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Ramanujan Sums-Wavelet Transform in the Context of ECG Signal Analysis

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Abstract. In the last fifteen years, Ramanujan Sums (RS) were widely used in signal processing applications like signal analysis, pattern recognition, feature extraction, image compression, etc. Ramanujan Sums Wavelet Transform (RSWT) is the combination of the Ramanujan Sums transform and the wavelet transform, introduced by Chen et al. [1] in 2013. In this transform, the multiresolution representation property of the wavelet transform and energy conservation property of the Ramanujan Sums transform are blended to extract more useful information from the signal under study. We exploit this feature of the RSWT to analyze ECG waveforms. The new representation of the ECG signal under the RS transform followed by the wavelet transform provides a multitude of ways of analyzing ECG signals, especially in the determination of QRS complexes and several intervals and amplitudes of the P, Q, R, S, and T waves. The advantage of the RSWT is that usual wavelet transform analysis reduces to a special case of the RSWT and hence the proposed method gives a generalized analysis tool that helps in a deeper understanding of the ECG features. Experimental results with the simulated ECG waveform show that the proposed method outperforms the existing ECG signal analysis using wavelet transform alone.

KEY WORDS: Ramanujan Sums, wavelet transform, ECG signal analysis.

1 Introduction

Srinivasa Ramanujan introduced a summation known today as the Ramanujan Sum in 1918 [2]. The q^{th} Ramanujan Sum is denoted by $c_q(n)$, and it is the sum of the n^{th} powers of the primitive q^{th} roots of unity. Ramanujan used this sum to derive many important infinite series expansions in number theory [3–5]. These sums are integer-valued, periodic, and orthogonal [6, 7]. This property of RS attracted the signal processing community to look into it and apply these sums in signal processing applications [8, 9]. A brief description of Ramanujan Sums is given in Section 2. In this paper, we study the application of RSWT in ECG signal analysis.

An electrocardiogram (ECG) measures and records the electrical activity of the heart. Analysis of ECG records is widely used for diagnosing many cardiac diseases. ECG signal is periodic with the fundamental frequency determined by the heartbeat. Significant features of ECG waveforms are the P, Q, R, S, and T waves, the duration of each wave, and certain time intervals such as the P-R, S-T, and Q-T intervals. Most of the clinically useful information in the ECG signal is found in the intervals and amplitudes defined by the characteristic wave peaks and boundaries [10]. The QRS complex is the most important characteristic waveform of the ECG signal. A variety of algorithms can be seen in the literature for QRS detection [11–14]. All these features are explicit in the detailed analysis of the signal. Hence, the development of accurate and robust methods for ECG signal analysis is an important area of research in signal processing.

The recorded ECG signal is often contaminated by noise and artifacts that can be within the frequency band of interest and appear with similar characteristics as the ECG signal itself. The sources of contamination may be due to power line interference, electrode pope or contact noise, patient-electrode motion artifacts, electromyographic noise, baseline wandering, etc. [15]. To extract useful diagnostic information from the noisy signal, we need to preprocess raw ECG. The proposed method conducts a preprocessing by transforming the signal to the RS domain and then applying wavelet transform of the RS transformed signal. In the RS transformation energy of the signal is preserved because of its superior energy compaction capability.

This paper deals with the multiresolution analysis of ECG signal using the Ramanujan Sums Wavelet Transform (RSWT) introduced by Chen et al. [1]. The new transform is the combination of the Ramanujan Sums transform and the wavelet transform. The multiresolution representation of the wavelet transform and the energy conservation property of the Ramanujan Sums transform are blended to extract more useful information from the ECG signal under study. First of all, the signal is transformed into the Ramanujan Sum domain by multiplying the ECG signal with the RS $c_q(n)$ for $q \in [1, Q]$. This transformed signal is analyzed at different levels using the wavelet transform with the basis Daubechies wavelet $db6$ and for $q \in [1, Q]$. This allows multiresolution analysis of the ECG signal for a given q. Apart from the usual wavelet transform analysis we have a variety of signal analysis results that can give more information about the hidden features in the ECG signal.

The rest of the paper is organized as follows. An overview of RS is given in Section 2. Wavelet transform and RSWT are explained in Sections 3 and 4. A short review of ECG signal analysis theory is given in Section 5. Results and discussion are given in Section6. Conclusions are included in Section 7.

2 Ramanujan Sums

The Ramanujan Sum $c_q(n)$ has been used by mathematicians to derive many important infinite series expansions for arithmetical functions in number theory [5]. Interestingly, this sum has many properties which are attractive from the point of view of digital signal processing. Srinivasa Ramanujan defined the q^{th} Ramanujan Sum by

$$c_q(n) = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} W_q^{kn} = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} W_q^{-kn}\,, \tag{1}$$

where $W_q = e^{-i2\pi/q}$, $i = \sqrt{-1}$ and (k,q) denotes the gcd of k and q. Here the sum runs over those k satisfying $(k,q) = 1$ means that we are considering all the integers which are co-prime to q in the summation.

For example, if $q = 8$ then $k \in \{1, 3, 5, 7\}$ so that

$$c_8(n) = e^{i2n\pi/8} + e^{i6n\pi/8} + e^{i10n\pi/8} + e^{i14n\pi/8}\,.$$

In number theory, the number of integers less than or equal to q and co-prime to q is called the Euler's totient function $\phi(q)$ [5]. Since $1, 3, 5, 7$ are co-prime to 8, $\phi(8) = 4$.

So the sum given in Eq. (1) has precisely $\phi(q)$ terms and it is clear that $c_q(0) = \phi(q)$. Also

$$c_q(n+q) = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} e^{i2n\pi k/q}.e^{i2\pi k} = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} e^{i2n\pi k/q} = c_q(n)\,.$$

That is $c_q(n)$ is periodic with period q.

If $(k,q) = 1$, we have $(q-k,q) = 1$. Therefore,

$$(W_q^k)^* = W_q^{-k} = W_q^{-(q-k)} = W_q^k\,,$$

where $*$ is the complex conjugate. This implies that the summation (1) is real-valued and it can also be written as:

$$c_q(n+q) = \sum_{\substack{k=1 \\ (k,q)=1}}^{q} \cos\frac{2n\pi k}{q}\,. \tag{2}$$

From Eq. (2), $c_q(n) = c_q(-n)$ shows that $c_q(n)$ is symmetric. Thus $c_q(n)$ is a real, symmetric, and periodic sequence in n.

For $0 \leq n \leq q-1$, first few Ramanujan sequences are

$$\begin{aligned}
c_1(n) &= 1 \\
c_2(n) &= 1, -1 \\
c_3(n) &= 2, -1, -1 \\
c_4(n) &= 2, 0, -2, 0 \\
c_5(n) &= 4, -1, -1, -1, -1
\end{aligned}$$

Any two Ramanujan Sums $c_{q_1}(n)$ and $c_{q_2}(n)$ are orthogonal in the sense that

$$\sum_{n=0}^{m-1} c_{q_1}(n) c_{q_2}(n) = 0, \quad q_1 \neq q_2 \, .$$

where $m = lcm(q_1, q_2)$. Note that $c_q(n)$ is integer-valued and more properties of $c_q(n)$ can be seen in [16].

3 Wavelet Transform

In wavelet analysis, the analyzing function is split into components which are small waves of short duration and finite energy called wavelets. Mathematically, 1D wavelet is defined as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right),$$

where $a > 0$ is the scaling (dilation) parameter and $b \epsilon \mathbb{R}$ is the location parameter and $\psi(t)$ is a mother wavelet. A mother wavelet is a small wave (oscillation) whose translation and dilation are used for signal analysis. We can translate the wavelet by changing b for a given scale a. In general, a mother wavelet is not an arbitrary function. It must satisfy certain mathematical criteria such as compact support, finite energy, and zero mean property [17–19]. The most commonly used mother wavelets in the biological signal analysis are the Daubechies mother wavelets.

Wavelet transform is an integral transform that converts the signal $x(t)$ under consideration into another representation in a more useful form. It is an inner product of the signal $x(t)$ and the mother wavelet $\psi(t)$. For a given scaling parameter a and translation parameter b wavelet transform is generally defined as

$$W(a,b) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|a|}} \psi^*\left(\frac{t-b}{a}\right) dt = \int_{-\infty}^{\infty} x(t) \psi^*_{(a,b)}(t) dt \, , \quad (3)$$

where $*$ is the complex conjugate. For every pair (a, b), the transform coefficient $W(a, b)$ measures how much the scaled wavelet is similar to the function at location $t = \frac{b}{a}$.

In signal processing terminology, wavelet transform is the convolution of a signal with a scaled wavelet. For each scale, we obtain an array of the same length N as the 1D signal has. A wavelet transform basically transforms a signal from the time domain to the joint time-scale domain and hence the wavelet transform coefficients are 2D. Continuous and discrete wavelet transform are the two types in use depending on whether we are considering a continuous signal or a discrete version of it.

3.1 Continuous wavelet transform

Continuous wavelet transform (CWT) is a wavelet transform in which both input and output are continuous. CWT is a function of two parameters defined in the same way as the wavelet transform in Eq. (3). CWT with discrete coefficients can be obtained by choosing $a = 2^{-m}$ and $b = n2^{-m}$ in equation (3), where m and n are integers.

$$W\left(a = 2^{-m}, b = n2^{-m}\right) = \frac{1}{\sqrt{2^{-m}}} \int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t - n2^{-m}}{2^{-m}}\right) dt\,,$$

$$i.e.,\ W(n,m) = 2^{\frac{m}{2}} \int_{-\infty}^{\infty} x(t)\psi^*(2^m t - n) dt\,.$$

In CWT with discrete coefficients, we find wavelet coefficients for every (m, n) pair of integers.

3.2 Discrete wavelet transform

Th discrete wavelet transform (DWT) is any wavelet transform for which the wavelets are discretely sampled. The DWT comes from the CWT with discrete coefficients, but the mathematical part is simplified largely. In DWT, instead of continuously varying parameters, we analyze the signal with a small number of scales with a varying number of translations at each scale. DWT can be implemented by passing the signal through an analysis filter bank consisting of a low pass filter and a high pass filter followed by a decimation operation.

4 Ramanujan Sums-Wavelet Transform

Using the q^{th} Ramanujan Sum $c_q(n)$, Ramanujan showed that several arithmetical functions in number theory can be expressed in the form

$$x(n) = \sum_{q=1}^{\infty} \alpha_q c_q(n), \quad n \geq 1\,. \tag{4}$$

This is called the Ramanujan Sum expansion of $x(n)$, where the coefficient [20, 21]

$$\alpha_q = \frac{1}{\phi(q)} \left(\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} x(n) c_q(n) \right). \tag{5}$$

The coefficient α_q in Eq. (5) is called the Ramanujan Sum transform or Ramanujan-Fourier Transform (RFT) [20–22]. We use the expansion formulae based on Eq. (4) and Eq. (5) to represent finite duration $1D$ signals. The $1D$ RS transform of a signal $x(t)$ with N samples is defined as

$$\alpha_q = \frac{1}{\phi(q)} \frac{1}{N} \sum_{n=1}^{N} x(n) c_q(n) \,.$$

In this paper, to analyze ECG signals a new transform obtained by combining the wavelet transform with the RS transform is used. The new transform is called the Ramanujan Sums-Wavelet Transform (RSWT) and it is defined by Chen et al. [1] as:

$$RSW(n, a, q) = \frac{1}{\sqrt{|a|}} \sum_{t=1}^{N} x(t) c_q(t) \psi^* \left(\frac{t-n}{a} \right) \tag{6}$$

$$= \frac{1}{\sqrt{|a|}} \sum_{t=1}^{N} f(t, q) \psi^* \left(\frac{t-n}{a} \right) \tag{7}$$

where $x(t)$ is the input 1D signal, $f(t, q) = x(t)c_q(t)$, and $\psi(.)$ is the wavelet basis. From Eq. (7), it can be seen that RSWT is actually the DWT of $f(t, q)$. When $q = 1, c_1(n) = 1$ for all n, and hence RSWT, in this case, reduces to the DWT of $x(t)$. That is, the wavelet transform analysis can be performed as a special case of RSWT analysis.

5 ECG Signal–Theory

Electrocardiography (ECG) is a diagnostic method, which enables the recording of the heart muscle electrical activity. The ECG is measured using the surface electrodes attached to the limbs or chest. An ECG signal trace consists of an electrical signal within the frequency range of $0.05-250$ Hz with a small voltage range of $0.01 - 5$ mV [23]. A typical ECG tracing of a normal cardiac cycle consists of a P wave, a QRS complex, and a T wave as shown in Figure 1. Significant features obtained from ECG analysis are the duration of P, Q, R, S, and T waves and certain time intervals such as P-R, S-T, Q-T, and R-R intervals [10, 23]. The normal heartbeat per minute is 60–100 bpm. For the heartbeat rate of 72 bpm, different amplitudes and intervals of ECG features are given in Table 1.

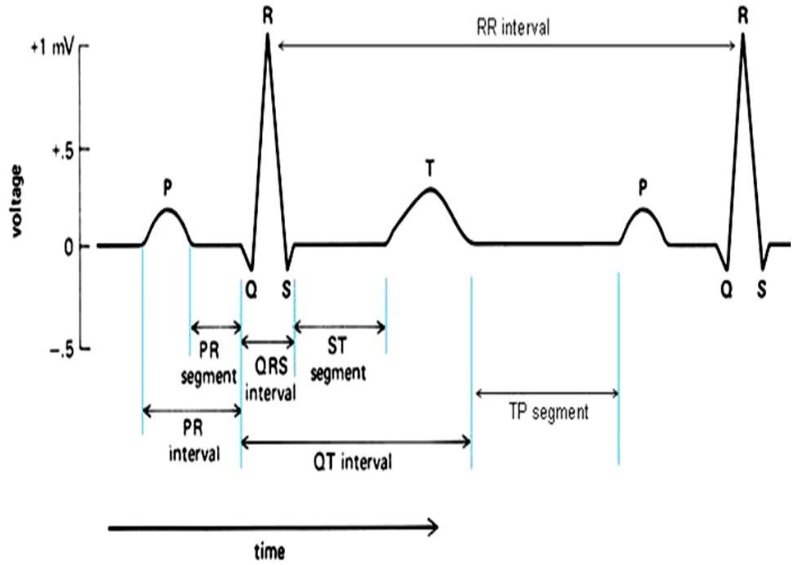

Figure 1. A typical one cycle ECG signal with intervals and segments.

Table 1. Normal ECG features for the heart beat 72 bpm

Wave	Amplitude	Interval	Duration
P	0.25 mV	P-R	0.16 s
Q	0.025 mV	S-T	0.18 s
R	1.6 mV	P	0.09 s
T	0.35 mV	QRS	0.11 s

6 Results and Discussion

A simulated sinus rhythm is considered for analysis. Ramanujan Sum transform is applied to the signal by multiplying it with the periodic sequence $c_q(n)$ for $q \in [1, Q]$ over the length of the input signal. After multiplying the signal $x(t)$ with the periodic sequence $c_q(n)$, wavelet transform analysis is done at different scales with the wavelet basis function $db6$. Here we have chosen the Daubechies6 ($db6$) mother wavelet because it is similar to the real ECG signal. The RSWT coefficients for each q, at different scales and location, gives the details and approximation of the input signal. Ordinary wavelet analysis results for the input ECG signal is given in Figure 2, where $d1, d2, d3, d4,$ and $d5$ are the details and $a5$ is the approximation at level five. RSWT analysis results with $q \in [1, 6]$ at five levels are shown in Figure 3. The RSWT analysis with $q = 1$ corresponds to ordinary wavelet transform analysis. Further investigation of the

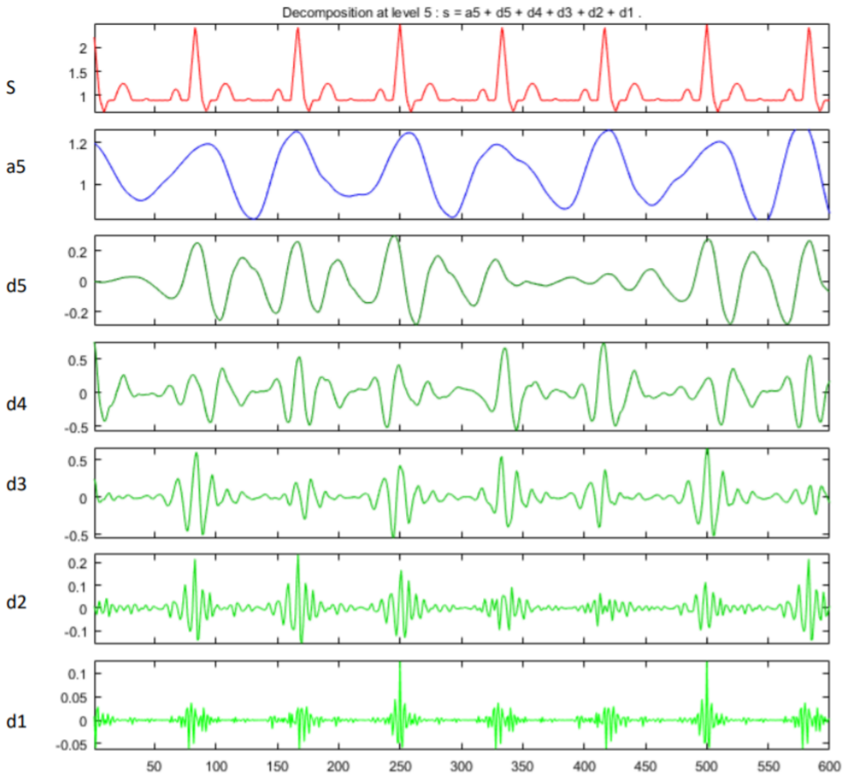

Figure 2. Original signal S, details d1–d5 and the approximation a5 under the wavelet transform.

diagnostic information is possible with the RSWT analysis for $q > 1$. The various abnormalities associated with heart diseases that are missed in the ordinary wavelet analysis are more visible in further investigation done with RSWT by changing the values of $q \in [1, Q]$. Hence the proposed method is robust and efficient in its performance as compared to the analysis done by applying wavelet transform alone.

7 Conclusions

This paper introduces Ramanujan Sums-wavelet transform in the ECG signal analysis with the Ramanujan Sums $c_q(n),\ q \in [1, Q]$, and the mother wavelet Daubechies6 ($db6$). Among all analysis results at a given scale, the result corresponding to $q = 1$ is the ordinary wavelet transform and the remaining analysis results provide chances of more accurate analysis in the determination of abnormalities associated with various heart diseases. Further clinical studies are

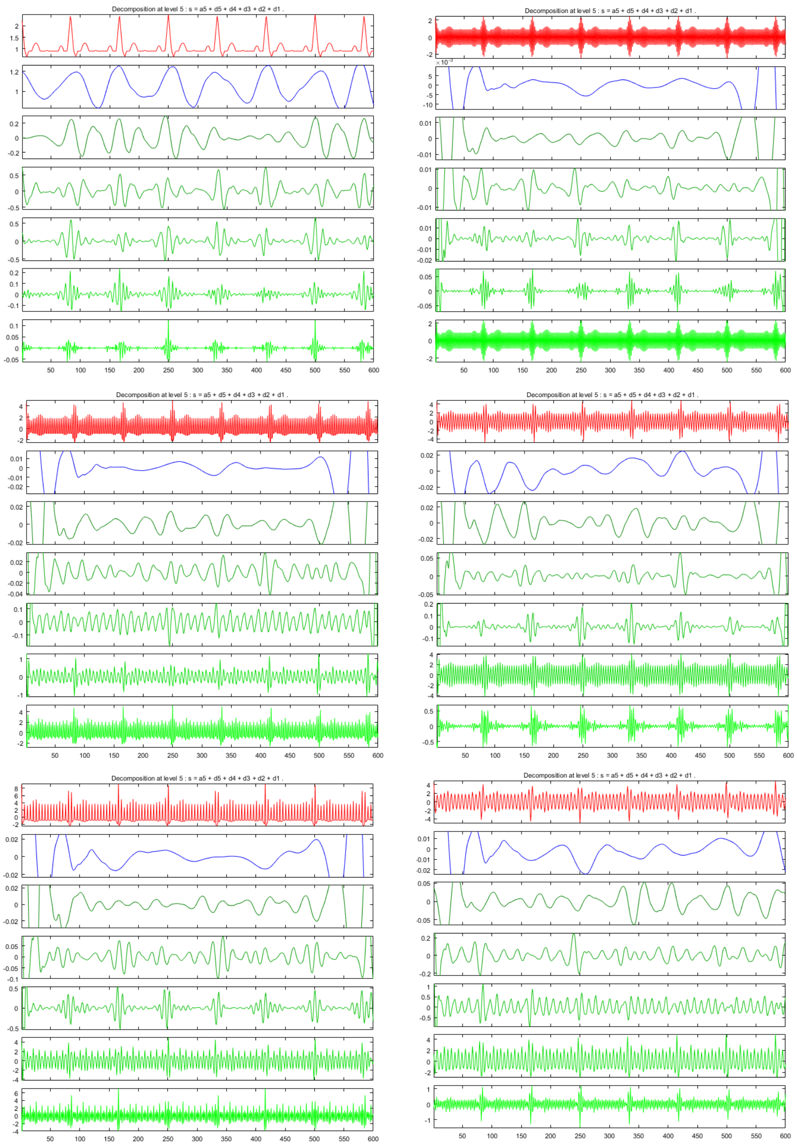

Figure 3. **Top left corner to the right:** ECG signal analysis results with the RSWT with different $c_q(n)$ for $q = 1, 2, 3, 4, 5, 6$. For $q = 1$ it is the usual wavelet transform.

needed to understand the power of RSWT analysis in identifying hidden details in an ECG report. Thus RSWT ECG signal analysis outperforms ordinary wavelet transform analysis in its ability to disclose more hidden information in the input ECG signal.

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