{\sigma}_-(0) \rangle$ and assuming the atom is initially in the upper or lower level, one can write Eq. (53) in the form

$$W(t) = 2\Gamma_1(t)\rho_{aa}(0) + 2\Gamma_3(t) - 1. \tag{55}$$

We wish to investigate atomic inversion in the weak and strong driving light limits because it shows remarkably different features in each limit. If the two-level atom is initially in the lower level, $\rho_{aa}(0) = 0$. Thus, with the aid of Eqs. (26) and (28), the atomic inversion (55) takes the form

$$W(t) = -1 + \frac{2\Omega^2}{\gamma^2 + 2\Omega^2}\left[1 - e^{-3\gamma t/4}\big(\cosh \mu t + \frac{3\gamma}{4\mu}\sinh \mu t\big)\right]. \tag{56}$$

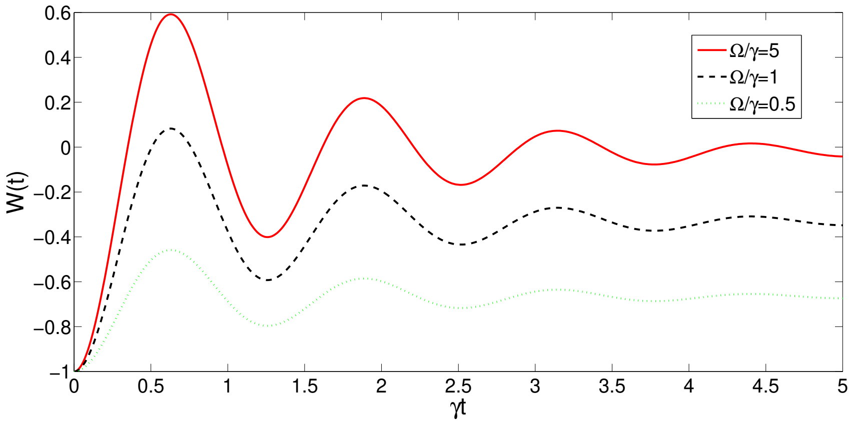

Figure 4. Plots of the atomic inversion ($W(t)$) [Eq. (56)] vs. γt.

If the atom is initially in the upper level, $\rho_{aa}(0) = 1$. Hence, on account of Eqs. (24) (26), and (28), the atomic inversion becomes

$$W(t) = -1 + 2e^{-3\gamma t/4}\big(\cosh\mu t - \frac{\gamma}{4\mu}\sinh\mu t\big) + \frac{2\Omega^2}{\gamma^2 + 2\Omega^2}\Big[1 - e^{-3\gamma t/4}\big(\cosh\mu t + \frac{3\gamma}{4\mu}\sinh\mu t\big)\Big]. \tag{57}$$

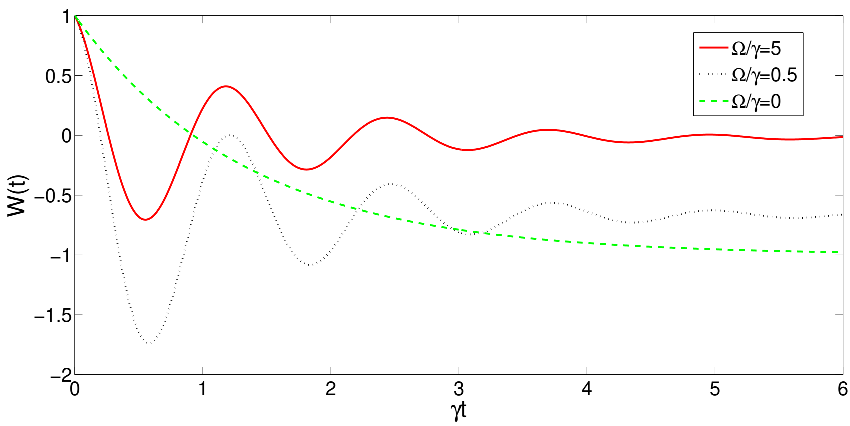

Figure 5. Plots of the atomic inversion ($W(t)$) [Eq. (57)] vs. γt.

6 Photon Statistics

In the previous two sections, we have studied the power spectrum and the second order correlation function for the fluorescent light emitted by the two-level atom. We now wish to determine the mean and variance of the photon number.

According to (27) the probability for the two-level atom to be in the upper level is given by

$$\rho_{aa}(t) = \Gamma_1(t)(\langle\hat{\sigma}_z(0)\rangle + 1)/2 + \Gamma_2(t)(\langle\hat{\sigma}_-(0)\rangle + \langle\hat{\sigma}_+(0)\rangle) + \Gamma_3(t) \quad (58)$$

and at steady state

$$\rho_{aa}(\infty) = \frac{\Omega^2}{\gamma^2 + 2\Omega^2}. \quad (59)$$

Furthermore, if the atom is initially in the lower level, then expression (58) becomes

$$\rho_{aa}(t) = \frac{\Omega^2}{\gamma^2 + 2\Omega^2}\left[1 - e^{-3\gamma t/4}\left(\cosh\mu t + \frac{3\gamma}{4\mu}\sinh\mu t\right)\right] \quad (60)$$

and for $\Omega \gg \frac{\gamma}{4}$

$$\rho_{aa}(t) = \frac{1}{2}[1 - e^{-3\gamma t/4}\cosh\Omega t] \quad (61)$$

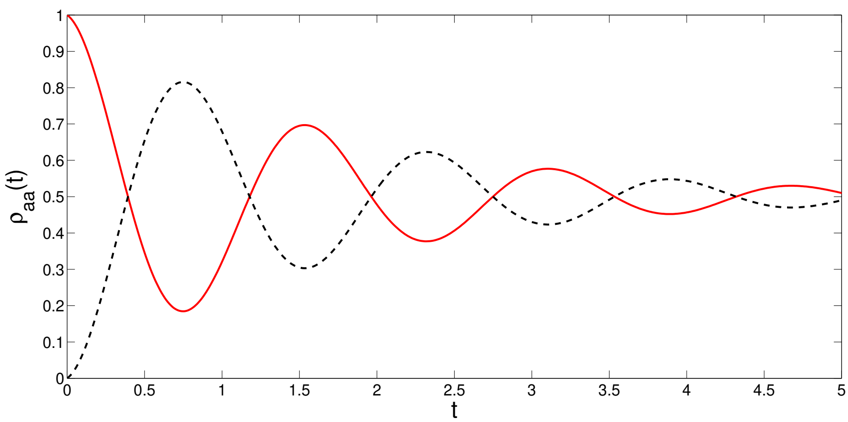

Figure 6. Plots of Eq. (61)(dotted curve) and Eq. (63) (red solid curve) versus $t$for $\Omega = 4$ and $\gamma = 0.8$.

On the other hand, if the atom is initially in the upper level

$$\rho_{aa}(t) = \frac{\Omega^2}{\gamma^2 + 2\Omega^2}\left[1 - e^{-\gamma t/4}\left(\cosh\mu t + \frac{3\gamma}{4\mu}\sinh\mu t\right)\right] + e^{-\gamma t/4}\left(\cosh\mu t - \frac{\gamma}{4\mu}\sinh\mu t\right) \tag{62}$$

and for $\Omega \gg \frac{\gamma}{4}$

$$\rho_{aa}(t) = \frac{1}{2}[1 + e^{-3\gamma t/4}\cosh\Omega t]. \tag{63}$$

On the other hand, the mean number of photons emitted by a two-level atom is expressible as

$$\bar{n}(t) = \rho_{aa}(t) = \langle\hat{\sigma}_+(t)\hat{\sigma}_-(t)\rangle. \tag{64}$$

Now on account of (64) along with (61) and (63), the mean photon number of the fluorescent light has the form

$$\bar{n}(t) = \frac{1}{2}[1 - e^{-3\gamma t/4}\cosh\Omega t] \tag{65}$$

when the atom is initially in the lower level and

$$\bar{n}(t) = \frac{1}{2}[1 + e^{-3\gamma t/4}\cosh\Omega t] \tag{66}$$

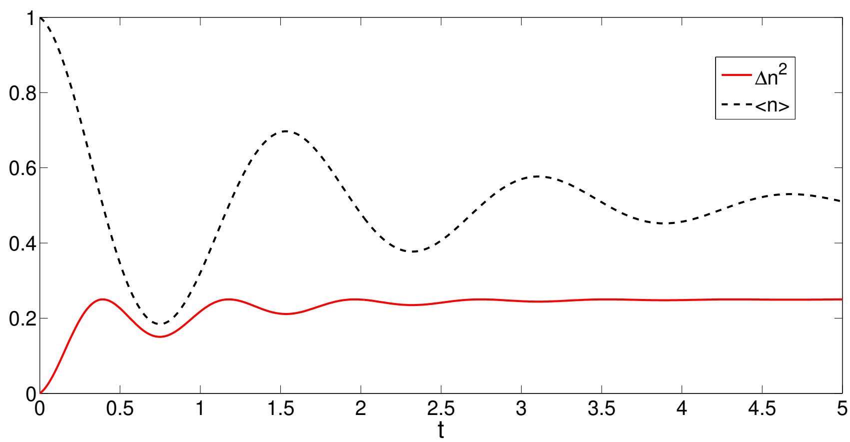

Figure 7. Plots of Eq. (65) (dotted curve) and Eq. (67) (red solid curve) vs. t for $\Omega = 4$ and $\gamma = 0.8$.

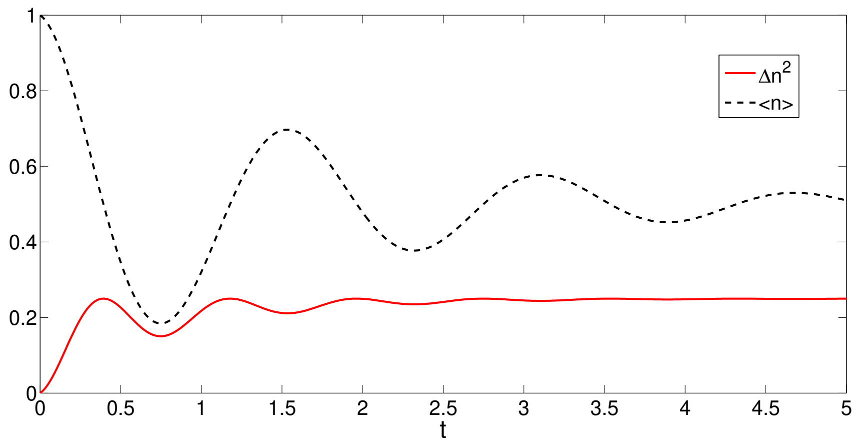

Figure 8. Plots of Eq. (66) (dotted curve) and Eq. (68) (red solid curve) vs. t for $\Omega = 4$ and $\gamma = 0.8$.

when the atom is initially in the upper level. Finally, using (64) we find the variance of the photon number to be

$$\Delta n^2 = \bar{n}(1-\bar{n}) = \frac{1}{2}[1 - e^{-3\gamma t/4}\cosh\Omega t] \tag{67}$$

when the atom is initially in the lower level and

$$\Delta n^2 = \frac{1}{2}[1 + e^{-\gamma t/4}\cosh\Omega t] \tag{68}$$

when the atom is initially in the upper level. We note that the photon statistics of the emitted light is sub-Poissonian.

7 Conclusion

The quantum properties of a light produced by a two-level atom in open space driven by coherent light and coupled with vacuum modes are thoroughly analyzed. Here we apply the master equation and the interaction Hamiltonian, to obtain the time evolution of the expectation values of the atomic operators. Employing the large time approximation scheme on these equation, we determined the solution of the expectation values of the atomic operators. With the aid of the resulting solution, we study the power spectrum, the atomic inversion, the mean photon number, and the normalised second-order correlation function of the fluorescent light in the weak and strong driving light limits. Moreover, we have found that the probability for the atom to be in the upper level.In addition, we showed that the width of the power spectrum of the fluorescent light is increased in the weak and strong driving light limits. Moreover, we obtained that the photons have a tendency to arrive at a detector separately rather than in pair. This shows that the fluorescent light thus exhibits the phenomenon of photon antibunching. Finally, we determined that the photon statistics of the emitted light is sub-Poissonian.

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