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Magnetized Strange Quark Matter in Lyra Geometry

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Abstract. In the present paper, we studied Magnetized-strange-quark-matter (MSQM) with the help of plane symmetric cosmological model in Lyra geometry. To analyze the results from the solution of field equations we used the relation of shear scalar and scalar expansion of the space-time as well as we have used the well accepted power law. We have computed dynamical cosmological parameters by using equation of state for strange quark matter and with the help of that we examined some physical and geometrical properties of the model in detail.

KEY WORDS: Cosmological Models, Magnetized-strange-quark-matter (MSQM), magnetic flux, Hubble parameter.

1 Introduction

The most of the researchers in the field of cosmology have curiosity about the behavior of the universe. One of them was Einstein, he developed theory of relativity that is general theory of gravitation and got very remarkable attention because of its success for constructing cosmological models and for understanding the evolution and origin of the universe. But this theory is not much successful to explain the late time acceleration of the universe. So it was a necessity to find alternative theories of gravitations. Some researchers have been introduced alternative gravitational theories to explain the late time acceleration of the universe. There are number of alternative theories and already checked by some researchers [1–5]. But we are interested in Lyra geometry since alternative idea of Riemannian geometry suggested by Lyra [6] with the new idea of a gauge function which resolves the problem of the non-integrability. Serially investigations were done by the several authors [7–10] and they have discussed cosmological models in Lyra's geometry with a constant gauge-vector in the time direction. After modification of Riemannian geometry by the Lyra, different sides of the field equations of Einstein theory have been computed by many researchers [11, 12]. Here, we have used field equations of Einstein theory in Lyra manifold and studied Magnetized-strange-quark-matter in plane symmetry. More-over, here our main intention is to observe the effects of Lyra generalization of Riemannian geometry on thermo dynamical properties of gravitation system. For achieving this goal, we are applying the power law and EoS for strange quark matter.

In the present work, we have been taken the MSQM distribution for plane symmetry in Lyra geometry. It is known that the most noteworthy component of the universe is magnetic field. Recently, the strong magnetic field got much attraction for the research in research community, because of its special properties of dense quark matter, neutron star matter and on stability of SQM [13–16]. Quark matter in strong magnetic field studied by [17, 18] and they found when the electromagnetic scale becomes the order of the nuclear scales, then some substantial changes appear in the properties of strange matter. Also it is known that the study of MSQM in the presence of magnetic field is the good acceptance.

MSQM discussed in modified theory [19] and they observed the universe starts with big bang and finished with Big Rip. Magnetic field is one of the most remarkable parts of the universe. Nowadays the magnetic field attracted lot of attention because of its effect and impact on the properties of SQM. The EoS for SQM by using a density-independent bag pressure in quark confinement is considered as [20],

$$p = \frac{\rho - 4B_c}{3} \,,$$

where bag constant – B_c is a vacuum energy density; it can be interpreted as the difference between the energy densities of the perturbative vacuum and physical ones, which has a constant value. The unit of bag constant is MeV/(fm)^3 and it lies in the range 60–80 MeV/(fm)^3. [21].

Here, we have considered a plane-symmetry cosmological model having the form given in [22, 23],

$$ds^2 = -A^2(dx^2 + dy^2)B^2dz^2 + dt^2 \,.$$

Here $A = A(t)$, $B = B(t)$, t is time.

Pawar et al. have studied plane symmetry cosmological model and they found universe starts expansion with big bang at initial time and expansion stops at large time [24]. Some researchers observe expanding and nonsingular universe in plane symmetry cosmological model and they found model is not attained isotropy for large time [25]. Also some researchers have been obtained plane symmetry model in alternative theory of gravitation and they observed the mean anisotropy parameter is constant throughout the evolution [26]. Recently Sahoo et al. [27] discussed MSQM in modified theory of gravity with different form of deceleration parameter in which they have considered equation of state for SQM. They have given notable observations about pressure and density towards bag constant. We motivate from the above research work and got the idea of the present paper. Here, we consider plane symmetry universe filled with magnetized-strange-quark-matter (MSQM) in Lyra manifold. The geometrical and physical behavior of the model is also analyzed. This works aims to study magnetized-strange-quark-matter (MSQM) in Lyra manifold.

We distributed our work in section wise, like section B is about field equations for Lyra's manifold. Section C is about metric (plane symmetry) and field equations for Magnetized strange quark matter in Lyra's manifold. In section D and E, we have computed the solution of field equations and analyzed the results. Conclusion of this work described in section F.

2 Field Equations of Lyra Manifold

The Lyra manifold is the modification of general relativity (GR). The field equations for Lyra's manifold is given by [7]

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\varphi_i\varphi_j - \frac{3}{4}g_{ij}\varphi_k\varphi^k = -T_{ij}\,, \tag{1}$$

where φ_i are displacement fields and the other notations have same meaning as in Riemannian geometry (here we have chosen $8\pi G = c = 1$). Here

$$\varphi_i = (0,0,0,\beta)\,, \tag{2}$$

where β is depend on time.

Also, the source of energy for MSQM is taken as

$$T_{ij} = (\rho + p + h^2)u_i u_j + \Big(\frac{h^2}{2} - p\Big)g_{ij} - h_i h_j\,, \tag{3}$$

where $u^i = (0,0,0,1)$ represents four-velocity vector and satisfying $u_i u_j = 1$. Also the magnetic flux h^2 is taken from x-direction with $h_i u^i = 0$. Now we can have the magnetic field in the yz-plane.

Here p is the proper pressure and ρ is the energy density.

The corresponding field equations become

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\varphi_i\varphi_j - \frac{3}{4}g_{ij}\varphi_k\varphi^k = -(\rho + p + h^2)u_i u_j - \Big(\frac{h^2}{2} - p\Big)g_{ij} + h_i h_j\,. \tag{4}$$

3 Metric and Field Equations

Our work motivated for a plane-symmetric cosmological model since it gives an opportunity for the study of in-homogeneity. For that, the metric is given by

$$ds^2 = -A^2(dx^2 + dy^2) - B^2dz^2 + dt^2 . \tag{5}$$

Here the metric potentials A, B are functions of cosmic time.

Now using eqn. (4), eqn.(2) and eqn.(3) for eqn. (5), we have

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -p + \frac{h^2}{2} , \tag{6}$$

$$\frac{\dot{A}^2}{A^2} + \frac{2\ddot{A}}{A} + \frac{3}{4}\beta^2 = -p + \frac{h^2}{2} , \tag{7}$$

$$\frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} - \frac{3}{4}\beta^2 = \rho + \frac{3}{2}h^2 . \tag{8}$$

Here $\dot{A} = \frac{dA}{dt}$, $\dot{B} = \frac{dB}{dt}$.

The dynamical parameters for plane symmetry are given by:

Average scale factor: $a(t) = (A^2B)^{\frac{1}{3}}$; (9)

Spatial volume: $V = a^3(t) = A^2B$; (10)

Directional and average Hubble parameter:

$$H_x = \frac{\dot{A}}{A} = H_y , \quad H_z = \frac{\dot{B}}{B} , \tag{11}$$

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\Big(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\Big); \tag{12}$$

Dynamical scalar expansion and shear scalar:

$$\theta = 3H ; \tag{13}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}\Big(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\Big)^2 ; \tag{14}$$

The average anisotropic parameter: $\Delta = \frac{1}{3}\sum_{i=1}^{3}\Big(\frac{H_i - H}{H}\Big)^2$, (15)

(H_i) are directional Hubble parameters.

We considered system for $\kappa = 1$.

4 The Solution of the Field Equations

System of Eqn. (6)–(8) reduces to three independent equations with six unknowns A, B, ρ, p, h^2, β. So for highly non-linear differential equations we require following physically significant conditions. For this,

i) We have used the relation of shear scalar and expansion scalar of the metric given by Collins [28], so that we have,

$$A = B^n \,; \tag{16}$$

ii) We have used the EoS for SQM, $p = \frac{\rho - 4B_c}{3}$, where B_c – bag constant.

iii) We have used the relation between the scalar field β and the average scale factor $a = a(t)$, $\beta = \beta_0 a^k$, where β_0 and k are positive constant

Now from Eq. (6), (7) and (16) we obtained the metric potentials

$$A = (ct+d)^{\frac{3n}{(q+1)(2n+1)}} \quad \text{and} \quad B = (ct+d)^{\frac{3}{(q+1)(2n+1)}} \,, \tag{17}$$

where c, d are arbitrary constants.

From Eqn. (17), Eqn. (5) can be written as

$$ds^2 = -(ct+d)^{\frac{6n}{(q+1)(2n+1)}}(dx^2+dy^2) - (ct+d)^{\frac{6}{(q+1)(2n+1)}} dz^2 + dt^2 \,. \tag{18}$$

5 Discussion of Some Physical and Dynamical Parameters

These parameters are very useful for the discussion of the properties of the cosmological model and it helps to extend cosmological theory in Lyra geometry. We have described some parameters for the metric given by Eqn. (18)

Average scale factor: $$a(t) = (ct+d)^{\frac{1}{q+1}} \,; \tag{19}$$

Spatial volume: $$V = a^3(t) = (ct+d)^{\frac{3}{q+1}} \,; \tag{20}$$

Directional and average Hubble parameters:

$$H_x = H_y = \frac{3nc}{(q+1)(2n+1)}(ct+d)^{-1} \tag{21}$$

$$H_z = \frac{3c}{(q+1)(2n+1)}(ct+d)^{-1} \,, \tag{22}$$

$$H = \frac{c}{(q+1)}(ct+d)^{-1} \,; \tag{23}$$

Dynamical scalar expansion: $$\theta = \frac{3c}{(q+1)}(ct+d)^{-1} \,; \tag{24}$$

Shear scalar: $$\sigma^2 = \frac{9c^2(n-1)^2}{2(q+1)^2(2n+1)^2}(ct+d)^{-2} \,; \tag{25}$$

Average anisotropic parameter: $$\Delta = \frac{2(n-1)^2}{2(n+1)^2} \,. \tag{26}$$

Now from Eq. (6), (7), (8) and (18) we obtained the value of ρ, p, h^2, β as

$$\rho = \Big(1 - \frac{3(n+1)}{2(q+1)(2n+1)}\Big)3c(ct+d)^{-1} + \frac{9nc^2(n-1)}{2(q+1)^2(2n+1)^2}(ct+d)^{-2} - \frac{3}{2}\beta_0^2(ct+d)^{\frac{2k}{q+1}} + 2B_c\,, \quad (27)$$

$$p = \Big(1 - \frac{3(n+1)}{2(q+1)(2n+1)}\Big)c(ct+d)^{-1} + \frac{3nc^2(n-1)}{2(q+1)^2(2n+1)^2}(ct+d)^{-2} - \frac{1}{2}\beta_0^2(ct+d)^{\frac{2k}{q+1}} - \frac{2}{3}B_c\,, \quad (28)$$

$$h^2 = \Big(\frac{3(n+1)}{2(q+1)(2n+1)}\Big)2c(ct+d)^{-1} + \frac{3nc^2(n+5)}{(q+1)^2(2n+1)^2}(ct+d)^{-2} + \frac{1}{2}\beta_0^2(ct+d)^{\frac{2k}{q+1}} - \frac{4}{3}B_c\,, \quad (29)$$

$$\beta = \beta_0(ct+d)^{\frac{k}{q+1}}\,. \quad (30)$$

From above solutions we can analyzed the nature of the cosmological model Eqn. (18).

We have the following observations (all parameters in arbitrary units):

- The metric potential A and B are constant at initial time and increasing function of time.
- At initial time spatial volume is finite. It is increasing function of cosmic time t. As time increases volume increases by choosing recent observed value of deceleration parameter $q = -0.53$. (Figure 1).

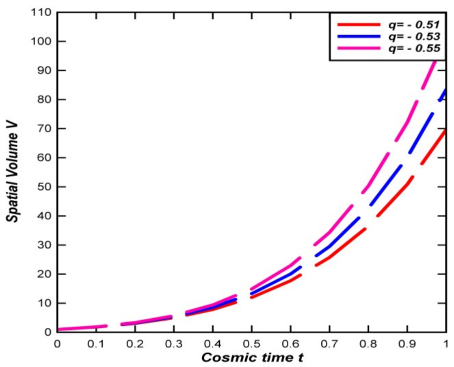

Figure 1. Spatial volume against cosmic time t for values of $c = d = 1$, by varying deceleration parameter q = -0.51, -0.53, -0.55.

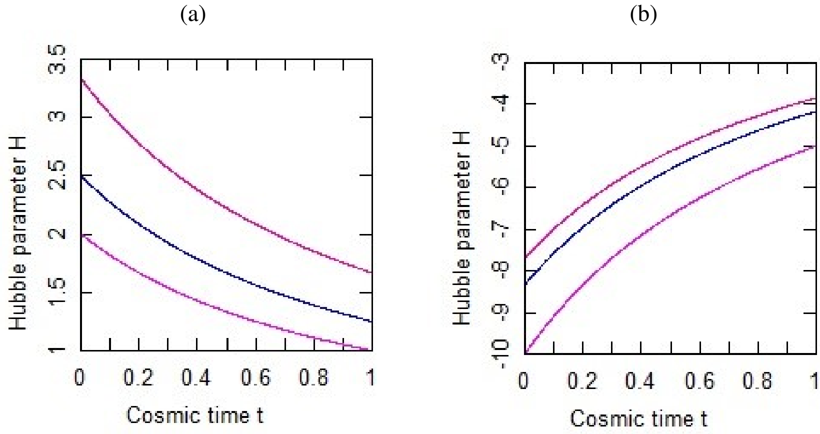

Figure 2. Hubble parameter against cosmic time t for values of $c = d = 1$, by approaching to deceleration parameter from left **(a)** as well as from right **(b)** in the neighborhood of deceleration parameter q. ($q < -1$ and $q > -1$, respectively).

- The directional Hubble parameter, average Hubble parameter are decreased from left and increased from right in the neighborhood of $q = -1$ (Figure 2).
- Also, $(\sigma/\theta)^2 \neq 0$, $n \neq 1$.

From the above result we have obtained the model is anisotropic throughout the evolution except $n = 1$ (Figure 3) hence we have good agreement with the results of the researchers [26].

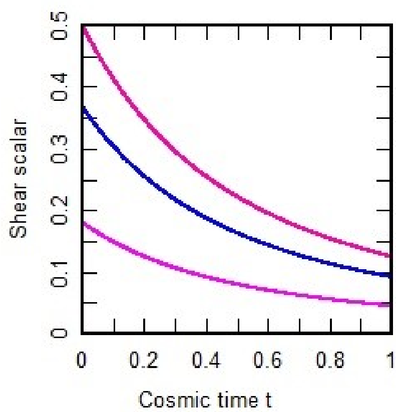

Figure 3. Shear scalar against cosmic time t for values of $c = d = 1$, $q < -1$, and varying $n = 2, 3, 4$.

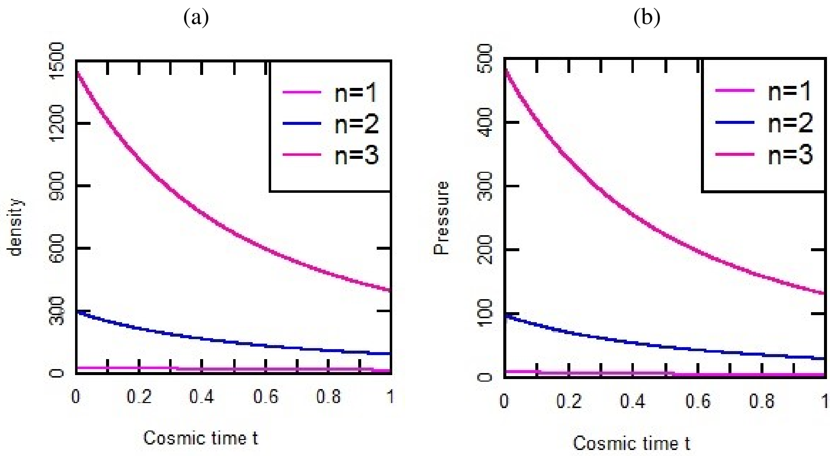

Figure 4. Density **(a)** and pressure **(b)** against cosmic time t for the values of $c = d = 1$, $q < -1$,$\beta_0 = 1$, $\beta c = 1$, $k = 1$ and varying $n = 2,\ 3,\ 4$.

- Scalar expansion (θ) and shear scalar (σ^2) diverge with $t \rightarrow 0$. The parameters σ^2 and θ are decreasing function of time t and vanish as $t \rightarrow \infty$. Our results well corresponded with the results of [29, 30].
- As time increases density, pressure and magnetic flux decreases (Figure 4 and Figure 5). There is no initial singularity since at initial time all quantity becomes constant.

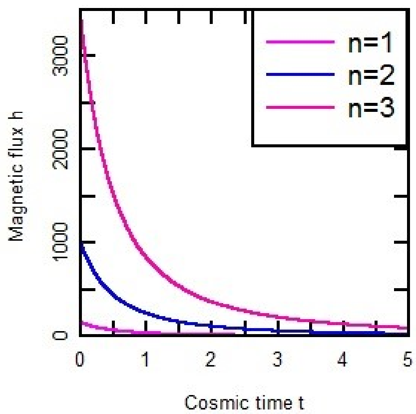

Figure 5. Magnetic flux against cosmic time t for values of $c = d = 1$, $q < -1$, $\beta_0 = 1$, $\beta c = 1$, $k = 1$ and varying $n = 2,\ 3,\ 4$.

6 Conclusions

Here we studied the magnetized strange quark matter in Lyra manifold by using plane symmetry universe. In order to solve field equations we have used relation between shear scalar and expansion scalar of the metric, EoS for SQM. It is good agreement that we found magnetic flux is effective and nonzero for plane symmetry cosmological model and it is the time dependent function. This is compatible with the result of [19]. There is no physical singularity for the model which we have discussed in detail. It means initially the universe has finite volume and then it expands as time increases. We have analogous results for Lyra geometry as discussed in other alternative theory in [31]. The Hubble parameters (directional as well as an average) are time dependent since they are function of cosmic time t. The expansion scalars are also the function of time t. All these parameters gives finite value at initial time but it decrease with increasing time t, and for infinite time it will be constant. We have good agreement with results of [26]. Also, the approach of deceleration parameter found to be negative value and it match with present behavior of universe. The anisotropic parameter is zero for $n = 1$. Otherwise it is varying constant throughout the evolution of the universe. Shear scalar is decreasing function of time t and vanished at $n = 1$. From the above discussion we can say that the model is isotropic only for $n = 1$ and anisotropic for $n \neq 1$. Also we observed that, as time t increases density and pressure decreases. There is no initial singularity since at initial time all quantities becomes constant. In the present paper we have focused on magnetic flux and we got effective magnetic flux which is decreasing function of cosmic time t.

Equation of state of dark energy plays measure role to distinguish the phases of accelerated expansion of the universe. We observed that in the ΛCDM model the cosmological constant $\omega \simeq -1$. But in a different dark energy theories observed different values of cosmological constant ω and found that the expansion of the universe is accelerating for any equation of state with $\omega < -1/3$. In the present work we have observed the value of the cosmological constant ω defined in the equation of state ($\omega = p/\rho$). From the value of density, pressure (Eqn. (**??**), Eqn. (**??**)) and deceleration parameter $q < -1$, we found that $\omega < -1/3$ for large time t. The present discussed model shows that the expansion of the universe is accelerating which agreed with [32].

Role of the bag constant in the expansion of the universe is important. We have observed the pressure and energy density approaches to bag constant for large time t ($t \rightarrow \infty$). In particular, pressure $p \rightarrow -(2/3)B_c$ and energy density $\rho \rightarrow 2B_c$ as $t \rightarrow \infty$. Negative sign for pressure indicate the expansion of the universe in late time. Also, Negative pressure and SQM along with active magnetic flux shows existence of the dark energy. This result agreed with the recent observation by P.K. Sahoo et al. [27] and the observations of the type Ia Supernovae by Riess et al. [33].

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