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Gauged Floreanini-Jackiw Type Chiral Boson in Lagrangian Approach

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Abstract. We have shown how to systematically derive the complete set of gauge transformation of Gauged Floreanini-Jackiw type chiral boson and its Gauge invariant version in the Lagrangian approach. It is found that the formalism has the ability to demonstrate a direction how to extend the phase space using auxiliary fields to restore the gauge invariance of a theory. We have also found that the prescription is equally effective in both the usual phase space and the extended phase space of a theory.

KEY WORDS: Lagrangian Approach, Gauged Floreanini-Jackiw type chiral boson, Gauge symmetry, Wess Zumino Term, FJ Type Kinetic Term, Constraint.

1 Introduction

Gauge invariance is one of the most fundamental and important concepts in modern theoretical physics. In particle physics it means local phase transformation of matter fields followed by appropriate transformation of gauge fields. Gauge symmetric related study has been done through Hamiltinian approach. But this formalism does not give the Lorentz covariant partition function. This drawback has remedy in Lagrangian approach. This is an advantage of Lagrangian formalism [1–3] over Hamiltonian formalism [4–9]. This formalism has developed by Shirzad [1]. So implementation of this formalism on the Gauged Floreanini-Jackiw type chiral boson in order to test whether a given model has a gauge symmetry or it lacks in it. Thus finding out the appropriate gauge transformation for that theory would be interesting subject of study. It would be more important to test this formalism in the extended phase space needed to restore the gauge invariance to see whether this formalism works there in an appropriate manner as it did in the usual phase space [2]. The Gauged Floreanini-Jackiw type chiral boson is an anommalous model. One justification for taking an anomalous model into consideration is to ensure whether this scheme is capable of testing the absence of gauge symmetry when it is lacking in a given model. Thus the use of this prescripton on anomalous model when it is demonstrating the lack of gauge symmetry, the anomalous model is immediately modified in such a way through which the gauge symmetry is restored and hence apply the formalism to determine the correct gauge transformations in that situation.

The aim of this paper is therefore, to study the gauge non-symmetric (anomalous) model with the prescription based on Lagrangian formulation and also determine the explicit form of the gauge transformatioins from the lagrangian equations of motion [1,3] through the Lagrangian formalism. We find the greatest number of equations including accelerations for a singular lagrangian by means of differentiating the lagrangian constraints and a number of identities among the Euler derivatives emerge. Through this principle, it is possible to determine and count different types of degrees of freedom; these are gauge, and constraint ones. developed by Shirzad in [1]. We are intended to investigate whether Shirzad's formalism enables one to verify the presence or absence of gauge symmetry in a given theory. We have also got the idea of Wess Zumino term which is needed to make the theory invariant.

Here we have considered the lagrangian of Gauged Floreanini-Jackiw type chiral boson [10, 11]. Application of lagrangian approach on this model, shows that the model does not posses gauge symmetry. It is modified in such a way that the gauge symmetry gets restored and hence apply the formalism to find out the correct gauge transformations in that situation.

The paper is organised as follows, in Section 2. investigations have been carried out through lagrangian formulation over Gauged Floreanini-Jackiw type chiral boson to test whether gauge symmetry posses or not. The model under consideration does not have gauge symmetry. In Section 3, the model is made gauge invariant by adding appropriate Wess Zumino term. Then investigation has been done and hence determine the gauge transformation generators in this case. End section 4 is devoted to conclusion.

2 Gauged Floreanini-Jackiw Type Chiral Boson

Free chiral boson is interesting because it is the fundamental component of string theory. The free chiral boson interacts with the $U(1)$ gauge field, and this interacting field theoretical model is called gauged chiral boson [11]. The theory of interacting chiral boson with FJ type kinetic term is known as Gauged Floreanini-Jackiw type chiral boson [10, 11]. The lagrangian density of our present consideration is

$$L_{GF} = \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) + \frac{1}{2}(\dot{A}_1 - A_0')^2 + \frac{ae^2}{2}(A_0^2 - A_1^2) - \frac{1}{2}e^2(A_0 - A_1)^2. \quad (1)$$

In general the Euler lagrange equations of motion for a lagrangian is

$$L_k(q, \dot{q}) = w_{kl}\ddot{q}_l + \alpha_k = 0, \quad (2)$$

where w_{kl} is the Hessian matrix and given by

$$w_{kl} = \frac{\partial^2 L}{\partial \dot{q_k} \partial \dot{q_l}}, \tag{3}$$

and α_j of equation (1) is

$$\alpha_k = \frac{\partial^2 L}{\partial q_k \partial \dot{q_l}} \dot{q_k} - \frac{\partial L}{\partial q_k}. \tag{4}$$

For a singular lagrangian $det[w_{kl}] = 0$.

Using (1), we get equations of motion for the field ϕ, A_0 and A_1 are

$$L_\phi = 2(\dot{\phi}' - \phi'') + 2e(A_0' - A_1'), \tag{5}$$

$$L_{A_0} = A_0'' - \dot{A_1}' - 2e\phi' - ae^2 A_0 - e^2 A_1 + e^2 A_0, \tag{6}$$

$$L_{A_1} = \ddot{A_1} - \dot{A_0}' + ae^2 A_1 + 2e\phi' + e^2 A_1 - e^2 A_0. \tag{7}$$

For this system the matrices w and α are

$$w = \begin{pmatrix} 0\,0\,0 \\ 0\,0\,0 \\ 0\,0\,1 \end{pmatrix} \delta(z - x), \tag{8}$$

$$\alpha = \begin{pmatrix} 2\dot{\phi}' - 2\phi'' + 2e(A_0' - A_1{}') \\ A_0'' - \dot{A_1}' - 2e\phi' + e^2 A_0 - e^2 A_1 - ae^2 A_0 \\ -\dot{A_0}' - e^2 A_0 + e^2 A_1 + 2e\phi' + ae^2 A_1 \end{pmatrix} \delta(x - y). \tag{9}$$

It is discovered that Hessian matrix w has rank three. Hence there exist a null eigenvector which is given by

$$\lambda^1(z) = (0, 1, 0)\delta(z - x) \tag{10}$$

and we get the primary lagrangian constraint multiplying equation (1) by λ^1

$$\gamma^1(x, t) = (A_0'' - \dot{A_1}' - 2e\phi' - ae^2 A_0 - e^2 A_1 + e^2 A_0)(x, t). \tag{11}$$

To preserve consistency of the primary constraint with time, the time derivative of γ^1 is added with L_{GF} that gives L_1

$$L_1(x, t) = (L_{GF} + \frac{\partial}{\partial t} \gamma^1)(x, t). \tag{12}$$

The matrices w_1 and α_1 are calculated as follows:

$$w^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{\partial}{\partial x} \end{pmatrix} \delta(z-x), \tag{13}$$

$$\alpha^1 = \begin{pmatrix} 2\dot{\phi}' - 2\phi'' + 2e(A_0' - A_1{}') \\ A_0'' - \dot{A_1}' - 2e\phi' + e^2 A_0 - e^2 A_1 - ae^2 A_0 \\ -\dot{A_0}' - e^2 A_0 + e^2 A_1 + 2e\phi' + ae^2 A_1 \\ \dot{A}_0'' - 2e\dot{\phi}' - ae^2 \dot{A}_0 - e^2 \dot{A}_1 + e^2 \dot{A}_0 \end{pmatrix} \delta(x-y). \tag{14}$$

A straight forward calculation shows that w_1 also has a null eigenvector

$$\lambda^2(z) = (e, 0, \frac{\partial}{\partial x}, 1)\delta(z-x). \tag{15}$$

Multiplying the equation (12) from left by λ^2, we get

$$\gamma^2(x,t) = \int dx \lambda^2_{i_2} L_1(x,t) = e^2(A_1' - A_0') + 2e\phi'' - 2e\dot{\phi}' + ae^2(A_1' - \dot{A}_0) + e^2(\dot{A}_0 - \dot{A}_1), \tag{16}$$

which is non vanishing one. This γ^2 is nothing but the secondary constraint for this system. For preserving consistency we have added the time derivative of secondary constraint to the equation (12) and obtain

$$L_2(x,t) = (L_1 + \frac{\partial}{\partial t}\gamma^2)(x,t). \tag{17}$$

It contains of w^2 and α^2

$$w^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial x} & -ae^2 + e^2 & -e^2 \end{pmatrix} \delta(z-x), \tag{18}$$

$$\alpha^2 = \begin{pmatrix} 2\dot{\phi}' - 2\phi'' + 2e(A_0' - A_1{}') \\ A_0'' - \dot{A_1}' - 2e\phi' + e^2 A_0 - e^2 A_1 - ae^2 A_0 \\ -\dot{A_0}' - e^2 A_0 + e^2 A_1 + 2e\phi' + ae^2 A_1 \\ \dot{A}_0'' - 2e\dot{\phi}' - ae^2 \dot{A}_0 - e^2 \dot{A}_1 + e^2 \dot{A}_0 \\ 2e\dot{\phi}'' + e^2(\dot{A_0}' - \dot{A_1}') + ae^2 \dot{A_1}' \end{pmatrix} \delta(z-x). \tag{19}$$

The matrix w^2 also is discovered to give another null eigenvector

$$\lambda^3(z) = (0, 0, \frac{\partial}{\partial x}, 1, 0)\delta(z-x) \tag{20}$$

Multiplying the equation (17) from left by λ^3 , we obtain

$$\gamma^3(z,t) = e^2(A_1' - A_0') + ae^2(A_1' - \dot{A}_0) + e^2(\dot{A}_0 - \dot{A}_1) + 2e\phi'' - 2e\dot{\phi}' \\ = \gamma^2(x,t). \tag{21}$$

Equation (21) indicates that the algorithm is ended here. So multiplication of λ^3 with L_2 does not gives rise to any new constraint. The local gauge symmetry of the action (1) are encoded in the identity (21). One can express γ^3 in the form of equation in a following manner

$$\gamma^3(x,t) = (eL_\phi + \frac{\partial}{\partial t}L_{A_0} + \frac{\partial}{\partial x}L_{A_1})(x,t). \tag{22}$$

The general form of identity [1] is

$$\sum_{s=0}^{n} \frac{d^s}{dt^s}(\phi_{si}L_i) = 0, \tag{23}$$

here ϕ_{si} are some functions of coordinate and their derivatives.

To get the non vanishing $\phi' s$ we equate equation (22) with (23) and we find that

$$\phi_{0,1}(x,z) = e\delta(z-x), \quad \phi_{0,1}(x,z) = \delta(z-x), \quad \phi_{0,2}(x,z) = \frac{\partial}{\partial x}\delta(z-x).$$

For the system (1) the general gauge transformation [1] of the fields are

$$\delta q_i = \sum_{s=0}^{n}(-1)^s \frac{d^s f}{dt^s}\phi_{si}, \tag{24}$$

where $f(t)$ is an arbitrary function of time. The gauge transformation formula (24) gives the following gauge transformation for the field ϕ, A_0 and A_1:

$$\delta\phi = ef(x,t), \tag{25}$$

$$\delta A_0 = -\frac{\partial}{\partial t}f(x,t), \tag{26}$$

$$\delta A_1 = -\frac{\partial}{\partial x}f(x,t). \tag{27}$$

The variation of L_{GF} under the above transformation is

$$\delta L_{GF}(x,t) = -\sum_{s=0}^{n}\frac{d^s(\phi_{si}L_i)}{dt^s}(x,t) \\ = -[eL_\phi + \frac{\partial}{\partial t}L_{A_0} + \frac{\partial}{\partial x}L_{A_1}]f(x,t) \\ = (e^2(A_1' - A_0') + ae^2(A_1' - \dot{A}_0) \\ + e^2(\dot{A}_0 - \dot{A}_1) + 2e\phi'' - 2e\dot{\phi}')f(x,t). \tag{28}$$

Since δL_{GF} does not vanish, The lagrangian density (1) is therefore, not invariant under the transformation (25), (26) and (27). Here also it appears that the formalism has tested correctly that the lagrangian has no gauge invariance in the usual phase space.

3 Gauged Floreanini-Jackiw Type Chiral Boson Made Gauge Invariant

In the previous section it is found that the lagrangian is not gauge invariant. Let us add the appropriate Wess Zunino term to the lagrangian (1) which mandatorily extend the phase space of the theory and investigation has been done whether the process can infer that gauge invariance has restored in the extended phase space.

Lagrangian density in $(1+1)$ dimension is

$$\begin{aligned} L_{EGF} = \dot{\phi}\phi' - \phi'^2 + 2e\phi'(A_0 - A_1) + \frac{1}{2}(\dot{A}_1 - A_0')^2 + \frac{ae^2}{2}(A_0^2 - A_1^2) \\ - \frac{1}{2}e^2(A_0 - A_1)^2 + \frac{1}{2}(a-1)(\dot{\theta}^2 - \theta'^2) \\ + e(a-1)(A_1\theta' - A_0\dot{\theta}) + e(A_0\theta' - A_1\dot{\theta}). \end{aligned} \tag{29}$$

The field θ represents an auxiliary field. The equations of motion for the field ϕ, A_1, A_0, θ are

$$L_\phi = 2(\dot{\phi}' - \phi'') + 2e(A_0' - A_1'), \tag{30}$$

$$L_{A_0} = A_0'' - \dot{A_1}' - 2e\phi' - ae^2A_0 - e^2A_1 + e^2A_0 - e\theta' + e(a-1)\dot{\theta}, \tag{31}$$

$$L_{A_1} = \ddot{A}_1 - \dot{A_0}' + ae^2A_1 + 2e\phi' + e^2A_1 - e^2A_0 + e\dot{\theta} - e(a-1)\theta', \tag{32}$$

$$L_\theta = (a-1)(\ddot{\theta} - \theta'') + e(a-1)(A_1' - \dot{A}_0) + e(A_0' - \dot{A}_1). \tag{33}$$

For the lagrangian (29), w and α are

$$w = \begin{pmatrix} 0\,0\,0 & 0 \\ 0\,0\,0 & 0 \\ 0\,0\,1 & 0 \\ 0\,0\,0 & a-1 \end{pmatrix} \delta(z-x), \tag{34}$$

$$\alpha = \begin{pmatrix} 2\dot{\phi}' - 2\phi'' + 2e(A_0' - {A_1}') \\ A_0'' - \dot{A_1}' - 2e\phi' + e^2A_0 - e^2A_1 - ae^2A_0 - e\theta' + e(a-1)\dot{\theta} \\ -\dot{A_0}' - e^2A_0 + e^2A_1 + 2e\phi' + ae^2A_1 + e\dot{\theta} - e(a-1)\theta' \\ -(a-1)\theta'' + e(a-1)(A_1' - \dot{A}_0) + e(A_0' - \dot{A}_1) \end{pmatrix} \delta(x-y) \tag{35}$$

We see that Hessian matrix w posses the following null eigenvector:

$$\lambda^1(z) = (0,1,0,0)\delta(z-x). \tag{36}$$

and the primary lagrangian constraint is obtained here in the same way as we have in previous section by multiplying λ^1 with (29) from the left

$$\gamma^1(x,t) = (A_0'' - \dot{A_1}' + e^2 A_0 - e^2 A_1 - ae^2 A_0 - 2e\phi' + e(a-1)\dot{\theta} - e\theta')(x,t). \quad (37)$$

In order to preserve consistency of the above primary constraint we add the time derivatives of γ^1 with L_{EGF} and obtain $L_1(x,t)$ as follows:

$$L_1(x,t) = (L_{EGF} + \frac{\partial}{\partial t}\gamma^1)(x,t). \quad (38)$$

The above $L_1(x,t)$ contains w_1 and α_1 as usual which are given by

$$w^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & (a-1) \\ 0 & 0 & -\frac{\partial}{\partial x} & e(a-1) \end{pmatrix} \delta(z-x) \quad (39)$$

$$\alpha_1 = \begin{pmatrix} 2\dot{\phi}' - 2\phi'' + 2e(A_0' - A_1') \\ A_0'' - \dot{A_1}' - 2e\phi' + e^2 A_0 - e^2 A_1 - ae^2 A_0 - e\theta' + e(a-1)\dot{\theta} \\ -\dot{A_0}' - e^2 A_0 + e^2 A_1 + 2e\phi' + ae^2 A_1 + e\dot{\theta} - e(a-1)\theta' \\ -(a-1)\theta'' + e(a-1)(A_1' - \dot{A}_0) + e(A_0' - \dot{A}_1) \\ \dot{A_0}'' + e^2\dot{A}_0 - e^2\dot{A}_1 - ae^2\dot{A}_0 - 2e\dot{\phi}' - e\dot{\theta}' \end{pmatrix} \delta(x-y). \quad (40)$$

The matrix w^1 gives the following null eigenvector:

$$\lambda^2(z) = (e, 0, \frac{\partial}{\partial x}, -e, 1)\delta(z-x). \quad (41)$$

Multiplying the equation (38) from left by λ^3, we obtain

$$\gamma^2(z,t) = 0. \quad (42)$$

The above vanishing condition establishes that multiplication of λ^2 with L_1 will not give new constraint. Naturally, rank of equation for acceleration will not be increased. The process is ended here. We write equation (42) in the following manner:

$$\gamma^2(x,t) = (\frac{\partial}{\partial t}L_{A_0} + \frac{\partial}{\partial x}L_{A_1} + eL_\phi - eL_\theta)(x,t). \quad (43)$$

One needs to compare equation (43) with (23) to obtain ϕ's and that results the following:

$$\phi_{0,1}(z,x) = e\delta(z-x), \qquad \phi_{1,2}(z,x) = \delta(z-x),$$
$$\phi_{0,3}(z,x) = \frac{\partial}{\partial x}\delta(z-x), \qquad \phi_{0,4}(z,x) = -e\delta(z-x).$$

The gauge transformation formula (24) gives the following transformation for the field ϕ, A_0 and A_1, θ:

$$\delta\phi = ef(x,t), \tag{44}$$

$$\delta A_0 = -\frac{\partial}{\partial t} f(x,t), \tag{45}$$

$$\delta A_1 = -\frac{\partial}{\partial x} f(x,t), \tag{46}$$

$$\delta\theta = -ef(x,t). \tag{47}$$

The variation of L_{EGF} is under the above transformations (44), (45) and (46), (47) is found out to be

$$\begin{aligned}\delta L_{EGF}(x,t) &= -\sum_{s=0}^{n} \frac{d^s(\phi_{si}L_i)}{dt^s}(x,t) \\ &= -[\frac{\partial}{\partial t}L_{A0} + \frac{\partial}{\partial x}L_{A1} + eL_\phi - eL_\theta)]f(x,t) = 0.\end{aligned} \tag{48}$$

It shows that in the extended phase space the lagrangian (29) is invariant under the above gauge transformation. Therefore, it is shown that the formalism is capable of verifying the gauge symmetric property of this system in the extended phase space also.

4 Conclusion

The lagrangian formalism developed in [1] is found instrumental to study the gauge symmetric property of a theory. We have considered the lower dimensional model of Gauged Floreanini-Jackiw type chiral boson [10,11]. The model had no gauge symmetry to start with. It has been verified by lagrangian formalism successfully. Then the model has made gauge invariant in the extended phase space with the inclusion of appropriate Wess Zumino term. Through the use of lagrangian formalism, the investigations were carried out on the gauge invariant version of this model. It is found that the formalism is not only useful in the usual phase space of the theory but also it is equally powerful in the extended phase space too [2]. Lagrangiuan approach provide a systematic derivation of gauge transformation generators in both usual and extended phase space. One important thing of this formalism is that one can have a guess about the Weiss-Zumino term which is needed to bring back the symmetry of a gauge non invariant theory.

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