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Bianchi Type I Quark and Strange Quark Cosmological Models in $f(G)$ Theory of Gravitation

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Abstract. The present research aims at investigating the Bianchi type I cosmological model in the presence of quark and strange quark matter which exist in the first second of the early universe in the framework of $f(G)$ theory of gravitation. The field equations are solved by assuming volumetric exponential and power law expansions. It is also assumed that the shear is proportional to expansion scalar of the models. Physical parameters like shear scalar, expansion scalar, deceleration parameter are also discussed in details.

KEY WORDS: LRS Bianchi type I, $f(G)$ gravity, Quark matter.

1 Introduction

General Relativity (GR) is self consistent and observationally tested theory of gravitation. However, it possesses singularities such as Black hole, Big-Bang etc. It is also inconsistent with observational data at infrared scales [1]. Further, the recent observational data indicates that the universe is accelerating and expanding. The accelerated expansion of the universe is explained by assuming hypothetical term dark energy in the background of GR theory. It is believed that pervasion of dark energy is 96 percent part of the universe. But it is still unknown and do not have definite proof of existence in the laboratories. Another approach of explaining accelerated expansion is to modify the Einstein Hilbert action. One of the most popular modifications of GR is based on introduction of function of the Ricci scalar, i.e. $f(R)$ in Einstein-Hilbert action. It is known as $f(R)$ theories of gravitation. Bucchdahl [2] has introduced the $f(R)$ theories. All the well established results of GR are preserved in the $f(R)$ theories. However, $f(R)$ is not the end of modifications. Recently, the $f(G)$ theory of gravitation is introduced. The $f(G)$ is obtained by introducing the Gauss-Bonnet curvature invariants G in the Einstein-Hilbert action. In the $f(G)$, f is Lagrangian density function of$G$. The curvature invariant G can avoid ghost contribution and useful into the regularization of the gravitational action [3]. Recently, various cosmological models have been constructed in the $f(G)$ theory for various physical fluids. Capozziello et al. [4] have discussed Noether symmetry approach in the framework of the $f(G)$ cosmology. Myrzakulov et al. [5] have studied cosmological solution on the Λ CDM model in the $f(G)$ gravity. Dadhich [6] has coupled four dimensional space times with Gauss-Bonnet gravity. Bamba et al. [7] have explored bouncing cosmology in the $f(G)$ gravity. Kang et al. [8] have obtained static spherically symmetric star in Gauss-Bonnet gravity. Katore et al. [9] have discussed string bulk viscous cosmological models in the $f(G)$ theory of gravitation.

According to the standard model of particle physics, universe began with Big Bang at extremely high temperature. It starts to expand and cool down. When the temperature of the universe is decrease to certain value, the universe undergoes a series of phase transitions. After a few seconds of Big Bang, quark gluon phase of the universe is formed. In this phase, quark, antiquarks and gluons combined to form hadrons. Hadrons are responsible for formation of the present Baryonic matter. The quark matter along with up quark (u) and down quark (d) contains strange quarks (s^*). Witten [10] has proposed that strange matter may be actual ground state of baryon mater at high densities. He also pointed out that two flavor $u - d$ quark would lower its energy by undergoing β decay into a strange quark matter until the Fermi energy of all flavor becomes equal and the energy per baryon drops. When u, d and s^* quarks are equal in number with electron then strange matter becomes absolutely stable state with neutral charge [11]. In MIT bag model stability of strange quark matter (SQM) depends on Bag constant (B) [12]. Various models have been proposed to study strange quark matter. Stability of strange matter is studied by Torres and Menezes [13]. Mahanta and Biswal [14] have constructed string cloud and domain walls with quark matter in Lyra geometry. Katore et al. [15] have studied Bianchi type II, VIII and IX metric with quark matter coupled to domain wall in General Relativity. Yilmaz et al. [16] have studied Bianchi type I and V with quark and strange quark matter in the $f(R)$ theory of gravity.

Friedmann-Robertson-Walker models satisfactorily describes present day universe. But at small scale, the universe is not like FRW models. Also we do not except that the early universe has the properties like the present era. In order to describe the early universe, the homogeneous and anisotropic models are constructed. In this respect Bianchi types cosmological models play a vital role. Among the Bianchi types, Bianchi type I is the simplest anisotropic model. It is anisotropic at an early time and tends to isotropic at later time. Bianchi type I cosmological models have been widely studied in the framework of various theories of gravitation. Katore et al. [17] have investigated Bianchi type I in $f(R)$

theory of gravitation. Chawla and Mishra [18] have obtained exact solution of Einstein field equations for Bianchi type I viscous fluid cosmological models. Yadav et al. [19] have presented viscous model with cosmological term for Bianchi type I in General Relativity. Bianchi type I models in General Relativity are studied by Saha et al. [20]. Singh and Chaubey [21] have presented wet dark fluid Bianchi type I universe in General Relativity. They have also studied modified generalized tensor theory for Bianchi type I, V, VI0 space time with Λ term and scalar field φ [22].

It is found that quark matter has not been considered in the framework of Gauss-Bonnet gravity. In this paper, an attempt has been made to study the existence of quark matter and discuss the effect of temperature. The above discussion motivated us to work for the present paper. The paper is organized as follows: in Section 2, we present field equation in $f(G)$ theory for Bianchi type I space time. In Section 3, we review thermodynamics. Sections 4 and 5 deal with the exact solution for exponential and power law volumetric expansion respectively. In Section 6, we conclude our results.

2 Metric and Field Equations

The action of the $f(G)$ gravity is given by the following equation:

$$S = \frac{1}{2K}\int [R + f(G)]\sqrt{-g}d^4x + S_\phi(g_{ij}, \phi)\,, \tag{1}$$

where g_{ij} is the metric tensor, $K^2 = 8\pi G_N$, G_N is the constant of Newtonian. S_ϕ is the action of matter. The matter is minimally coupled to the metric tensor g_{ij} which means $f(G)$ is a purely metric theory of gravity. ϕ represents the matter field. The action is the function of metric as well as matter. The $f(G)$ is an arbitrary function of G which is given by

$$G = R^2 - 4R_{ij}R^{ij} + R_{ij\mu\nu}R^{ij\mu\nu}\,, \tag{2}$$

where R is the Ricci scalar, R_{ij} stands for Ricci tensor and $R_{ij\mu\nu}$ denotes Riemannian tensors. Varying the action (1) with respect to the metric g_{ij} we obtain the field equations as

$$\begin{aligned} R_{ij} - \frac{1}{2}Rg_{ij} + \delta\Big[R_{i\mu j\nu} + R_{\mu j}g_{\nu i} - R_{\mu\nu}g_{ji} - R_{ij}g_{\nu\mu} + R_{i\nu}g_{j\mu} \\ + \frac{1}{2}R(R_{ij}g_{\mu\nu} - g_{i\nu}g_{j\mu})\Big]\nabla^\mu\nabla^\nu + (Gf_G - f)g_{ij} = KT_{ij}\,. \end{aligned} \tag{3}$$

Here ∇^μ denotes the covariant derivative and f_G stand for the derivative of $f(G)$ with respect to G. We consider the LRS Bianchi type I line element given by

$$ds^2 = dt^2 - L^2dx^2 - M^2(dy^2 + dz^2)\,, \tag{4}$$

where L and M are metric potentials and functions of cosmic time t. Bianchi type I is more general cosmological models than the isotropic FRW models. It is important in the description of the evolution of the early phases of the universe. It is homogeneous model whose spatial section is flat but the expansion or contraction rate is directional dependent [19]. Very recently Hatkar et al. [23] have studied Kasner type Bianchi I with magnetized string cosmological models in $f(R,T)$ theory of gravitation. The Ricci scalar R and Gauss-Bonnet (GB) invariant for Bianchi type I is found to be

$$R = -2\left[\frac{L_{44}}{L} + 2\frac{M_{44}}{M} + 2\frac{L_4 M_4}{LM} + \frac{M_4^2}{M^2}\right], \tag{5}$$

$$G = 8\left[\frac{L_{44}M_4^2}{LM^2} + 2\frac{L_4 M_4 M_{44}}{LM^2}\right], \tag{6}$$

where subscript '4' denotes differentiation with respect to t. Due to the experimental results, Yilmaz et al. [16] have taken energy momentum tensor of the quark matter in the following perfect fluid form:

$$T_{ij} = (\rho + P)\,\mu_i \mu_j - P g_{ij}, \tag{7}$$

where u^i is the four velocity vector, $\rho = \rho_q + B$, $P = P_q - B$ are the total energy density and total pressure, respectively. The ρ_q represents strange quark energy density and P_q stands for strange quark pressure. Here, B is the bag constant. We will use the equation of state $P = \gamma\rho$, $0 \leq \gamma \leq 1$, since quark behaves like perfect fluid. It is assumed that in the QCD model vacuum exclude quarks whereas when volume contains quarks, vacuum must be expelled. In this trading, the cost is energy. This energy per unit volume is the bag constant (B). The energy associated with the presence of quark in volume$V$ is BV. Also there is energy due to the kinetic motion of the quark in the volume. The value of the bag constant lies in the range 60–80 MeV(fm)$^{-3}$. At zero temperature, the value of bag constant is 150 MeV^4 [24]. The quark matter bag model assumes that quarks are free to move in the colorless region only. Since quark energy has been a contribution of two types of energies i.e. energy due to expulsion of vacuum of QCD from volume V and energy associated with kinetic motion of the quark. The bag constant must exceed a certain minimum value. In the MIT bag model, the value of the bag constant is ~ 55 MeV(fm)$^{-3}$ where as in lattice calculation, it is estimated as ~ 55 MeV(fm)$^{-3}$ [12]. On the basis of bag models the equation of state for strange quark matter is modeled as [25]

$$P_q = \frac{1}{3}\rho_q \tag{8}$$

Burgio and Schulze [26] have pointed out that the maximum mass value whether B is density dependent or not, is dominated by the quark EoS at densities where the bag constant is much smaller than the quark kinetic energy. Katore and

Hatkar [27] have studied strange quark matter coupled to string cloud in Lyra geometry.

Let us define expansion scalar (θ), and shear scalar (σ) required for further derivation:

$$\theta = u^i_{;i} = \frac{L_4}{L} + 2\frac{M_4}{M} \tag{9}$$

$$\sigma = \sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{\sqrt{3}}\Big(\frac{L_4}{L} - \frac{M_4}{M}\Big)\,. \tag{10}$$

The average scale factor (a) and the volume (V) are related by

$$V = a^3 = LM^2\,. \tag{11}$$

Now, let us write the field equation (3) using equations (4)–(7) explicitly as

$$\frac{M_4^2}{M^2} + 2\frac{M_{44}}{M} - 16\frac{M_{44}M_4}{M^2}(f_G)_4 - 8\frac{M_4^2}{M^2}(f_G)_{44} + Gf_G - f = -KP\,, \tag{12}$$

$$\frac{L_{44}}{L} + \frac{M_{44}}{M} + \frac{L_4M_4}{LM} - 8\Big(\frac{L_{44}M_4}{LM} + \frac{M_{44}L_4}{ML}\Big)(f_G)_4 - 8\frac{L_4M_4}{LM}(f_G)_{44} + Gf_G - f = -KP\,, \tag{13}$$

$$\frac{M_4^2}{M^2} + 2\frac{L_4M_4}{LM} - 24\frac{L_4M_4^2}{LM}(f_G)_4 + Gf_G - f = K\rho \tag{14}$$

We have three equations in five unknown (L, M, P, ρ, f). To obtain exact solution of the system, we need more conditions. We can assume mathematical or physical relation between the unknown variables. We assume that the expansion (θ) is proportional to the shear scalar (σ). The ansatz with the help of equations (9) and (10) leads to

$$L = M^n\,, \tag{15}$$

where $n = \dfrac{1 + 2m_1\sqrt{3}}{1 - m_1\sqrt{3}}$ is proportionality constant. The physical importance of the stated direct mathematical relation is explained by Thorne [28]. According to the observations of the velocity-red shift relation for extragalactic sources; the Hubble expansion of the universe is isotropic today within approximately thirty percent. It is also found that the constant $\sigma/\theta = \text{const.}$ is satisfied for spatially homogeneous metric [29]. Secondly we assume two specific following function of the volume and subsequently use these to evaluate two different models:

$$V = c_1 e^{3mt}\,, \tag{16}$$

$$V = c_2 t^{3l}\,, \tag{17}$$

where c_1, c_2, l and m are constants. Use of the volumetric laws for obtaining solution is simple and gives physically viable models. Singh and Beesham [30] have used volumetric laws to obtain solution of the field equations in General relativity for anisotropic dark energy. Katore and Hatkar [31] have also obtained solutions using volumetric laws in the frame work of Lyra geometry for magnetized anisotropic dark energy.

3 Thermodynamics of the Model

The issues like origin, structure, constitutions related to the universe and have attracted the researcher from long time. Thermodynamics play an important role in the analysis of the behavior of the early universe. In addition to this, CMB radiation and black holes are invested using thermodynamics. Black hole has laid relation between gravity theories and thermodynamics [32–34]. Jacobson [35] has reproduced gravity theory from thermodynamical system. Recently, Singh and Singh [36] have investigated the Friedmann–Robertson–Walker space time for analysis of thermodynamical behavior of the model. We assume that the universe satisfies the first law of the thermodynamics. The union of the first and the second law of thermodynamics is applied to co-moving volume as

$$\tau dS_1 = d(\rho V) + PdV , \tag{18}$$

where τ and S_1 represent the temperature and entropy, respectively. In thermodynamics system, we get

$$S_1 = \frac{P + \rho}{\tau} V + d_1 , \tag{19}$$

where d_1 is the integration constant. Let us denote the entropy density by $\overline{S}$ and for simplicity taking the integration constant d_1 zero we obtain temperature and entropy density respectively as

$$\tau = \rho^{\frac{\gamma}{1+\gamma}} , \tag{20}$$

$$\overline{S} = (1 + \gamma) \rho^{\frac{1}{1+\gamma}} . \tag{21}$$

It is clear from the expressions (20) and (21)that the temperature and entropy density are depend on the quark matter and equation of state (EOS) parameter γ. We are interested in investigating the compatibility of the results obtained earlier and our findings on the background of the $f(G)$ theory.

4 Exponential Law Model

Now using equations (15) and (16), we obtain the metric potentials as

$$L = c_1^{\frac{n}{n+2}} e^{\frac{3mnt}{n+2}} , \tag{22}$$

$$M = c_1^{\frac{1}{n+2}} e^{\frac{3mt}{n+2}} . \tag{23}$$

Using equations (6), (22) and (23), the Gauss–Bonnet term (G) is obtained as

$$G = \frac{648(2n+1)m^4 n^2}{(n+2)^4} . \tag{24}$$

Using the equations (22) and (23) in equations (12) and (13), it leads us to the following expression of the $f_G(G)$:

$$f_G(G) = \frac{(n+2)t}{24m} + e^{-\frac{3m(2-n-n^2)t}{(1-n)(n+2)}} . \tag{25}$$

It is necessary to note that the integration constants appeared in the subsidiary steps of the differential equation is taken zero. Using equations (12)–(14), (22)–(25), we have the following expressions for pressure and energy density of quark matter (perfect fluid) are found to be:

$$P = l_1 + l_2 e^{-l_3 t} , \tag{26}$$

$$\rho = l_1(1-n) + l_4 e^{-l_3 t} , \tag{27}$$

where

$$l_1 = \frac{9m^2}{(n+2)K} ,$$

$$l_2 = \frac{648m^4 n(2-n-n^2)(n-2)}{(1-n)^2(n+2)K} ,$$

$$l_3 = \frac{3m(2-n-n^2)}{(1-n)^2(n+2)K} ,$$

$$l_4 = \frac{1944m^4 n(2-n-n^2)}{(1-n)^2(n+2)^4 K} .$$

From equations (26) and (27), it is clear that when $t \to \infty$, P, and ρ tend to constants. It should be noted that the obtained result is different from the one that obtained in $f(R)$ and $f(R,T)$ theories of gravitation for Bianchi type I metric [24, 37].

With the help of equations (8), (12)–(14), (26)–(27), the strange quark density and pressure of the model are obtained as

$$\rho_q = l_1(1-n) + l_4 e^{-l_3 t} - B , \tag{28}$$

$$P_q = \frac{1}{3}\left(l_1(1-n) + l_4 e^{-l_3 t} - B\right) . \tag{29}$$

The metric potentials L and M tends to infinity as $t \to \infty$ and P, $\rho \to$ const. Thus the universe will be steady state in the far future.

Making use of equation (27) in equations (20) and (21), we have the temperature and entropy density as

$$\tau = \left(l_1(1-n) + l_4 e^{-l_3 t}\right)^{\frac{\gamma}{1+\gamma}} , \tag{30}$$

$$\overline{S} = (1+\gamma)\left(l_1(1-n) + l_4 e^{-l_3 t}\right)^{\frac{1}{1+\gamma}} . \tag{31}$$

The quark matter pressure and energy density are functions of time t alone. They are not depends on the EOS parameter γ which we found in the $f(R)$ theory [16]. The pressure and density of quark matter and strange quark matter are differing by bag constant. The function $f(G)$ is increasing with increasing time whereas Gauss-Bonnet curvature invariant G is constant. The model have singularity for $n = -2$ and $n = 1$. The temperature and entropy density are decreasing functions of time. The result is consistent with Coley [38]. For $\gamma = 0$, the temperature becomes unity. According to the generalized second law of thermodynamics, the total entropy of the universe is increasing with time during cosmological evolution, i.e. the total entropy of the system is positive $\overline{S} \geq 0$ [36].

In this model we find that $\overline{S} \geq 0$ (Figure 1) which shows that the second law of thermodynamics is satisfied. Moreover, experimental results indicate that quark matter can occur at high temperature [39]. From Figures 2 and 3, it is clear that quark matter is present when the temperature is high and as temperature decreases, quark matter eventually transforms into strange quark matter. Here, the condition $P = 0$ and $\tau = 0$ cannot be obtained for any value of constants m and n. However, P and τ tends to zero as $t \to 0$, i.e. for large time quark matter converted into stable strange quark matter.

Note that the graphs are plotted for particular values of constants and other physical parameters. One may approach different conclusion by choosing different

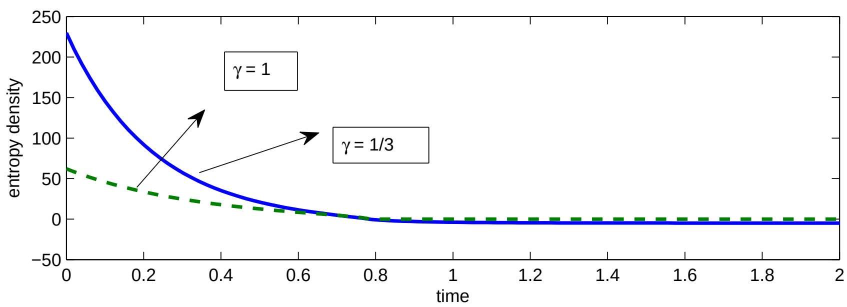

Figure 1. (Color online) Variation of entropy with cosmic time obtained using equation (31) for different values of γ.

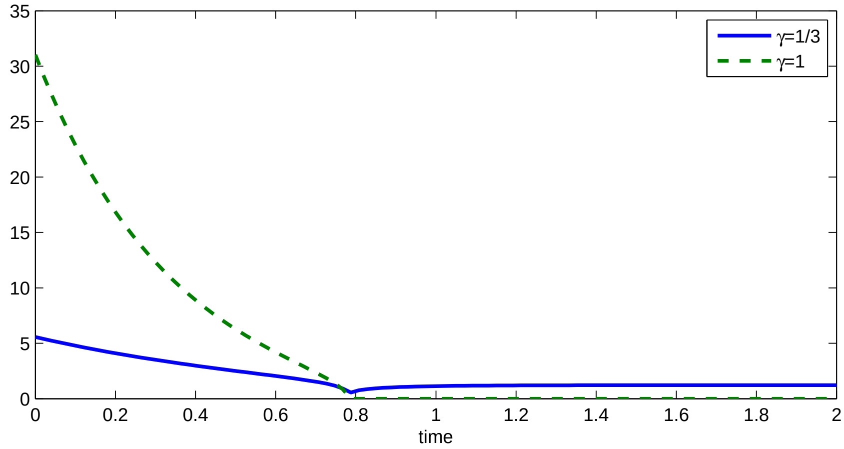

Figure 2. (Color online) Plot of temperature with cosmic time obtained using equation (30).

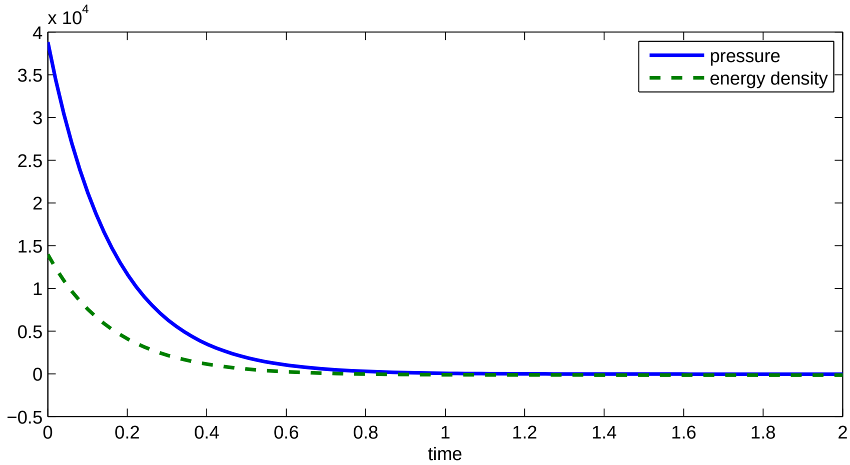

Figure 3. (Color online) Plot of pressure (solid line) and energy density (dotted line) against cosmic time obtained from equation (26) and (27).

values of constants. We also would like to clear that using the graphs we observe the behavior of the certain quantities and not measuring the exact values. So the we have not mentioned the units of any quantity. From Figure 4, it is clear that energy density is decreasing function of time. For $n = -1$, it is negative. Variation of temperature and entropy with cosmic time is shown in Figures 2 and 1 for Zeldovich fluid ($\gamma = 1$) and for radiation dominated universe ($\gamma = 1/3$). It is also observed that in the early stages of evolution of the universe, temperature

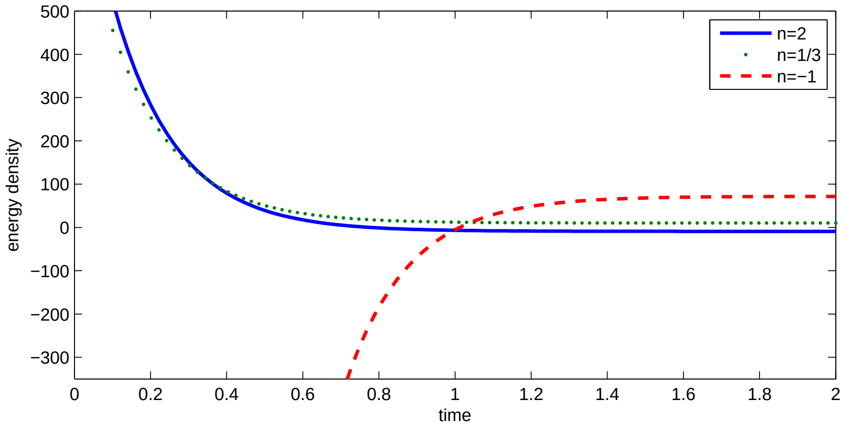

Figure 4. (Color online) Plot of energy density against cosmic time for different values of constant n.

of the universe is larger for $\gamma = 1$ than that of $\gamma = 1/3$ whereas entropy is larger for $\gamma = 1/3$ than that of $\gamma = 1$.

Using equations (9), (10), (22) and (23) the expression for expansion scalar (θ), shear scalar (σ), and deceleration parameter (q) are obtained as

$$\theta = 3m\,, \tag{32}$$

$$\sigma = \frac{\sqrt{3}m(n-1)}{n+2}\,, \tag{33}$$

$$q = -1\,. \tag{34}$$

From equations (32) and (33), the expansion and shear scalar are constant. Therefore, the universe is expanding with constant rate. The sign of deceleration parameter describes whether the universe is accelerating or decelerating. Negative value of the deceleration parameter implies, the universe is accelerating whereas positive value stands for decelerating universe. Here, deceleration parameter is negative, i.e. the universe is accelerating. It is pointed out that in Bianchi type I (Kasner type) models even small amount of anisotropy leads to unacceptably large helium abundances. In fact limits on the shear can be obtained in order for the predictions of the helim-4. Note that these limits are larger than those obtained from measurements of the isotropy of the CMB temperature distribution [40–42]. In this sense as σ/θ is constant throughout the evolution of the universe. It is zero, for $n = 1$ thus, the universe is isotropic for $n = 1$and for other values of n, it is anisotropic. The result is consistent with the large angle anomalies observed in CMB radiation [43].

5 Power Law Model

From equations (15) and (17), straightforward calculation gives us the metric potentials as

$$L = c_2^{\frac{n}{n+2}} t^{\frac{3ln}{n+2}} , \tag{35}$$

$$M = c_2^{\frac{1}{n+2}} t^{\frac{3l}{n+2}} . \tag{36}$$

From equations (6), (35) and (36), the Gauss–Bonnet term (G) is obtained as

$$G = \frac{216l^3 n(3ln - 3n + 6l - 6)}{(n+2)^4 t^4} . \tag{37}$$

Making use of equations (35), (36), (12) and (13) and solving the differential equations we obtain $f_G(G)$ as

$$f_G(G) = c_3 + c_4 t^{1-k_1} + \frac{k_2 t^2}{2(1+k_1)} , \tag{38}$$

where

$$k_1 = \frac{6l + 2n^2 + 2n - 3ln - 3ln^2 - 4}{(n+2)(1-n)} ,$$

$$k_2 = \frac{18l^2 - 6l + 3ln - 9l^2 n + 3ln^2 - 9l^2 n^2}{72l^2(1-n)} ,$$

and c_3, c_4 are constants. The expressions for quark matter pressure (P) and energy density (ρ) in case of perfect fluid using equations (12)–(14), (35)–(38) are obtained in the following form:

$$KP = \frac{k_4}{t^2} + k_5 t^{-k_1-3} , \tag{39}$$

$$K\rho = \frac{k_6}{t^2} + k_7 t^{-k_1-3} , \tag{40}$$

where

$$k_3 = \frac{3l}{n+2} , \qquad k_4 = \frac{27l^2 - 6ln - 12l}{(n+2)^2} - \frac{216l^3 k_2}{(1+k_1)(n+2)^3} ,$$

$$k_5 = \frac{216l^3 n c_4(1-k_1)}{(n+2)^3} , \qquad k_6 = (2n+1)k_3^2 - \frac{24nk_2k_3^2}{1+k_1} ,$$

$$k_7 = 24nc_4k_3^2(1-k_1) .$$

From equations (39) and (40), we observe that when $t \to \infty$, the P, ρ becomes zero, i.e. the quark matter transforms to another matter form. Moreover, we see that the metric potentials tends to infinity as $t \to \infty$ and $\rho \to 0$ with $t \to \infty$. Thus the universe will be deemed to have a cold ending. It is analogous with the result obtained in the $f(R)$ gravity [17]. With the help of equations (8), (12)–(14), (35)–(38), we obtain energy density and pressure of strange quark matter in the following form:

$$K\rho_q = \frac{k_6}{t^2} + k_7 t^{-k_1-3} - B\,, \tag{41}$$

$$KP_q = \frac{1}{3}\left(\frac{k_6}{t^2} + k_7 t^{-k_1-3} - B\right). \tag{42}$$

The temperature and entropy with the help of (20), (21) and (41) are found to be

$$\tau = \left(\frac{1}{K}\left(\frac{k_6}{t^2} + k_7 t^{-k_1-3}\right)\right)^{\frac{\gamma}{1+\gamma}}\,, \tag{43}$$

$$\overline{S} = (1+\gamma)\left(\frac{1}{K}\left(\frac{k_6}{t^2} + k_7 t^{-k_1-3}\right)\right)^{\frac{1}{1+\gamma}}\,. \tag{44}$$

The field equations are satisfied subjected to the condition $l = n + 2/3$. In this model energy density, temperature and entropy are decreasing functions of time (Figure 5). The Gauss–Bonnet curvature invariant G and $f(G)$ are functions of time t. G is decreasing whereas $f(G)$ is increasing with increasing time. The quark matter pressure and energy density are not depends on EOS parameter γ as obtained by Yilmaz et al. [16]. The pressure and energy density of quark matter and strange quark matter differ by bag constant. As obtained in exponential expansion model, in this case, we also have $\overline{S} \geq 0$, i.e. the total entropy of the

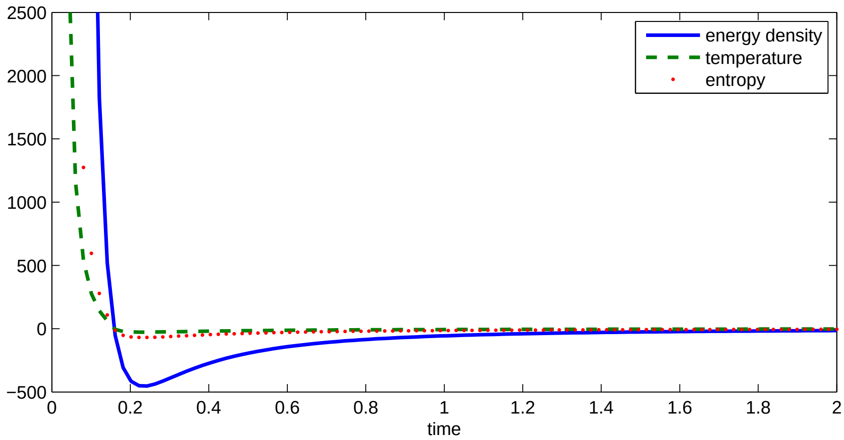

Figure 5. (Color online) Plot of energy density (solid line), temperature (dashed line) and entropy (dotted line) with cosmic time.

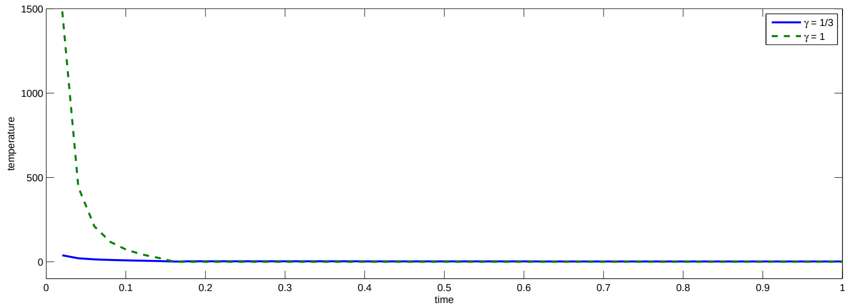

Figure 6. (Color online) Plot of temperature against cosmic time obtained form equation for different values of γ.

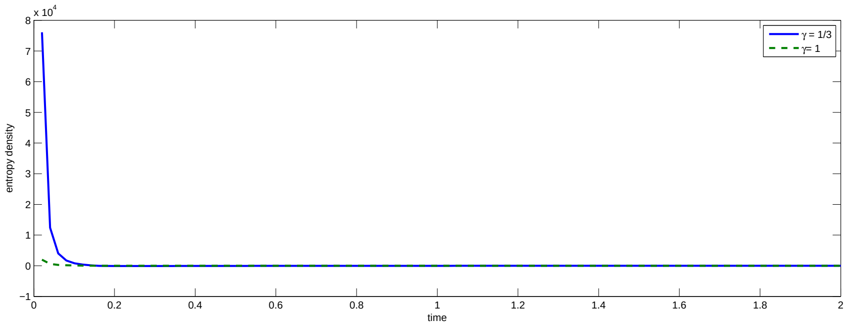

Figure 7. (Color online) Variation of entropy with cosmic time obtained from equation for different value ofγ.

universe is increasing. The model have singularity at $t = 0$, $n = -2$ and $n = 1$. Here, it should be note that the condition $P = 0$ and $\tau = 0$ cannot be satisfied for any value of constants l and n. However, the temperature and pressure are functions of cosmic time. They are very high at the early stages of evolution of the universe showing the existence of quark matter. As temperature decreases with increasing time quark matter transforms into stable strange quark matter. It is also observed that in the early stages of evolution of the universe, temperature of the universe is larger for $\gamma = 1$ than that of $\gamma = 1/3$ whereas entropy is larger for $\gamma = \frac{1}{3}$ than that of $\gamma = 1$. (Figures 6 and 7).

Using equations (9), (10), (41) and (42), the expressions of expansion scalar (θ), shear scalar (σ) and deceleration parameter (q) are found to be

$$\theta = \frac{3l}{t}\,, \tag{45}$$

$$\sigma = \frac{\sqrt{3}(n-1)}{(n+2)t}\,, \tag{46}$$

$$q = \frac{1-l}{l} \; . \tag{47}$$

It is clear that, the expansion and shear scalar are decreasing functions of time. The rate of expansion of the universe was higher in the early stages of evolution than the present. The universe is accelerating for $l > 1$ and $l < 0$ and decelerating for $0 < l < 1$. In this case the$\frac{\sigma}{\theta}$ is constant throughout the evolution of the universe which is similar to the exponential model in the previous section. It is zero for $n = 1$, the universe is isotropic for $n = 1$and for other values of n, it is anisotropic.

6 Conclusion

We have presented quark and strange quark matter models in $f(G)$ theory of gravitation for Bianchi type I metric. Two volumetric expansions namely exponential and power law models are explored. The crucial points observed in the investigation are as follows:

1. In both the models, the ratio σ/θ is constant and independent of cosmic time. It is depend on n. The universe is anisotropic throughout the evolution of the universe which support the large angle anomalies observed in CMB radiation [43]. When $n < 1$, σ/θ is negative and when $n = 1$, σ/θ is positive and greater than the present upper limit 10^{-5} of σ/θ obtained by Collins et al. [44].
2. In the exponential expansion model, the P, ρ becomes constant at large time. The universe is accelerating and expanding. Thus the universe will be steady state in the future [40–42].
3. In the power law model, the P, ρ vanishes at large time, i.e. the quark matter transforms to another matter form. The universe will be deemed cold ending in the future [40–42].
4. The second law of thermodynamics is satisfied, i.e. the universe have increasing entropy.
5. An important point is that the function $f(G)$ is obtained by solving the field equations.
6. The novelty of the work is that in both the models (exponential and power law models) it is observed that quark matter is transformed into strange quark matter (near zero temperature and pressure).

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