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Cosmological Models of Universe with Variable Deceleration Parameter in Lyra's Geometry

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Abstract. In this paper, we discuss Cosmological models based on Lyra's geometry in the presence of particle creation. The effect of variable deceleration parameter on the displacement field (β) and energy density (ρ) has been inspected and exact solution is obtained for FRW model for a specific form of particle creation function.

KEY WORDS: Lyra's geometry, particle creation, variable deceleration parameter.

1 Introduction

Einstein introduced the cosmological constant into his field equations and Einstein's field equations allow only non-static cosmological models for non-zero energy density. In Einstein's general theory of relativity, gravitation is described in terms of geometry. In 1998, Weyl [1] proposed a more general theory in which electromagnetism is also described geometrically. However, this theory which is based on non-integrability of length transfer was not accepted. Then in 1951, Lyra [2] came up with a modification of Riemannian geometry which may also be considered as modification of the geometry of Weyl's. He introduced the structureless manifold with a gauge function. This removed the non-integrability condition of the length of a vector under parallel transport and also led to the inclusion of the cosmological constant in a more natural way from the geometry. Then Sen and Dunn [3] suggested a new scalar tensor theory of gravitation. As suggested by Halford [4, 5], the constant displacement vector field ϕ_i in Lyra's geometry and the cosmological constant in the general theory of relativity play the same role.

The cosmological models formulated on Lyra's geometry have been examined by many authors [6] with an invariable displacement field vector. Nonetheless the constant nature of the diplacement field vector is a coincidence and no scientific proof explanantion exists for it. The Einstein's field equation is a system of higher order non-linear differential equations and we look for their solutions in order to use them in astrophysics and cosmology. The law of variation for Hubble's parameter may be used to give the solutions to these equations as suggested by Berman [7]. The inspection (Knop et al. [8]; Riess et al. [9]) of type Ia Supernovae enable to investigate expanssion of universe. These observations conclude that the expanssion of universe is accelerating. Thus the cosmological model with variable deceleration parameter be considered.

2 Field Equation

The Einstein's field equation based on the Lyra's geometry is written as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - 3\phi_k\phi^k = -8\pi G T_{ij}\,, \tag{1}$$

where ϕ is the displacement vector.

Based on [10] energy momentum tensor in the presence of creation of matter is given as

$$T_{ij} = (\rho + p + p_c)u_i u_j - (p + p_c)g_{ij}\,, \tag{2}$$

where ρ and p are the energy density and pressure respectively, p_c is the creation pressure, u_i – the field-four velocity, and g_{ij} is the metric tensor. The time like displacement vector is given by

$$\phi_i = (\beta(t), 0, 0, 0)\,. \tag{3}$$

For the FRW metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right]\,,$$

where $k = 1, 0, -1$, the field equation (1) with (2) and (3) becomes

$$3H^2 + \frac{3k}{R^2} - \frac{3\beta^2}{4} = \chi\rho\,, \tag{4}$$

$$2\dot{H} + 3H^2 + \frac{k}{R^2} + \frac{3\beta^2}{4} = -\chi(p + p_c)\,, \tag{5}$$

where $\chi = 8\pi G$ and $H = \dot{R}/R$ is the Hubble's function.

Equations (4) and (5) lead to the continuity equation

$$\chi\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + 3\left[\chi(\rho + p + p_c) + \frac{3\beta^2}{2}\right]H = 0\,, \tag{6}$$

where the creation pressure is given as

$$p_c = -(1+\gamma)\frac{\rho}{N}\frac{dN}{dt}\frac{1}{3H}\,,$$

where N is the particle creation function.

Now considering a baratropic equation of state

$$p = \gamma\rho, -1 \leq \gamma \leq 1\,.$$

Eliminating $\rho(t)$ from equations (4) and (5), we obtain

$$2\dot{H} + 3(1+\gamma)H^2 + (1+3\gamma)\frac{k}{R^2} + (1-\gamma)\frac{3\beta^2}{4} = (1+\gamma)\frac{\dot{N}}{N}\,. \tag{7}$$

There are two independent equations in four unknowns viz. $R(t)$, $\rho(t)$, $\beta(t)$, and $N(t)$.

2.1 Exact solution of field equation

According to [11] deceleration parameter q is defined as

$$q = -\frac{\ddot{R}R}{\dot{R}^2}\,, \tag{8}$$

where R is the scale factor of the universe and is the function of cosmic time t. The Hubble parameter is defined as $H = \dot{R}/R$. We have considered the varying deceleration parameter in the form [11]

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = -1 - \frac{(\alpha R^{\alpha})}{1+R^{\alpha}}\,, \tag{9}$$

where α is constant. Now integrating the equation (9) yields

$$H = \frac{\dot{R}}{R} = A(1+R^{\alpha})\,, \tag{10}$$

where A is an arbitrary constant of integration. A is taken to be positive, which ensures the positivity of the Hubble parameter.

Again integrating equation (10) yields the scale factor as

$$R = \left[e^{-A\alpha t} - 1\right]^{-\frac{1}{\alpha}}\,. \tag{11}$$

We shall now discuss the behaviour of model by considering the following form for the rate of particle creation [10]:

$$\frac{1}{N}\frac{dN}{dt} = aH\,, \tag{12}$$

where $a \geq 0$ and from equation (7) and (12), we get

$$\beta^2 = -\frac{4}{3}\left[\frac{2\dot{H} + 3H^2(1+\gamma) + k/R^2(1+3\gamma) - (1+\gamma)aH}{(1-\gamma)}\right]. \tag{13}$$

Now we take $e^{-A\alpha t}$ as M and using (10), (11), (12) in (13), we get

$$\beta^2 = \frac{4}{3(\gamma-1)}\left[\frac{AM}{M-1}\Big(\frac{2\alpha(M-1)^{\frac{1}{\alpha}}}{M} + \frac{3(1+\gamma)AM}{M-1} - a(1+\gamma)\Big) + \frac{k(1+3\gamma)}{(M-1)^{\frac{-2}{\alpha}}}\right]. \tag{14}$$

Now using equation (13) in equation (4), we get

$$\chi\rho = \frac{2\dot{H} + 6H^2 + 4k/R^2 - (1+\gamma)aH}{(1-\gamma)}. \tag{15}$$

With the use of (10), (11), (12) in (15), we get

$$\chi\rho = \frac{1}{1-\gamma}\left[\frac{2\alpha A}{(M-1)^{\frac{\alpha-1}{\alpha}}} + \frac{6A^2M^2}{(M-1)^2} + 4k(M-1)^{\frac{2}{\alpha}} - a(1+\gamma)\frac{AM}{M-1}\right]. \tag{16}$$

It is well known that the different values of the parameters will give rise different graph, so the variations of some parameters are shown, by taking particular values of the integrating constants as $\alpha = 1$, $\gamma = 0.1$, $a = 1$, $A = 1$, $k = 1$ and $\alpha = -5$, $\gamma = -0.1$, $a = 1$, $A = 1$, $k = 1$ in the following Figures 1 and 2.

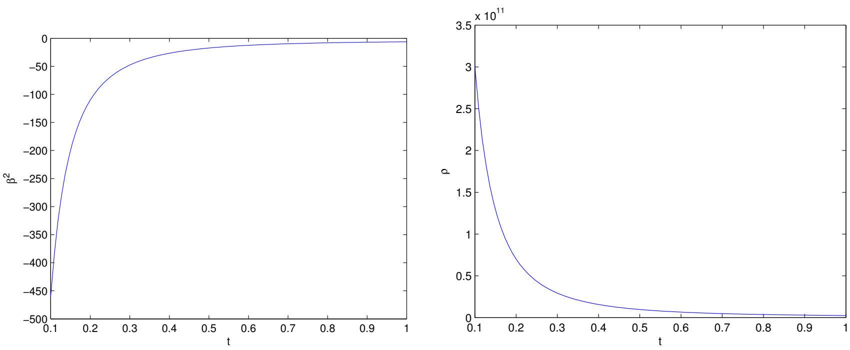

Figure 1. The variation of β^2 vs time t and energy density ρ vs time t, when $\alpha = 1$, $\gamma = 0.1$, $a = 1$, $A = 1$, $k = 1$

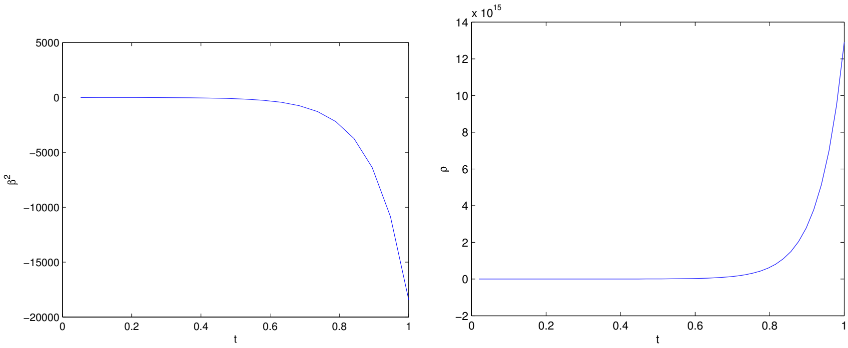

Figure 2. The variation of β^2 vs time t and energy density ρ vs time t, when $\alpha = -5$, $\gamma = -0.1$, $a = 1$, $A = 1$, $k = 1$

3 Conclusion

In this paper we have discussed cosmological models in Lyra's geometry for variable deceleration parameter using equation (9). The behaviour of the displacement field (β) and the energy density (ρ) have been inspected.

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References

[1] H. Weyl (1918) Reine Infinitesimalgeometrie. *Mathematische Zeitschrift* **2**(3) 384-411.

[2] G. Lyra (1951) Über-eine modifikation der Riemannschen Geometrie. *Mathematische Zeitchrift* **54** 52-64.

[3] D.K. Sen, K.A. Dunn (1971) A Scalar-Tensor theory of gravitation in a modified Riemannian manifold. *Journal of Mathematical Physics* **12**(4) 578-586.

[4] W.D. Halford (1970) Cosmological theory based on Lyra's geometry. *Australian Journal of Physics* **23** 863-869.

[5] W.D. Halford (1972) Scalar-Tensor theory of gravitation in Lyra Manifold. *Journal of Mathematical Physics* **13**(11) 1699-1703.

[6] A. Pradhan, J.P. Shahi, C.B. Singh (2006) Cosmological Models of Universe with Variable Deceleration Parameter in Lyra Manifold. *Brazilian Journal of Physics* **36**(4A) 1227-1231.

[7] M.S. Berman (1983) A Special Law of Variation of Hubble Parameter. *Il Nuovo Cimento B* **74**(2) 182-186.

[8] R.K. Knop, et al. (2003) New Constraints on Ω_M, Ω_Λ, and w from an Independent Set of 11 High-Redshift Supernovae Observed with the Hubble Space Telescope. *The Astrophysical Journal* **598**(1) 102-137.

[9] A.G. Riess, et al. (2004) Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraint on Dark Energy Evolution. *The Astrophysical Journal* **607**(2) 665-687.

[10] K. Desikan, S. Das (2017) A New Class of Cosmological Models in Lyra Geometry in the Presence of Particle Creation. *International Journal of Pure and Applied Physics* **13**(4) 303-309.

[11] N. Banerjee, S. Das (2005) Acceleration of the universe with a simple trigonometric potential. *General Relativity and Gravitation* **37**(10) 1695-1703.