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Leptonic Decay Widths and Decay Constants of ρ, ω, φ, ψ and Υ Mesons in a Dirac Formalism

B.K. Panda[1], **S. Panda**[2*], **S.N. Jena**[2], **A.K. Panda**[2]

[1]Department of Physics Aska Science College, Aska, Odisha, India
[2]P.G. Department of Physics, Berhampur University, Berhampur-760007, Odisha, India
[*]Corresponding author E-mail: sp.phy@buodisha.edu.in

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Abstract. In an approach predicted by the independent quark model with scalar- vector linear potential, we compute the leptonic decay widths and electromagnetic decay constants of vector mesons. We derive the quark-antiquark momentum distribution wave function from bound Quark eigenmode, Supposing a strong association between quark-antiquark momenta within the meson, so that the total momentum identically zero in the mesons center of mass frame. We obtain the leptonic decay width of ρ, ω, φ, ψ and Υ as 6.61 keV, 0.71 keV, 1.26 keV, 5.49 keV, and 1.31 keV and decay constants of ρ, ω, φ, ψ, and Υ as 0.19 GeV, 0.064 GeV, 0.074 GeV, 0.089 GeV and 0.021 GeV using our model parameter. These values are consistent with the said relevant experimental observations.

KEY WORDS: Decay constant of mesons, quarks, antiquarks, leptonic decay widths.

1 Introduction

While QCD is supposed to be the underlying theory of strong interaction among quarks and gluons, inside hadrons, the fact is that a large number of low-energy phenomena can not be made clear by first principles QCD [1] should serve as an introduction that the theory of strong interaction between quarks and gluons must be supplemented by some other mechanism. It is necessary to employ phenomenological models because that is the only alternative. To date, several promising models of chiral potentials have been developed. One that uses a chiral Dirac-like scalar-vector linear potential with quarks of different masses [2] is currently being used to research low-energy phenomena in the baryonic sector, such as octet-baryon masses [2], the properties of the electron [3], the nuclear magnetic moments [4], and charge radii, radiative decay [5–7]. Additionally, this model has proven to be a successful explanation for pion mass, the decay constant (ρ_n) and (ρ_0) mass splittings, and ordinary light meson radiative decay. The successful use of the model on both baryons and mesons in the light flavor sector allows the light flavor hybrid version of the bag model (CBM) [8] to be regarded as a simple and highly successful alternative to the cloudy bag model (CBM). The main objective of the current work is to broaden the validity of the formula for determining the leptonic decay width of heavy vector mesons in the charm and bottom flavor sectors to ρ, ω, φ, ψ and Υ. Van Royen-Weisskopf's calculations [9] have widely investigated the nonrelativistic behavior of heavy vector mesons in the charm and bottom flavor sectors with adequate radiative corrections. However, this framework would not suit ordinary vector mesons in the light flavor sector because their constituent quark dynamics have been more relativistic. These, along with any vector mesons like ρ, ω, φ, ψ and Υ will be the most well-suited for the model that makes use of the ansatz of the dominant confining interaction taken phenomenologically.

Our technique here is very comparable to that of Margolis and Mendel, [10], who utilized the positronium annihilation process (in the bag model) [11]. Because we presume that now the constituent quark-antiquark pair within the meson annihilates primarily to a single virtual photon, that also main ones a lepton pair, we will believe that these two components are indistinguishable, and thus can exist as a single constituent particle. Furthermore, since we believe that the center-of-mass motion does not play an important role in the dynamics of the system, we postulate that the decay dynamics are unaffected by the center-of-mass motion. In that case, it is possible to make a convincing argument that there is a very strong correlation between the quark-antiquark momentum and the total momentum, which allows the total momentum to be equal to zero in the center of mass frame of the vector meson. Accordingly, in the ground state of the decaying vector meson, the momentum distribution of the bound quark-antiquark pair represents the appropriate ground state for this configuration of SU(6) spin-flavor. Then, using the suitable Feynman diagram, the transition probability amplitude can indeed be successfully described as the free quark-antiquark pair annihilation amplitude integrated over the model momentum distribution. Clearly, in any reaction that involves a quark-antiquark annihilation, there is difficulty relating to energy conservation because the total kinetic energy of the annihilating quark-antiquark does not equal the mass-energy of the decaying meson at rest. Phenomenological models employ the leading-order calculation methods, even simple quantum field theory (QFT) field expressions. All describe the missing bound quark-antiquark annihilation in meson. Therefore, we will go along with the standard assumption that when quark-antiquark annihilation occurs, accompanied by the vanishing of the meson bound state, a differential amount of energy is provided to the photon.

Thus, we are satisfied with admitting the usual presumption that the difference amount of energy is indeed provided to the photon whenever meson quark-antiquark annihilation takes place, which also arises whenever a meson bound sate is discovered to have disappeared.

In Section 2, we discussed the basic framework of our model; in Section 3, we derive momentum distribution amplitude of the quark and antiquark in the vector meson ground state. In Section 4, we found the transition matrix elements for leptonic decay of vector mesons as well as the corresponding decay widths equation. Finally, Sections 5 and 6 contain the research outcomes and inferences.

2 Basic Framework

To fully understand the leptonic decay of vector mesons, we need to represent the decaying vector mesons initial state accurately. We must obtain a complete picture of the initial state of the decaying vector mesons with constituent quark and antiquark and their respective momenta and spin. Whereas the bound constituent quark and antiquark within the meson are in specific energy states, the quarks and antiquarks have no specific momenta. While momentum distribution amplitude of the constituent quark and antiquark in the meson before annihilation into a lepton pair is predictable, their amplitude at the instant of annihilation can not be calculated. In this instance, the bound quark orbital deduced in a model could be achieved with momentum space projection of the corresponding bound quark orbital. We derive this; it is necessary to make use of some of the simplifying assumptions. Here clarify the overview and conventions of the model we used in these calculations; it's appropriate to highlight some aspects. Following this model, light hadrons are visualized as a color-singlet assembly of a quark and an antiquark, which are independently bounded by an average flavor-independent potential of the form [2].

$$U_q(r) = \frac{1}{2}(1+\gamma_0)(a^2 r + U_0)\,, \qquad \text{with } \ a > 0\,. \tag{1}$$

An illustration of the confining interaction is used in the model as the phenomenological expression of the non-perturbative multigluon framework. Although interaction resulting from one gluon exchange and the quark pion interaction which is essential to restore chiral symmetry are presumed, a theoretical framework that accounts for these residual interactions has not yet been developed. Although these residual interactions are allowed to treat perturbatively in the model and are likely to affect the mass splitting in light hadron spectroscopy [2–7, 12], these effect on hadronic decay processes is reduced significantly. As a first approximation, the confining part of the interaction, phenomenologically defined by $U(r)$ in equation (1), is thought to produce zeroth-order quark dynamics inside the meson core, which could have updated information of vector meson's leptonic decay

$$\mathfrak{L}_q^0(x) = \bar{\psi}_q(x)\Big[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - U_q(r) - m_q\Big]\psi_q(x). \tag{2}$$

Then there is the subsequent Dirac equation which admits

$$[\gamma^0 E_q - \vec{\gamma}\cdot\vec{p} - m_q - U_q(r)]\psi_q(\vec{r}) = 0. \tag{3}$$

The static solutions of positive and negative energy in zeroth order could be achieved in the meson's ground state

$$\psi_{q\xi}^{(+)}(\vec{r}) = N_q \begin{pmatrix} \phi_q(\vec{r}) \\ -i\frac{\vec{\sigma}\cdot\vec{r}}{\xi_q}\phi_q'(\vec{r}) \end{pmatrix} \chi_\xi, \tag{4}$$

$$\psi_{q\xi}^{(-)}(\vec{r}) = N_q \begin{pmatrix} -i\frac{\vec{\sigma}\cdot\vec{r}}{\xi_q}\phi_q'(\vec{r}) \\ -i\phi_q(\vec{r}) \end{pmatrix} \bar{\chi}_\xi\,. \tag{5}$$

Here N_q is the normalisation constant. The two components spinors are available, $\chi\uparrow$ and $\chi\downarrow$ denote

$$\chi\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

$$\text{and} \quad \bar{\chi}\uparrow = \begin{pmatrix} 0 \\ -i \end{pmatrix}, \qquad \bar{\chi}\downarrow = \begin{pmatrix} i \\ 0 \end{pmatrix}. \tag{7}$$

Meanwhile, the radial angular part $\phi_q(\vec{r})$ of the upper component solution corresponds to quark flavor q and is defined by

$$\phi_q\,(\vec{r}) = \frac{i}{\sqrt{4\pi}}\left(\frac{f_q(\vec{r})}{r}\right), \tag{8}$$

where a Schrödinger-type equation is satisfied with the reduced radial component $f_q(\vec{r})$

$$f_q{''}\,(r) + \xi_q(E_q - m_q - a^2 r - U_0)f_q(\vec{r}) = 0 \tag{9}$$

with $\xi_q = E_q + m_q$. It is possible to transform Eq. (9) into a comfortable dimensionless form

$$f_q{''}\,(p) + [\epsilon_q - \rho]f_q(\rho) = 0. \tag{10}$$

Here, we take $\rho = r/r_{0q}$ is a dimensionless variable with $r_{0q} = (a^2\xi_q)^{-\frac{1}{3}}$ and

$$\epsilon_q = \left(\frac{4\xi_q}{a^4}\right)^{\frac{1}{3}}(E_q - m_q - U_0). \tag{11}$$

N_q is given by in Eqs. (5) and (6)

$$N_q^2 = \frac{3(E_q + m_q)}{4E_q + 2m_q - U_0}\,. \tag{12}$$

The ground state of the individual quark binding energy E_{qn} in zeroth order is derived from the eigenvalue condition (11), as equation (10) provides the essential eigenvalue equation, the solution of which would be given $\epsilon_q = 2.33811$ by the standard numerical method. By calculating the leptonic decay widths of vector meson, vector meson masses as well as the momentum distribution of quarks inside the vector meson can be estimated.

3 Momentum Distribution Wavefunction

Here we discuss the field theoretical calculation of the leptonic decay of vector mesons. The initial state of decaying vector mesons in terms of a constituent of the quark and antiquark, which are suitable for their particular momenta and spin, is therefore conveniently defined. Although there are no explicit momenta in the exact energy state in the meson, the constituent quark and antiquark still can be distributed in the meson immediately before their annihilation by a lepton pair. Based on some of the simplification assumptions; the momentum probability amplitudes should be identified. The respective binding quark state of the model as in equations (4) and (5) can be achieved by an appropriate spatial momentum projection. The approach mentioned in the previous studies [7] in the framework can rightly be described as radiative decays.

If the amplitude of $G_q(\vec{p},\xi\xi')$ to find the bound quark of flavor q in its eigenmode $\psi_{q\xi}^{(+)}(\vec{r})$ in such a state momentum state $\vec{p}$ and spin-projection ξ' then

$$G_q(\vec{p},\xi,\xi') = \frac{V_q^+ G_q(\vec{p},\xi')}{\sqrt{2E_p}} \int d\vec{r} \psi_{q\xi}^{(+)}(\vec{r})\, e^{-i(\vec{p}\cdot\vec{r})}\,, \tag{13}$$

where $E_p = \sqrt{p^2 + m_q^2}$ and $V(\vec{p},\xi')$ is the usual free Dirac spinor,

To facilitate equation (13) the current independent quark model, we take radial apart of the bound quark orbital $\psi_{q\xi}^{(+)}(\vec{r})$ in Gaussian form, like harmonic potential wave functions. So that

$$f_q(r) = A_q \left(\frac{r}{r_{0q}}\right) e^{\left(-\beta_q r^2/2\right)} \tag{14}$$

with normalization constant $A_q = \left(\frac{2}{\pi r_{0q}^2}\right)^{1/4}$ and $\beta_q = \frac{1}{8r_{0q}^2}$ that produces results in

$$G_q(\vec{p},\xi,\xi') = G_p(p)\delta_{\xi\xi'} \tag{15}$$

$$\text{if} \qquad G_p(p) = \left(\frac{N_q}{\xi_q}\right)\left(\frac{\pi}{\beta_q}\right)(E_p + E_q)\left(1 + \frac{m_q}{E_p}\right)^{1/2} e^{(-p^2/2\beta_q)}\,. \tag{16}$$

If the $G_q(p)$ probability distribution function for quark q moves within the meson with momentum $\vec{p}$. We assume that in the center of mass frame of the vector meson, the total momentum of the quark and antiquark is zero. We are expanding the concept of Margolis and Mendel [10] to reflect them. Amplitude $G_v\left(\vec{p_1}, \vec{p_2}\right)$ is used to find convincing dynamic distribution. $\vec{p_1}$ quark along with $\vec{p_2} = -\vec{p_1}$ antiquark momentum inside the vector meson is taken as

$$G_v\left(\vec{p_1}, \vec{p_2}\right) = \sqrt{G_q\left(\vec{p_1}\right) G_q(\vec{p_2})}\,. \tag{17}$$

The momentum distribution wave function of the individual quarks is equal, and finally, the effective momentum distribution function of the meson we write as follows:

$$G_v\left(\vec{p_1}, \vec{p_2}\right) = |G_p(\vec{p_1})| \tag{18}$$

With the information of $G_v\left(\vec{p_1}, \vec{p_2}\right) = |G_p(\vec{p_1})|$ and taking $b_q^{\dagger}\left(\vec{p_1},\, \xi_1\right)$ and $\bar{b}_q^{\dagger}\left(-\vec{p_1},\, \xi_2\right)$ are 'quark-antiquark creation operators respectively, the ground state of neutral vector meson in its rest frame for a particular spin projection S_v generally expressed as

$$|V(S_v)\rangle = \sqrt{3} \sum_{(q,\xi_1,\xi_2)\epsilon S_v} \int d\vec{p_1} G_q\left(\vec{p_1}\right) C^{S_v}_{\xi_1\xi_2} \varsigma_q^V b_q^{\dagger}\left(\vec{p_1}, \xi_1\right) \bar{b}_q^{\dagger}\left(-\vec{p_1},\, \xi_2\right) |0\rangle\,. \tag{19}$$

Here is the summation, with flavor coefficient ς_q^V and the spin configuration coefficient $C^{S_v}_{\xi_1\xi_2}$ represents the appropriate SU(6) spin-flavor structure of the particular vector-meson V with its spin projection S_v. Due to the single color configuration of the decaying meson, the strong coefficient is $\sqrt{3}$.

4 Leptonic Decay Width and Decay Constant

As shown in Figure 1, we shall use the Feynman diagram in proleptical studies [9, 10]. We assume the primary contribution to the leptonic decay of neutral vector meson derived from the annihilation of the bound quark-antiquark. In the meson, quark pairs into single-photon, which eventually annihilates the pair of leptons and obtain the expression of leptonic decay width and decay constants of

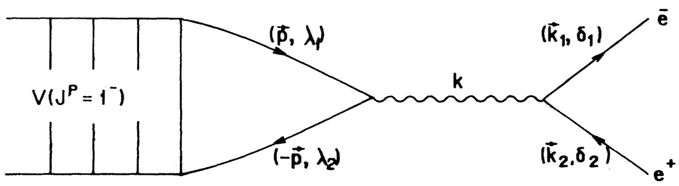

Figure 1. One-photon contribution to the leptonic decay of vector mesons.

vector meson. So, in the S matrix element, we can write it in the configuration space by specifying $D_{\mu\nu}(x_2 - x_1)$ is the photon propagator as

$$S_{fi} = \langle e^-(k_1, \delta_1)e^+(k_2, \delta_2)|(-ie^2) \int d^4x_1 d^4x_2 \times |\{\bar{\psi}_e^{(-)}(x_2)\gamma^\mu \psi_e^{(-)}(x_2)\} D_{\mu\nu}(x_2 - x_1) \times \{\sum_q e_q \bar{\psi}_q^{(+)}(x_1)\gamma^\mu \psi_q^{(+)}(x_1)\} V(S_v)\rangle. \quad (20)$$

According to equation (8), the decaying vector meson state is used the quark lepton field expansion are in the form of

$$\psi_q(x) = \sum_\xi \frac{1}{(2\pi)^{3/2}} \int \frac{d\vec{p}}{\sqrt{2E_p}} \times \left[b_q(\vec{p},\,\xi)U_q(\vec{p},\,\xi)e^{-ipx} + \bar{b}_q^\dagger(\vec{p},\,\xi)V_q(\vec{p},\,\xi)e^{ipx}\right] \quad (21)$$

and

$$\psi_e(x) = \sum_\delta \frac{1}{(2\pi)^{3/2}} \int \frac{d\vec{k}}{\sqrt{2E_k}} \times \left[d_e(\vec{k},\,\delta)U_q(\vec{k},\delta)e^{-ikx} + \bar{d}^\dagger{}_e(\vec{k},\delta)V_e(\vec{k},\delta)e^{ikx}\right]. \quad (22)$$

In a similar form, one can get S_{fi}, as

$$S_{fi} = -i2\pi^4\delta^4\left(k_1 + k_2 - \hat{O}M_v\right) M_{sv}\left(k_1, k_2, \delta_1, \delta_2\right), \quad (23)$$

where $\hat{O} \equiv (1, 0, 0, 0)$ and $(E_{p1} + E_{p2}) \simeq M_v$ and $M_{sv}(k_1, k_2, \delta_1, \delta_2)$ is the transition matrix element for the decay process, which is invariant. Simplifying Eq. (23). Now just as in the previous one, the current model can be represented in studies [13, 14] of the vector mesons leptonic decay width as

$$\Gamma(V \to e^+e^-) = \left(\frac{4}{9}\right)\left(\frac{\alpha_{\text{c.m}}^2}{\pi^3}\right)\left(\frac{\langle e_V\rangle}{M_V}\right)^2 \left|\int_0^\infty dpp^2 G_q(p)\left(2 + \frac{M_q}{E_p}\right)\right|^2. \quad (24)$$

The specific average values of charge for each vector meson is represented $\langle e_q\rangle_V$ here, such as

$$\langle e_q\rangle_{\rho,\omega,\phi} = \left(\frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{1}{3}\right) \quad \text{and} \quad \langle e_q\rangle_{\psi,\gamma} = \left(\frac{2}{3}, \frac{1}{3}\right).$$

We are putting Eq. (16) into Eq. (24), will then be leptonic decay width calculated from Eq. (18) the form is written as

$$\Gamma(V \to e^+e^-) = \left(\frac{4}{9}\right)\left(\frac{1}{\pi\alpha_q}\right)^{3/2}\left(\frac{\xi_{\text{c.m}}}{M_V}\right)^2\left(\frac{N_q}{\xi_q}\right)^2 \langle e_q\rangle_V^2 I_V^2, \quad (25)$$

where

$$I_V = \int_0^\infty dpp^2 \sqrt{1 + \frac{m_q}{E_p}} \left(2 + \frac{m_q}{E_p}\right)(E_p + E_q)e^{-p^2/2\beta_q}. \quad (26)$$

The term f_V in the current model can be taken from the normal one. Defining the relationship as defined for the electromagnetic constant in terms of $\Gamma(V \to e^+e^-)$ from the normal defining relationship as specified

$$f_V = \left[\frac{3\Gamma(V \to e^+e^-)}{4\pi\alpha^2_{\text{c.m}} M_V}\right]^{1/2}. \quad (27)$$

The leptonic decay of vector meson, given as $\Gamma(V \to e^+e^-)$ or f_V can be used to measure their expressions (26) and (27).

5 Outcomes

In this section, we use the equations (25), (26), and (27) deduced in Section 4 to calculate the leptonic decay widths of mesons ρ, ω, φ, ψ and Υ and also their decay constants. The computations primarily concern the model's potential parameters (a, U_0), as well as the quark masses (m_u, m_d, m_s, m_c and m_b). The calculated meson masses were like the observed ones. In a sense, our goal here is to perform a parameterless computation of the decay widths, using the potential parameters extracted previously from applications of the current model to the hadron sectors [2–7, 12].

In the current model, first of all, we have fixed the potential parameters

$$(a,\, U_0) = (1.401,\, -1.510)\ \text{GeV}$$

with an acceptable quark mass option

$$(m_u = m_d) = (001)\ \text{GeV} \quad (28)$$

$$\text{and} \quad (m_s = m_c = m_b) = (0.202, 1.46,\, 5.5)\ \text{GeV}. \quad (29)$$

The quark binding energy (E_q) is the order of the current quark masses, the scale factor (r_{0q}) and the normalization constant (N_q^2) analogous expressions in Section 2 and the consistent quantities required for determining the ground state splitting of ρ, ω, φ, ψ and Υ mesons as of Section 2

$$\begin{aligned}(E_u = E_d,\, r_{0u} = r_{0d},\, N_u^2 = N_d^2)& \\ = (2.6823\ \text{GeV},\ 0.5741\ \text{GeV}^{-1},\ 0.6588\ \text{GeV}),& \quad (30)\\ (E_s,\, E_c,\, E_b) = (2.7486,\, 5.5362,\, 8.4304)\ \text{GeV}& \\ (r_{0s},\, r_{0c},\, r_{0b}) = (0.5568,\, 0.3082,\, 0.2450)\ \text{GeV}^{-1}& \\ (N_s^2,\, N_c^2,\, N_b^2) = (0.6857,\, 0.7897,\, 0.9039)\ \text{GeV}.& \quad (31)\end{aligned}$$

Leptonic decay widths and the corresponding electromagnetic decay constants of the vector mesons ρ, ω, φ, ψ and Υ are determined using model potential parameters. The resulting quantities were taken from Eqs. (28) to (31). The vector meson masses M_V involved in this equation are taken as the observed, and using these model parameters, integral I_V in Eq. (26) with the help of the Gaussian quadrature technique

$$\begin{aligned}(I_\rho = I_\omega,\, I_\varphi) &= (1.3184,\, 1.2401)\ \text{GeV}^4,\\ (I_\psi,\, I_\Upsilon) &= (0.3798, 0.2400)\ \text{GeV}^4.\end{aligned} \tag{32}$$

Equations (30) to (31) and (32) yield the leptonic decay widths $\Gamma\,(V \to e^+e^-)$ and electromagnetic decay constants f_V for the vector mesons ρ, ω, φ, ψ and Υ from Eqs. (25) and (27) respectively for this model.

Table 1 presents both the findings for ρ, ω, φ mesons and the corresponding experimental values [15] relative to the other models [10, 16, 17] The estimated results also presented for vector meson ψ and Υ compared to the experimental values corresponding to it [15]. We discover the calculated results of mesons, almost in comparison to those achieved by Margolis and Mendel in the bag model in the similar calculation totally co-related quark-antiquark momenta, that are slightly higher than the corresponding experimental values and the results of Jena et al. [16].‘ The results for these mesons are lower than the experimental values for Bag model calculations [17] with fully uncorrelated quark-antiquark momenta. Our observations are reasonably consistent with the experimental values of mesons, and heavy mesons match the experimental values [15].

The ratio of R_V to the different vector mesons ρ, ω, φ, ψ and Υ determined from the estimated leptonic decay width values as defined in Table 1 and the associated mean electric charges $\langle e_q \rangle_V$.

Table 1. Leptonic decay widths $\Gamma(V \to e^+e^-)$ and the decay constants and f_V respective presented along with the results of Refs. [17–19] and Ref. [20], as well as the experiment [15]

	V	Current calculation	Ref. [15]	Ref. [17]/ Ref. [18]	Experimental data [15]	f_V Ref. [19]	f_V Ref. [20]
	ρ	6.61	6.26(8.10)	7.80	7.04±0.06		
	ω	0.71	0.67(0.87)	0.84	0.60±0.02		
Γ keV	φ	1.26	1.58(1.84)	1.69	1.26±0.14		
	ψ	5.49			5.55±0.14		
	Υ	1.31			1.34±0.018		
	ρ	0.19	0.19(0.22)	0.21	0.204±0.008		0.16
	ω	0.064	0.06(0.07)	0.07	0.059±0.001	0.510	0.16
f_V GeV	φ	0.074	0.08(0.09)	0.07	0.074±0.003		0.161
	ψ	0.089			0.0896±0.001		0.228
	Υ	0.021			0.0252±0.002		0.299

We find

$$\begin{aligned}(R_\rho,\, R_\omega,\, R_\varphi) &= (13.22,\ 12.78,\ 11.34)\ \text{keV}\\ (R_\psi,\, R_\Upsilon) &= (12.24,\, 4.05)\ \text{keV}\end{aligned} \tag{33}$$

as against the experimental ratio 12.0 ± 1.5 keV [21]. For all vector mesons including Υ meson, the estimated values R_v are found in fair agreement.

6 Inferences

Hayne and Isgur [18] developed a formalism based on an ad hoc quark-antiquark momentum distribution in the Gaussian form to extend the nonrelativistic quark model calculations beyond static approximation. However, we observed that the electromagnetic decay constants f_V accessed in our computations comply well with some of the findings in Ref [18]. Thus, unlike Hayne and Isgur, our model has the advantage of directly achieving the quark-antiquark momentum distribution in Gaussian form. Consequently, even within constraints of the present working approximations, the model over a simple computational framework that appropriately explains the leptonic decay width and decay constants of ρ, ω, φ, ψ and Υ meson.

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