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Dynamics of Generalized Ghost Pilgrim Dark Energy in General Relativity

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Abstract. This work is based on Generalized Ghost Pilgrim Dark Energy (GGPDE) in General Relativity (GR) in the framework of Bianchi type VI_0 space-time. To obtain exact solutions of the field equations, we have used simple parametrization of average scale factor $a(t) = \exp\{(\alpha t + \beta)^p\}$ as proposed by Mishra and Dua (Astrophys. Space Sci. (2021) 366:6). We have investigated the non-interacting and interacting GGPDE and dark matter (DM). Some well-known parameters like Hubble parameter, equation of state (EOS) parameter, deceleration parameter, etc are constructed for both the models. It is found that deceleration parameter indicates accelerated phase while EOS parameter represents a cosmological constant for both the models. The stability analysis and energy conditions for both the non-interacting and interacting models are investigated.

KEY WORDS: GGPDE, GR, Hubble parameter, EOS parameter, deceleration parameter.

1 Introduction

Recent astronomical observations [1–5] shows that we are living in an expanding and accelerating Universe. These observations suggests that the Universe is dominated by two dark components containing dark matter (DM) and dark energy. Dark matter, a pressure-less matter, is mainly used to explain galactic curves and structure formation of the Universe while DE, an exotic energy with large negative pressure is used to explain the cosmic acceleration of the Universe. Researchers are actively engaged in finding the nature of the dark energy and people have proposed various candidates to describe it. The simplest and most obvious candidate for DE is the cosmological constant Λ with the EOS parameter $\omega = p/\rho = -1$ where p is the pressure and ρ is the energy density of DE. But it suffers from the problems of fine-tuning and cosmic coincidence [6, 7]. To rectify the problem of DE, various DE models such as

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quintessence [8], phantom [9, 10], tachyon [11], dilaton [12], etc. models have been proposed by cosmologists from time to time.

To explain the present accelerated stage of the Universe, a form of DE named as Veneziano ghost DE has been proposed [13–16]. The energy density of the vacuum ghost field is proportional to $\Lambda^3_{\text{QCD}} H$ [17–19], where Λ_{QCD} is the QCD mass scale and H is the Hubble parameter. This GDE attracts the attention of researchers as its energy density ρ_{DE} depends linearly on the Hubble parameter H such as $\rho_{\text{DE}} = \tau H$ where τ is a constant with dimension [energy]^3 and is related to the QCD (Quantum Chromodynamics) mass scale. QCD describes the strong interaction in nature. The general vacuum energy of the Veneziano ghost field in QCD is of the form $H + O\left(H^2\right)$ [20]. The term H^2 occupies a significant position in the evolution of the early Universe, which acts as the early DE [21]. By taking the term H^2 into account [22], one can give better agreement with observational data compared to the usual GDE which is known as generalized ghost dark energy (GGDE). The energy density of generalized model is defined by $\rho_{\text{DE}} = \tau H + \eta H^2$ where η is a constant. Wei [23] proposed a new dark energy model called pilgrim DE (PDE) based on the speculation that black hole (BH) formation can be avoided through the strong repulsive force of the type of DE. GGDE has been modified in terms of PDE as $\rho_{\text{DE}} = \left(\tau H + \eta H^2\right)^u$ where u is a PDE parameter [24].

Recently, many authors have studied various dark energy cosmological models in different theories of gravities. Santhi et al. [25] have discussed GGPDE in Bianchi type I Universe. Jawad [26] have explored GGPDE in the framework of non-flat FRW Universe. Garg et al. [27] have investigated GGPDE in the gravitation theory of Saez-Ballester. Mishra and Dua [28] have worked out FLRW universe in Brans-Dicke gravity theory with a simple parametrization of average scale factor $a\left(t\right)$.

Bianchi type models are among the simplest models with anisotropic background to describe the early stages of the evolution of the Universe. The simplicity of the field equations and relative ease of solutions makes Bianchi space-times useful in constructing models of spatially homogeneous and anisotropic cosmologies.

Motivated by the above discussed recent works of various authors, we propose a study GGPDE in GR in the framework of Bianchi type VI_0 space-time. The plan of the manuscript is as follows: Metric and field equations are discussed in Section 2. In Section 3, we have obtained the solutions of field equations. The non-interacting and interacting models are discussed in Sections 4 and 5, respectively. The stability analysis and energy conditions are described in Section 6 and Section 7, respectively. Various parameters are discussed graphically in Section 8. The paper ends with concluding remarks in Section 9.

2 Metric and Field Equations

Bianchi type VI_0 space-time is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{-2x} dz^2 \,, \tag{1}$$

where the scale factors A, B and C are functions of cosmic time t only.

Einstein field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -\left(\bar{T}_{ij} + \acute{T}_{ij}\right) \,, \tag{2}$$

where R_{ij} is the Ricci tensor and R is the Ricci scalar.

The energy momentum tensor $\bar{T}^i_j$ for dark matter (DM) is

$$\bar{T}^i_j = \text{diag}\left[\rho_m, 0, 0, 0\right] \,, \tag{3}$$

where ρ_m is the energy density of DM.

The energy momentum tensor $\acute{T}^i_j$ for GGPDE is

$$\begin{aligned} \acute{T}^i_j &= \text{diag}\left[\rho_{\text{DE}}, -p_{\text{DE}_x}, -p_{\text{DE}_y}, -p_{\text{DE}_z}\right] \\ &= \text{diag}\left[1, -\omega_x, -\omega_y, -\omega_z\right] \rho_{\text{DE}} \\ &= \text{diag}\left[\rho_{\text{DE}}, -\omega_{\text{DE}}\rho_{\text{DE}}, -\omega_{\text{DE}}\rho_{\text{DE}}, -\omega_{\text{DE}}\rho_{\text{DE}}\right] \,, \end{aligned} \tag{4}$$

where ρ_{DE} is the energy density of GGPDE, p_{DE} is the pressure of GGPDE and $\omega_x = \omega_y = \omega_z = \omega_{\text{DE}}$ are the directional EOS parameters on x, y and z axes respectively and $\omega_{\text{DE}}\rho_{\text{DE}} = p_{\text{DE}}$.

The Einstein's field equations (2) for the metric (1) using Eqs. (3) and (4) takes the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\omega_{\text{DE}}\rho_{\text{DE}} \,, \tag{5}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = -\omega_{\text{DE}}\rho_{\text{DE}} \,, \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\omega_{\text{DE}}\rho_{\text{DE}} \,, \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \rho_m + \rho_{\text{DE}} \,, \tag{8}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \,. \tag{9}$$

Integrating Eq. (9) and assuming integrating constant to be unity, we get

$$B = C \,. \tag{10}$$

Using Eq. (10) in Eqs. (5)-(8), we get

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{A^2}=-\omega_{\rm DE}\rho_{\rm DE}\,, \tag{11}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=-\omega_{\rm DE}\rho_{\rm DE}\,, \tag{12}$$

$$\frac{\dot{B}^2}{B^2}+2\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=\rho_m+\rho_{\rm DE}\,. \tag{13}$$

The energy conservation equation is

$$\dot{\rho}_m+\dot{\rho}_{\rm DE}+3H\left(\rho_m+\rho_{\rm DE}+p_{\rm DE}\right)=0\,, \tag{14}$$

where overhead dot $(\dot{})$ denotes differentiation w.r. to cosmic time t.

Throughout the study, we have considered both the non-interacting and interacting DM and GGPDE.

The continuity equations for DM and GGPDE, where the DM component is interacting with GGPDE component through an interaction term Q are termed as

$$\dot{\rho}_m+3H\rho_m=Q\,, \tag{15}$$

$$\dot{\rho}_{\rm DE}+3H\left(\rho_{\rm DE}+p_{\rm DE}\right)=-Q\,, \tag{16}$$

where $Q>0$ indicates that energy flows from GGPDE to DM, $Q<0$ means that energy flows in opposite direction and $Q=0$ represents the non-interacting scenario. In general, Q is inversely proportional to time. Wei and Cai [29] proposed

$$Q=3bH\rho_m\,, \tag{17}$$

where $b>0$ is a coupling constant.

3 Solutions of Field Equations

The spatial volume V is given by

$$V=a^3=AB^2\,, \tag{18}$$

where a is the average scale factor.

The Hubble's parameter H is calculated as

$$H=\frac{\dot{a}}{a}=\frac{\dot{V}}{3V}=\frac{1}{3}\left(H_x+H_y+H_z\right)=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)\,, \tag{19}$$

where $H_x = \dot{A}/A$ and $H_y = H_z = \dot{B}/B$ are the directional Hubble parameters in the directions of x, y and z axes, respectively.

The deceleration parameter q is defined as

$$q = -a\ddot{a}/\dot{a}^2 \,. \tag{20}$$

The Anisotropy parameter A_p is defined as

$$A_p = \frac{1}{3H^2} \sum_{i=1}^{3} \left(\frac{H_i - H}{H} \right)^2 \,. \tag{21}$$

Equations (11)–(13) are a system of three field equations with five unknowns $A, B, \rho_m, \rho_{\text{DE}}$ and ω_{DE}. So, we are in search of two extra relations. These are as follows:

(i) The energy density of GGPDE is given by

$$\rho_{\text{DE}} = \left(\tau H + \eta H^2\right)^u \,, \tag{22}$$

where u is a PDE parameter.

(ii) Mishra and Dua [28] proposed a simple parametrization of average scale factor as

$$a\left(t\right) = \exp\left\{(\alpha t + \beta)^p\right\} \,, \tag{23}$$

where $\alpha, \beta > 0$ and $0 < p < 1$ are arbitrary constants.

From Eqs. (11) and (12), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{p_1}{V} \exp\left(\int \frac{\frac{-2}{A^2}}{\frac{\dot{B}}{B} - \frac{\dot{A}}{A}} dt \right) \,. \tag{24}$$

Following Adhav [30], we assume

$$\frac{\dot{B}}{B} - \frac{\dot{A}}{A} = \frac{2}{A^2} \,. \tag{25}$$

Using Eq. (25) in Eq. (24), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{p_1}{V} e^{-t} \,. \tag{26}$$

Integrating Eq. (26), we obtain

$$A = p_2 B \exp\left\{ p_1 \int \frac{e^{-t}}{\left[\exp\left\{(\alpha t + \beta)^p\right\}\right]^3} dt \right\} \,, \tag{27}$$

where p_2 is a constant of integration.

Equations (18), (23) and (27) together implies

$$A = \exp\left\{(\alpha t+\beta)^p\right\} p_2^{2/3} \exp\left\{\frac{2p_1}{3}\int \frac{e^{-t}}{\left[\exp\left\{(\alpha t+\beta)^p\right\}\right]^3} dt\right\} , \tag{28}$$

$$B = \exp\left\{(\alpha t+\beta)^p\right\} p_2^{-1/3} \exp\left\{\frac{-p_1}{3}\int \frac{e^{-t}}{\left[\exp\left\{(\alpha t+\beta)^p\right\}\right]^3} dt\right\} . \tag{29}$$

From Eqs. (28) and (29), we can arrive at a conclusion that the cosmic scale factors are increasing functions of t.

The Hubble parameter H is obtained as

$$H = \alpha p \left(\alpha t+\beta\right)^{p-1} . \tag{30}$$

H diminishes and ultimately approaches to a small value as the age of the Universe increases.

The deceleration parameter q is obtained as

$$q = -1 - \left(\frac{p-1}{p}\right)\left(\alpha t+\beta\right)^{-p} , \tag{31}$$

$q \to -1$ as $t \to \infty$. Moreover, $q > 0$ for $t < \left[\left(\frac{1}{p}-1\right)^{\frac{1}{p}} - \beta\right]/\alpha$ and $q < 0$ for $t > \left[\left(\frac{1}{p}-1\right)^{\frac{1}{p}} - \beta\right]/\alpha$. So, our model shows transition of early cosmic deceleration to present cosmic acceleration.

The GGPDE density ρ_{DE} is calculated as

$$\rho_{\text{DE}} = \left[\tau\alpha p\left(\alpha t+\beta\right)^{p-1} + \eta\alpha^2 p^2 \left(\alpha t+\beta\right)^{2p-2}\right]^u , \tag{32}$$

ρ_{DE} approaches to a small value as observed from Figure 2.

The Anisotropy parameter A_p is calculated as

$$A_p = \frac{2p_1^2 e^{-2t}}{9\alpha^2 p^2 \left(\alpha t+\beta\right)^{2p-2}\left[\exp\left\{(\alpha t+\beta)^p\right\}\right]^6} . \tag{33}$$

From Eq. (33), we can conclude that $A_p \to 0$ as $t \to \infty$. Thus, the obtained model of the Universe attains isotropy at late times.

4 Non-Interacting Model

The energy conservation equation for DM is

$$\dot{\rho}_m + 3H\rho_m = 0 . \tag{34}$$

Using Eq. (30) in Eq. (34), we get

$$\rho_m = \frac{\rho_0}{\exp\left[3\left(\alpha t+\beta\right)^p\right]}\,, \tag{35}$$

where ρ_0 is a constant of integration.

Figure 5 indicates that $\rho_m \to \infty$ as $t \to 0$ and $\rho_m \to 0$ as $t \to \infty$.

The energy conservation equation for GGPDE is

$$\dot{\rho}_{\rm DE} + 3H\left(\rho_{\rm DE}+p_{\rm DE}\right) = 0, p_{\rm DE} = \omega_{\rm DE}\rho_{\rm DE}\,. \tag{36}$$

Using Eqs. (30) and (32) in Eq. (36), we obtain

$$\omega_{\rm DE} = -1 + \frac{u\left(1-p\right)\left[\tau\left(\alpha t+\beta\right)^{p-2} + 2\eta\alpha p\left(\alpha t+\beta\right)^{2p-3}\right]}{3\left(\alpha t+\beta\right)^{p-1}\left[\tau p\left(\alpha t+\beta\right)^{p-1} + \eta\alpha p^2\left(\alpha t+\beta\right)^{2p-2}\right]}\,, \tag{37}$$

$\omega_{\rm DE} \to -1$ as the age of the Universe increases. Thus, our non-interacting model behaves like a cosmological constant model.

$$\begin{aligned} p_{\rm DE} &= \omega_{\rm DE}\rho_{\rm DE} \\ &= \left[\tau\alpha p\left(\alpha t+\beta\right)^{p-1} + \eta\alpha^2 p^2\left(\alpha t+\beta\right)^{2p-2}\right]^u \\ &\times\left\{-1 + \frac{u(1-p)[\tau(\alpha t+\beta)^{p-2} + 2\eta\alpha p(\alpha t+\beta)^{2p-3}]}{3(\alpha t+\beta)^{p-1}[\tau p(\alpha t+\beta)^{p-1} + \eta\alpha p^2(\alpha t+\beta)^{2p-2}]}\right\}. \end{aligned} \tag{38}$$

5 Interacting Model

The energy conservation equation for DM is

$$\dot{\rho}_m + 3H\rho_m = 3bH\rho_m\,. \tag{39}$$

Using Eq. (30) in Eq. (39), we get

$$\rho_m = \frac{\acute{\rho}_0}{\exp\left\{3\left(1-b\right)\left(\alpha t+\beta\right)^p\right\}}\,, \tag{40}$$

where $\acute{\rho}_0$ is an integrating constant.

Figure 9 indicates that $\rho_m \to \infty$ as $t \to 0$ and $\rho_m \to 0$ as $t \to \infty$.

The energy conservation equation for GGPDE is

$$\dot{\rho}_{\rm DE} + 3H\left(\rho_{\rm DE}+p_{\rm DE}\right) = -3bH\rho_m\,. \tag{41}$$

Using Eqs. (30) and (32) in Eq. (41), we get

$$\omega_{\rm DE} = -1 - \frac{b\rho_m}{\rho_{\rm DE}} + \frac{u(1-p)[\tau(\alpha t+\beta)^{p-2}+2\eta\alpha p(\alpha t+\beta)^{2p-3}]}{3(\alpha t+\beta)^{p-1}[\tau p(\alpha t+\beta)^{p-1}+\eta\alpha p^2(\alpha t+\beta)^{2p-2}]}\,, \tag{42}$$

where ρ_m is given by Eq. (40).

$\omega_{\mathrm{DE}} \to -1$ as time evolves. Thus, our interacting model behaves like a cosmological constant model.

$$p_{\mathrm{DE}} = \omega_{\mathrm{DE}}\rho_{\mathrm{DE}} = [\tau\alpha p(\alpha t+\beta)^{p-1} + \eta\alpha^2 p^2(\alpha t+\beta)^{2p-2}]^u \times\Big\{-1 - \frac{b\rho_m}{\rho_{\mathrm{DE}}} + \frac{u(1-p)[\tau(\alpha t+\beta)^{p-2}+2\eta\alpha p(\alpha t+\beta)^{2p-3}]}{3(\alpha t+\beta)^{p-1}[\tau p(\alpha t+\beta)^{p-1}+\eta\alpha p^2(\alpha t+\beta)^{2p-2}]}\Big\}. \quad (43)$$

6 Stability Analysis

We have examined the stability of both non-interacting and interacting models in this section. The square speed of sound is defined as $v_s^2 = \dot{p}_{\mathrm{DE}}/\dot{\rho}_{\mathrm{DE}}$. The sign of v_s^2 occupies a significant position for stability analysis of a background evolution of cosmic models. The positive value of v_s^2 indicates that the model is stable whereas the negative value indicates that the model is classically unstable [31]. Also, the casualty condition must be satisfied. It means that the sound speed is less than the speed of light.

The square speed of sound v_s^2 for non-interacting model is obtained as

$$\begin{aligned} v_s^2\dot{\rho}_{\mathrm{DE}} = &-\dot{\rho}_{\mathrm{DE}} + \frac{u(1-p)\alpha^u}{3}\Big[\frac{-\tau\alpha}{(\alpha t+\beta)^2} + 2\eta\alpha^2 p(p-2)(\alpha t+\beta)^{p-3}\Big] \\ &\times[\tau p(\alpha t+\beta)^{p-1} + \eta\alpha p^2(\alpha t+\beta)^{2p-2}]^{u-1} \\ &+ \frac{u(u-1)(1-p)\alpha^u}{3}\Big[\frac{\tau}{(\alpha t+\beta)} + 2\eta\alpha p(\alpha t+\beta)^{p-2}\Big] \\ &\times[\tau p(\alpha t+\beta)^{p-1} + \eta\alpha p^2(\alpha t+\beta)^{2p-2}]^{u-2} \\ &\times[\tau\alpha p(p-1)(\alpha t+\beta)^{p-2} + 2\eta\alpha^2 p^2(p-1)(\alpha t+\beta)^{2p-3}]. \end{aligned} \quad (44)$$

The square speed of sound v_s^2 for interacting model is obtained as

$$\begin{aligned} v_s^2\dot{\rho}_{\mathrm{DE}} = &-\dot{\rho}_{\mathrm{DE}} + 3b\acute{\rho}_0\alpha p(1-b)(\alpha t+\beta)^{p-1}\exp\{-3(1-b)(\alpha t+\beta)^p\} \\ &+ \frac{u(1-p)\alpha^u}{3}\Big[\frac{-\tau\alpha}{(\alpha t+\beta)^2} + 2\eta\alpha^2 p(p-2)(\alpha t+\beta)^{p-3}\Big] \\ &\times[\tau p(\alpha t+\beta)^{p-1} + \eta\alpha p^2(\alpha t+\beta)^{2p-2}]^{u-1} \\ &+ \frac{u(u-1)(1-p)\alpha^u}{3}\Big[\frac{\tau}{(\alpha t+\beta)} + 2\eta\alpha p(\alpha t+\beta)^{p-2}\Big] \\ &\times[\tau p(\alpha t+\beta)^{p-1} + \eta\alpha p^2(\alpha t+\beta)^{2p-2}]^{u-2} \\ &\times[\tau\alpha p(p-1)(\alpha t+\beta)^{p-2} + 2\eta\alpha^2 p^2(p-1)(\alpha t+\beta)^{2p-3}]. \end{aligned} \quad (45)$$

v_s^2 is negative throughout the evolution of the Universe as seen from Figures 7 and 11. Thus, both non-interacting and interacting models are unstable.

7 Energy Conditions

The Energy conditions, i.e. Weak Energy Conditions (WEC), Dominant Energy Conditions (DEC) and Strong Energy Conditions (SEC) are respectively given by

(I) $\rho_{\rm DE} \geq 0$; (II) $\rho_{\rm DE} + p_{\rm DE} \geq 0$; (III) $\rho_{\rm DE} + 3p_{\rm DE} \geq 0$.

The left-hand sides of (I), (II) and (III) for both non-interacting and interacting models based on Eqs. (32), (38) and (43) have been plotted in Figures 8 and 12 and found that (I) $\rho_{\rm DE} \geq 0$, (II) $\rho_{\rm DE} + p_{\rm DE} \geq 0$ and (III) $\rho_{\rm DE} + 3p_{\rm DE} \leq 0$. So, WEC (red line) and DEC (blue line) are satisfied whereas SEC (green line) is violated. The violation of SEC gives anti-gravitational effect for which Universe gets jerk and thus our model exhibits transition from the early deceleration to present cosmic acceleration. So, our model is in good agreement with recent cosmological observations.

8 Graphical Discussions

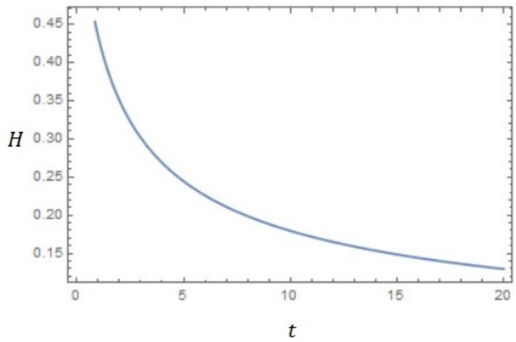

Figure 1. The time-dependent evolution of H for $\alpha = 1.4$, $p = 0.5$ and $\beta = 0.2$. H decreases and ultimately approaches to small value at late times.

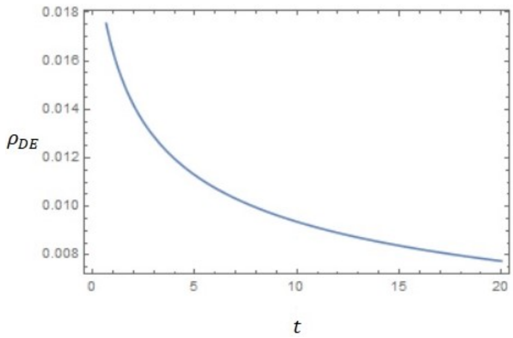

Figure 2. The time-dependent variation of $\rho_{\rm DE}$ for $\tau = 0.0004$, $\eta = 0.0005$, $\alpha = 1.4$, $p = 0.5$, $u = 0.5$ and $\beta = 0.2$. $\rho_{\rm DE}$ diminishes and tends to small value at the later age of the Universe.

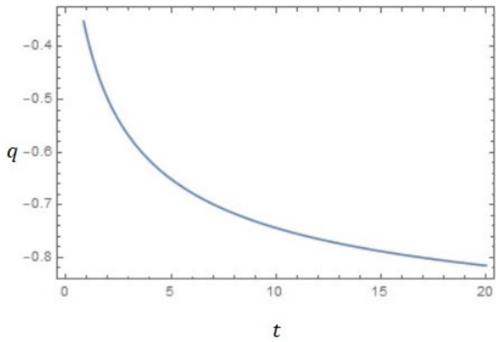

Figure 3. The plot of q verses t for $\alpha = 1.4, p = 0.5$ and $\beta = 0.2$. q changes its sign from positive to negative which indicates that our model represents transition of early cosmic deceleration to present cosmic acceleration.

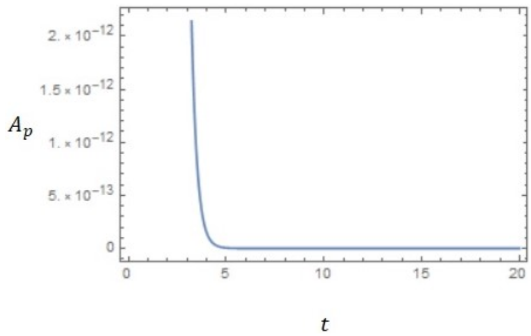

Figure 4. The graph of A_p verses t for $\alpha = 1.4, p = 0.5, p_1 = 0.03$ and $\beta = 0.2$. $A_p \to \infty$ as $t \to 0$ and $A_p \to 0$ as $t \to \infty$. Thus, our Universe approaches isotropy at late times.

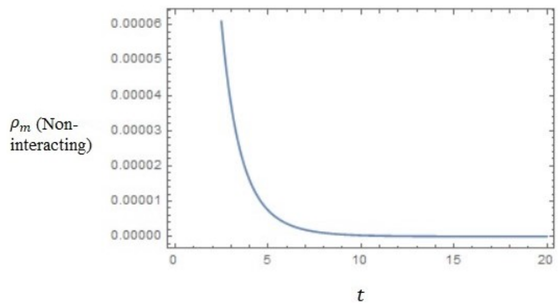

Figure 5. The plot of ρ_m (non-interacting case) verses cosmic time t for $\rho_0 = 0.04, \alpha = 1.4, \beta = 0.2$ and $p = 0.5$. ρ_m is a decreasing function of t and approaches to zero at the later stage of the Universe.

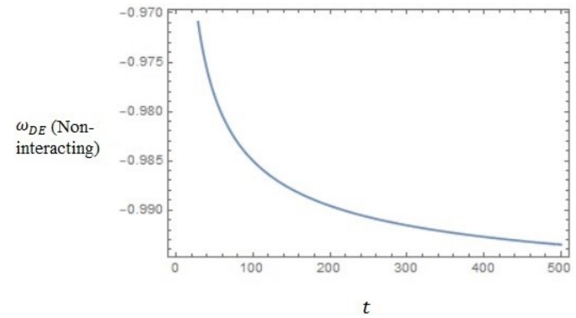

Figure 6. The graph of ω_{DE} (non-interacting case) verses t for $u = 0.5, \tau = 0.0004, \eta = 0.0005, \alpha = 1.4, p = 0.5$ and $\beta = 0.2$. $\omega_{\mathrm{DE}} \rightarrow -1$ as time evolves. Thus, our non-interacting model behaves like a cosmological constant model.

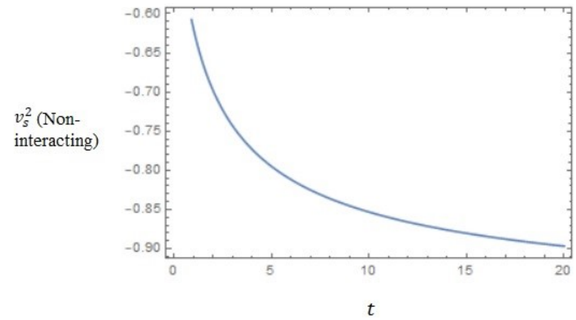

Figure 7. The time-dependent evolution of v_s^2 (non-interacting case) for $u = 0.5, \tau = 0.0004, \eta = 0.0005, \alpha = 1.4, p = 0.5$ and $\beta = 0.2$. v_s^2 is negative throughout the evolution of the Universe which indicates that our non-interacting model is unstable.

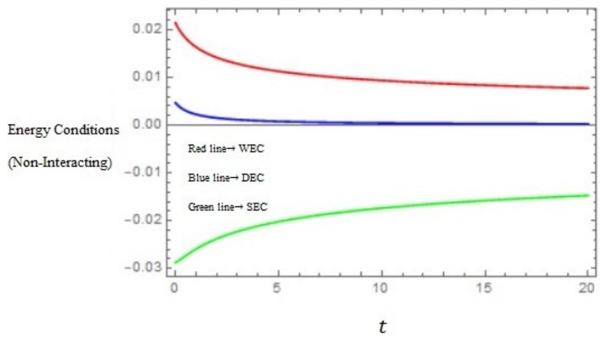

Figure 8. The plot of Energy Conditions (non-interacting case) verses cosmic time t for $u = 0.5, \tau = 0.0004, \eta = 0.0005, \alpha = 1.4, p = 0.5$ and $\beta = 0.2$. WEC (red line) and DEC (blue line) are satisfied whereas SEC (green line) is violated.

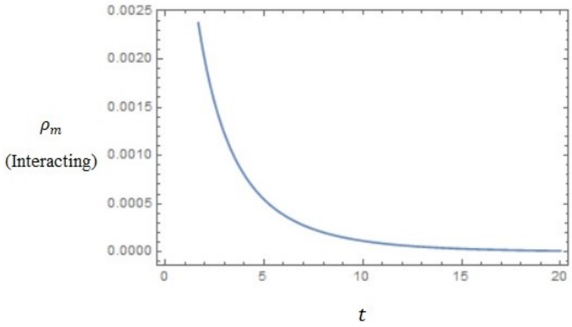

Figure 9. The time-dependent variation of ρ_m (interacting case) for $\acute{\rho}_0 = 0.04, b = 0.5, \alpha = 1.4, \beta = 0.2$ and $p = 0.5$. $\rho_m \to 0$ as $t \to \infty$ as seen from the above figure.

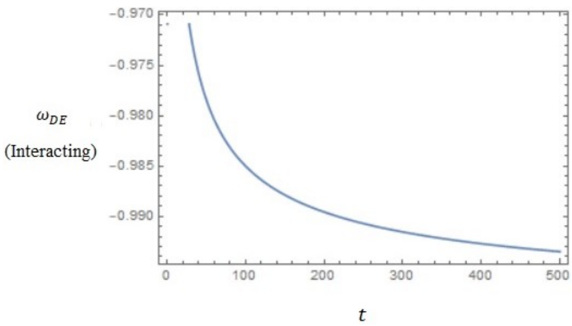

Figure 10. The graph of $\omega_{\rm DE}$ (interacting case) verses cosmic time t for $\acute{\rho}_0 = 0.04, b = 0.5, \alpha = 1.4, \beta = 0.2, \tau = 0.0004, \eta = 0.0005, u = 0.5$ and $p = 0.5$. $\omega_{\rm DE} \to -1$ as time evolves. Thus, our interacting model behaves like a cosmological constant model.

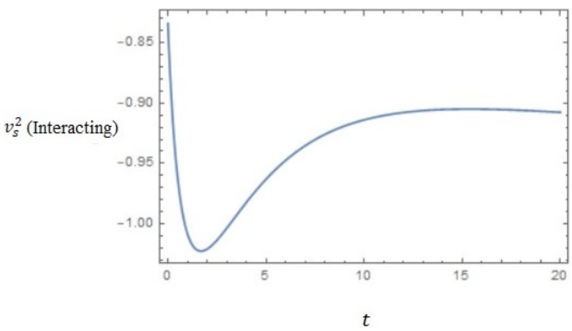

Figure 11. The plot of v_s^2 (interacting case) verses cosmic time t for $\acute{\rho}_0 = 0.04, b = 0.5, \alpha = 1.4, \beta = 0.2, \tau = 0.0004, \eta = 0.0005, u = 0.5$ and $p = 0.5$. v_s^2 is negative as observed from the above figure. Thus, our interacting model is unstable.

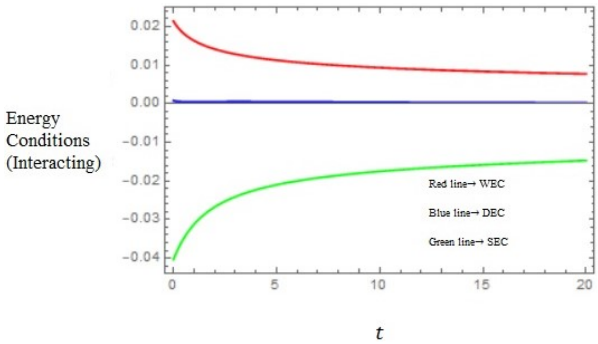

Figure 12. The time-dependent variation of Energy Conditions (interacting case) verses t for $\acute{\rho}_0 = 0.04, b = 0.5, \alpha = 1.4, \beta = 0.2, \tau = 0.0004, \eta = 0.0005, u = 0.5$ and $p = 0.5$. WEC (red line) and DEC (blue line) are satisfied whereas SEC (green line) is violated.

9 Conclusions

This investigation is about the determination of a spatially homogenous and anisotropic Bianchi type VI_0 space-time with GGPDE in GR. To determine the solutions of the system of field equations completely, we have used simple parametrization of average scale factor $a\left(t\right) = \exp\left\{(\alpha t + \beta)^p\right\}$ as proposed by Mishra and Dua [28]. We have discussed the non-interacting and interacting scenarios. The main features of both the models are point-wise listed below:

- The Hubble parameter H diminishes and ultimately approaches to a small value with the passage of cosmic time.
- The GGPDE density ρ_{DE} is a decreasing function of t and tends to a small value as time evolves.
- The deceleration parameter q yields two phases of the Universe. From Figure 3, it is observed that the sign of q changes from positive to negative. At the initial stage, the sign of q is positive which indicates the decelerating phase of the Universe and at late times, q is negative which describes the present accelerating phase of the Universe. Thus, our model shows early cosmic deceleration to present cosmic acceleration.
- The matter energy density $\rho_m \to 0$ as $t \to \infty$ as observed from Figures 5 and 9 for both the non-interacting and interacting models.
- $\omega_{\text{DE}} \to -1$ as seen from Figures 6 and 10 for both the non-interacting and interacting models. Thus, both the non-interacting and interacting models behaves like a cosmological constant.
- From Figures 7 and 11 it is observed that v_s^2 is negative for both the non-interacting and interacting models. It represents the unstable nature of the Universe.

- From the study of Energy Conditions namely WEC, DEC and SEC, we confirmed that WEC and DEC are satisfied but violates SEC. The violation of SEC gives anti-gravitational effect for which Universe gets jerk and thus our Universe exhibits transition from early decelerating to present accelerating Universe.

The interesting features which we found in this paper are that both the non-interacting and interacting models behaves like a cosmological constant. Thus, the work presented here may give a better understanding on the evolution of the Universe and satisfactory results on GGPDE which are in good agreement with the present-day observations.

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