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A Comparative Study of Cosmological Models in Barber Self Creation Theory of Gravitation

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Abstract. In this paper, we have investigated a comparative study of Bianchi type-VI_0 cosmological models in self creation theory formulated by Barber [1] under the influence of cosmic string, perfect fluid, thick domain wall, wet dark fluid and macroscopic body. Exact cosmological models in the theory are obtained with the help of relation between metric coefficient and equation of state. Also, we discuss stability, some kinematical properties and their graphical representation of the obtained models. The main part of the paper is that to compare the obtained results of cosmic string, Perfect fluid, thick domain wall, wet dark fluid, and macroscopic body in Barber self-creation theory. The models in above mentioned energy momentum tensor are similar and behave alike except macroscopic body. All these models studied here will be useful for a better understanding of self-creation cosmology and structure formation of the universe.

KEY WORDS: Bianchi type-VI_0, Barber self-creation theory, cosmic string, Perfect fluid, thick domain wall, wet dark fluid and macroscopic body.

1 Introduction

In an attempt to produce continuous creation theories, in 1982, Barber [1] has proposed two self-creation theories modifying the Brans and Dicke [2] and Brans [3] theory of gravitation and Einstein theory of general relativity. In 1987 Brans has pointed out that the first theory of Barber is not satisfactory because of violation of equivalence principle. According to Brans, Barber's first theory is in disagreement with experimental observations and is also not consistent in general. His second self-creation theory is a modification of Einstein's general theory of gravitation to variable G-theory which predicts local effect and is within the observational limit. In his postulate, the gravitational coupling of the Einstein field equations is allowed to be a variable scalar on the space time so that this scalar couples to the trace of energy momentum tensors.

The Barber field equation in second self-creation theory can be expressed as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\varphi^{-1}T_{ij}\,, \tag{1}$$

and

$$\Box\varphi = \varphi_{;k}^{'k} = \frac{8\pi\upsilon}{3}T\,, \tag{2}$$

where φ is Barber's scalar, T_{ij} is the energy momentum tensor, $\Box$ is the invariant d'Alembertian, T is the trace of energy momentum tensor T_{ij}, υ is a coupling constant to be determined from experiment and $0 \prec |\upsilon| \prec 1/10$. In the limit $\upsilon \to 0$, this theory approaches the Einstein theory in every respect. Due to the nature of the space time Barber's scalar φ is a function of t. Pimentel [4, 5], Venkateswarlu *et al*. [6], Shanti and Rao [7], Mohanty *et al.* [8, 9], Adhav *et al.* [10], Katore *et al.* [11] and Pawar *et al.* [12] are some of the authors who have investigated the various aspects of Barber's second self-creation theory by using various cosmological models.

The purpose of the present work is to obtain Bianchi type-VI_0 cosmological model in presence of cosmic string, perfect fluid, thick domain walls, wet dark fluid and macroscopic body. Our paper is organized as follows. In Section 2, we derive the metric and field equations in self-creation theory of gravitation; in Section 3 – solutions of cosmic string as a source; in Section 4 – solutions of perfect fluid as a source; in Section 5 – solutions of thick domain walls. Section 6 contains solutions of wet dark fluid and Section 7 – solutions of macroscopic body. Section 8 is mainly concerned with the physical and kinematical properties of the model and Section 9 with graphical representation of cosmic string, perfect fluid, thick domain walls and wet dark fluid. Section 10 contains the observational parameters of all the models. The last Section 11 contains comparisons and conclusion.

2 Metric and Field Equations

We consider the Bianchi type VI_0 space time in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dz^2\,, \tag{3}$$

where A, B, C are functions of t only.

The field equations in Barber second self-creation theory are given by

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{1}{A^2} = -8\pi\varphi^{-1}T_1^1\,, \tag{4}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2} = -8\pi\varphi^{-1}T_2^2\,, \tag{5}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{1}{A^2} = -8\pi\varphi^{-1}T_3^3\,, \tag{6}$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} - \frac{1}{A^2} = -8\pi\varphi^{-1}T_4^4\,, \tag{7}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0\,, \tag{8}$$

$$\varphi_{44}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\varphi_4 = \frac{8\pi\upsilon}{3}T\,, \tag{9}$$

where suffixes 4 denotes ordinary differentiation with respect to time t.

3 Solution of Cosmic String

The origin of our universe is one of the finest cosmological mysteries even today. The exact physical situation at early stage of the formation of our universe is still a subject of study. The concept of string theory was established to express events of the early stage of the evolution of the universe. The general relativistic treatment of strings was initiated by Vilenkin [13] and Letelier [14] and the gravitational effects of cosmic strings have been extensively discussed by Khadekar *et al.* [15], Kandalkar *et al.* [16], Adhav *et al.* [17], Pawar *et al.* [18], Vidyasagar *et al.* [19].

The energy momentum tensor for string cloud is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j\,, \tag{10}$$

where ρ is the rest energy density of the cloud of strings with massive particles attached to them, $\rho = \rho_p + \lambda$, ρ_p being the rest energy of the particles attached to the strings and λ the tension density of the system of strings. u_i describes the cloud four velocity and x_i represents the direction of strings. The equations (4) to (9) is a system of six independent equations with seven unknowns A, B, C, φ, υ, ρ, λ. In order to get deterministic solution we use the condition for Reddy string

$$\rho + \lambda = 0\,. \tag{11}$$

From equation (8), we get

$$B = \mu C\,, \qquad \mu = 1\,. \tag{12}$$

Also, the power law is given by

$$B = A^n\,. \tag{13}$$

Using (4) to (7), we get

$$\frac{A_{44}}{A} + k_1\frac{A_4^2}{A^2} - \frac{2}{A^2} = 0\,. \tag{14}$$

Solving equation (14), we get

$$A = k_3 t + k_4\,, \tag{15}$$

$$B = (k_3 t + k_4)^n\,, \tag{16}$$

$$C = (k_3 t + k_4)^n\,. \tag{17}$$

Using equations (15), (16) and (17) cosmological model in equation (3) takes the form

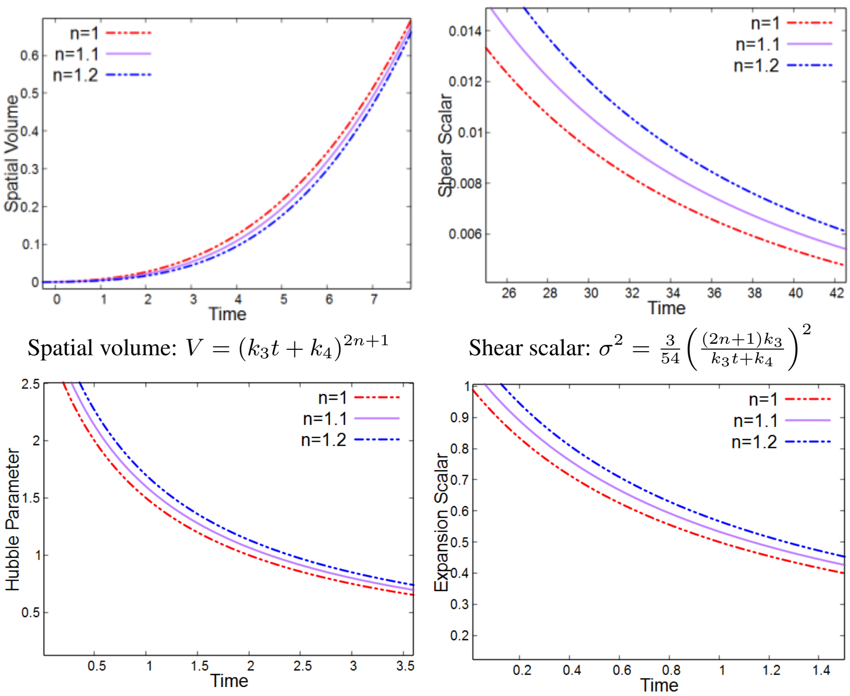
$$ds^2 = -dt^2 + (k_3 t + k_4)^2 dx^2 + (k_3 t + k_4)^{2n} e^{2x} dy^2 + (k_3 t + k_4)^{2n} e^{-2x} dz^2 \,. \quad (18)$$

From equation (4)–(9), we get

$$\varphi = \frac{k_7}{(k_3 t + k_4)^{2n}} + k_6 \,, \quad (19)$$

$$\lambda = \frac{k_8}{8\pi (k_3 t + k_4)^2} \left(\frac{k_7}{(k_3 t + k_4)^{2n}} + k_6 \right) , \quad (20)$$

$$\rho = \frac{k_9}{8\pi (k_3 t + k_4)^2} \left(\frac{k_7}{(k_3 t + k_4)^{2n}} + k_6 \right) . \quad (21)$$



Spatial volume: $V = (k_3 t + k_4)^{2n+1}$

Shear scalar: $\sigma^2 = \frac{3}{54} \left(\frac{(2n+1)k_3}{k_3 t + k_4} \right)^2$

Hubble parameter: $H = \frac{(2n+1)k_3}{k_3 t + k_4}$

Expansion scalar: $\theta = \frac{1}{3} \frac{(2n+1)k_3}{k_3 t + k_4}$

Figure 1.

Properties for cosmic string like:

$$\text{Spatial volume} \qquad V = (k_3 t + k_4)^{2n+1}\,; \tag{22}$$

$$\text{Expansion scalar} \qquad \theta = \frac{1}{3}\frac{(2n+1)k_3}{k_3 t + k_4}\,; \tag{23}$$

$$\text{Hubble parameter} \qquad H = \frac{(2n+1)k_3}{k_3 t + k_4}\,; \tag{24}$$

$$\text{Shear scalar} \qquad \sigma^2 = \frac{3}{54}\left(\frac{(2n+1)k_3}{k_3 t + k_4}\right)^2\,; \tag{25}$$

$$\text{Deceleration parameter} \qquad q = -\frac{2n}{2n+1}\,. \tag{26}$$

Graphical representation of spatial volume, shear scalar, Hubble parameter and expansion scalar with time of cosmic string are shown in Figure 1.

Figure 1 shows that volume increases with time. Volume $V \to \infty$ as $t \to \infty$. Hubble parameter decreases with increasing time. The expansion scalar and shear scalar are decreasing with increase in time and both vanishes as $t \to \infty$ for distinct value of n in presence of cosmic string. Also, a plot of average scale factor versus time is shown in Figure 2.

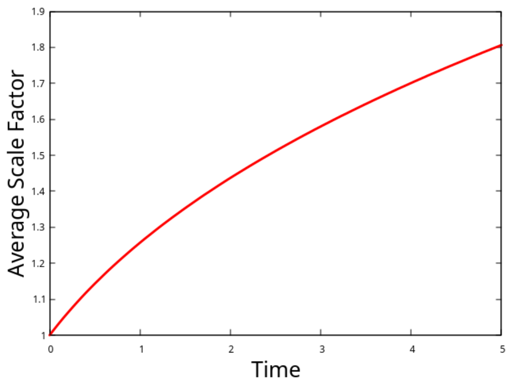


Figure 2. Plot of average scale factor versus time ($k_3 = k_4 = 1$).

4 Solution of Perfect Fluid

We have the perfect fluid energy momentum tensor as

$$T_{ij} = (\rho + p)\, u_i u_j + p\, g_{ij}\,, \tag{27}$$

together with

$$g_{ij} u^i u^j = -1\,, \tag{28}$$

where u^i is the four-velocity vector of the fluid, p and ρ are the proper pressure and energy density respectively.

From (27) and (28), the components of T_i^j in comoving coordinate are

$$T_1^1 = p\,, \qquad T_2^2 = T_3^3 = p\,, \qquad T_4^4 = -\rho\,. \tag{29}$$

Condition use radiation

$$\rho - 3p = 0\,. \tag{30}$$

Solving equations (4) to (9) with help of equations (29) and (30), we gain metric coefficients, cosmological model, physical properties and graphical illustration of the model. But all above results are same as the result obtained from cosmic string. Also, the value of proper pressure and energy density are

$$p = \frac{-k_8}{8\pi(k_3t+k_4)^2}\left(\frac{k_7}{(k_3t+k_4)^{2n}} + k_6\right), \tag{31}$$

$$\rho = \frac{k_9}{8\pi(k_3t+k_4)^2}\left(\frac{k_7}{(k_3t+k_4)^{2n}} + k_6\right). \tag{32}$$

5 Solution and Model of Thick Domain Walls

In this section we discuss the thick domain walls in the Bianchi type VI_0 space time given by (3). A thick domain wall may be considered as a solution like solution of the scalar field equations coupled with gravity. There are two methods of analyzing thick domain walls. One method is to solve gravitational field equations with an energy-momentum tensor describing a scalar field ψ with self-interactions contained in a potential $\upsilon(\psi)$ given by

$$\psi_i\psi_j - g_{ij}\left[\frac{1}{2}\psi_i\psi_j - \upsilon(\psi)\right]. \tag{33}$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho\left(g_{ij} + \omega_i\omega_j\right) + p\omega_i\omega_j\,, \tag{34}$$

$$\omega_i\omega^j = -1\,. \tag{35}$$

where ρ is the energy density of the walls, p is the pressure in direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction. Katore *et al.* [20], Chirde *et al.* [21], Bhoyar *et al.* [22] are some of the authors who have investigated several aspects of domain wall. Here we use the second approach to study the thick domain walls in scale covariant theory of gravitation.

From (34) with the help of (35), we get

$$T_1^1 = -p\,, \qquad T_2^2 = T_3^3 = T_4^4 = \rho\,. \tag{36}$$

Using radiation condition

$$p - 3\rho = 0\,. \tag{37}$$

Solving equations (4) to (9) with the help of equations (36) and (37), we obtain metric coefficients, cosmological model, physical properties and graphical illustration of the model. But all above results are same as the results obtained from cosmic string and perfect fluid. Also, the value of proper pressure and energy density are

$$p = \frac{k_8}{8\pi(k_3 t + k_4)^2}\left(\frac{k_7}{(k_3 t + k_4)^{2n}} + k_6\right)\,. \tag{38}$$

$$\rho = \frac{-k_9}{8\pi(k_3 t + k_4)^2}\left(\frac{k_7}{(k_3 t + k_4)^{2n}} + k_6\right)\,. \tag{39}$$

6 Solution and Model of Wet Dark Fluid

The nature of the dark energy component of the universe is one of the hidden mysteries of cosmology. The wet dark fluid (WDF) is a model for dark energy. This model is in the essence of the generalized Chaplygin gas Gorini *et al.* [23], where an energetic equation of state is offered with properties relevant for the dark energy problem. Here the motivation stems from an empirical equation of state proposed by Tait [24] and Hayward [25] to treat water and aqueous solution.

The equation of state for WDF is very simple.

$$P_{\text{WDF}} = \gamma\left(\rho_{\text{WDF}} - \rho_*\right)\,,$$

The pressures γ and ρ_* are taken to be positive and we restrict ourselves to $0 \le \gamma \le 1$.

To find the WDF energy density, we use the energy conservation equation

$$\rho_{\text{WDF}} = \frac{\gamma}{1+\gamma}\rho_* + \frac{c}{v\,(1+v)}\,,$$

where c is the constant of integration and v is the volume expansion WDF naturally includes two components, a piece that behaves as a cosmological constant as well as a standard fluid with an equation of state $p = \gamma\rho$. We can show that if we take $c > 0$, this fluid will not violate the strong energy condition $p + \rho \ge 0$.

$$p_{\text{WDF}} + \rho_{\text{WDF}} = (1+\gamma)\,\rho_{\text{WDF}} - \gamma\rho_* = (1+\gamma)\,\frac{c}{v^{(1+v)}} \ge 0\,.$$

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW case by Holman *et al.* [26]. T. Singh *et al.* [27] studied in Bianchi type I universe with wet dark fluid. Jain *et al.* [28] have been studied in detailed for

Einstein-Rosen universe with wet dark fluid. Also Chaubey [29], Mishra *et al.* [30], Samanta [31], Nimkar *et.al.* [32], Chirde *et.al.* [33], Deo *et al.* [34], Angit *et al.* [35], Dagwal [36] are some of the researchers who have investigated various aspects of wet dark fluid.

The energy momentum tensor of wet dark fluid is given by

$$T_{ij} = (\rho_{\text{WDF}} + p_{\text{WDF}})u_i u_j + p_{\text{WDF}} g_{ij} \,, \tag{40}$$

where u^i is the flow vector satisfying

$$g_{ij} u^i u^j = -1 \,. \tag{41}$$

By using comoving system of coordinates, we get

$$T_1^1 = T_2^2 = T_3^3 = p_{\text{WDF}} \,, \qquad T_4^4 = -\rho_{\text{WDF}} \,. \tag{42}$$

Condition use radiation

$$3p_{\text{WDF}} - \rho_{\text{WDF}} = 0 \,. \tag{43}$$

Solving equations (4) to (9) with the assistance of equations (42) and (43), we obtain metric coefficients, cosmological model, physical properties and graphical representation of the model. But all above results are identical to the results obtained from cosmic string, perfect fluid and thick domain walls also. The values of proper pressure and energy density are

$$p_{\text{WDF}} = \frac{-k_8}{8\pi(k_3 t + k_4)^2} \left(\frac{k_7}{(k_3 t + k_4)^{2n}} + k_6 \right) \,, \tag{44}$$

$$\rho = \frac{k_9}{8\pi(k_3 t + k_4)^2} \left(\frac{k_7}{(k_3 t + k_4)^{2n}} + k_6 \right) \,. \tag{45}$$

7 Solution and Model of Macroscopic Body

The energy momentum tensor of macroscopic body [37] is given by,

$$T^{ij} = (\varepsilon + p)u^i u^j - p g^{ij} \,. \tag{46}$$

Here p is the pressure, ε is the energy density and u_i represents the four velocity vectors of the distribution, respectively

$$T_1^1 = T_2^2 = T_3^3 = -p \,, \qquad T_4^4 = -(2p + \varepsilon) \,. \tag{47}$$

Bianchi type VI_0 does not exist in macroscopic body.

8 Properties of All Models

$$\text{Average scale factor :} \quad a(t) = (k_3 t + k_4)^{\frac{2n+1}{3}} \; ; \tag{48}$$

$$\text{Deceleration parameter :} \quad q = \frac{-2n}{2n+1} \; ; \tag{49}$$

$$\text{Jerk parameter :} \quad J = \frac{4n^2}{(2n+1)^2} \; .; \tag{50}$$

$$\text{Snap parameter :} \quad S = \frac{(2n-2)(2n-5)(2n-8)}{(2n+1)^3} \; ; \tag{51}$$

$$\text{Lark parameter :} \quad l = \frac{(2n-2)(2n-5)(2n-8)(2n-11)}{(2n+1)^4} \; . \tag{52}$$

9 Graphical Illustration

Graphical illustration of spatial volume, shear scalar, Hubble parameter and expansion scalar with time of cosmic string, perfect fluid, wet dark fluid and domain walls are shown in Figure 3.

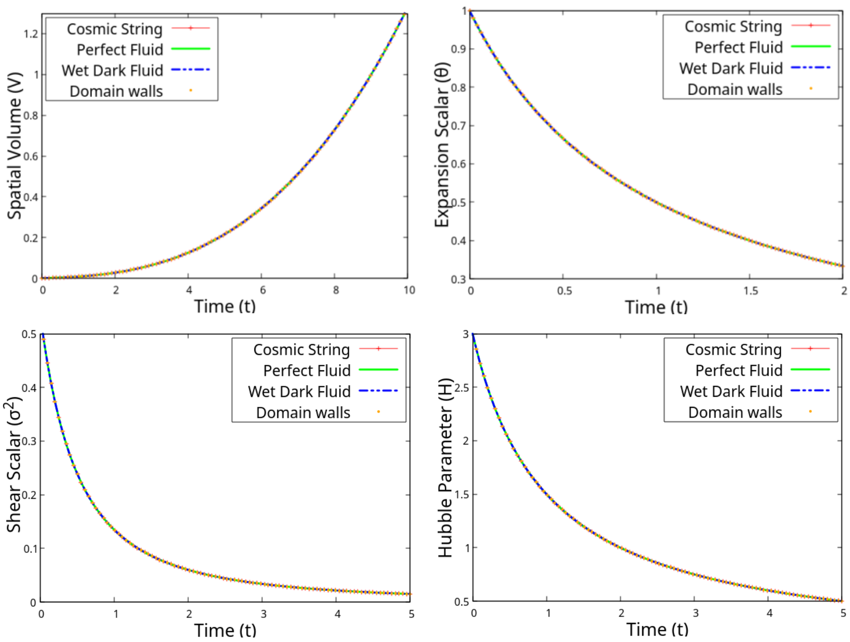


Figure 3.

Figure 3 shows that volume increases with time. Volume $V \to \infty$ as $t \to \infty$. Hubble parameter decreases with increasing time. The expansion scalar and shear scalar are decreasing with increase in time and both vanishes as $t \to \infty$.

Also the graph of average scale factor vs. time is shown in Figure 4

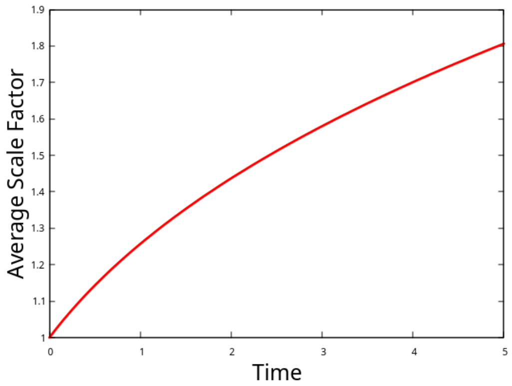


Figure 4. Plot of average scale factor versus time ($k_3 = k_4 = 1$).

10 Observational Parameters of All Models

Look-back time redshift: Look-back time is defined as the difference between present age of universe t_0 and the age of the universe t when a particular light ray at redshift z was emitted. It depends on the dynamics of the universe.

$$t_L = t_0 - t\,, \tag{53}$$

where t_0 is the present age of universe and z denotes redshift of light well measured quantity of a far distant object such as galaxies. The redshift of light is emitted due to expansion of universe. For given redshift z, the average scale factor of the universe $a(t)$ is related to the present scale factor of the universe $a_0(t)$

$$1 + z = \frac{a_0(t)}{a(t)}\,. \tag{54}$$

Equations (48), (53) and (54) give

$$t_L = \frac{k_3\,(2n+1)}{H_0}\left[1 - \frac{1}{(1+z)^{\frac{3}{2n+1}}}\right]\,. \tag{55}$$

Luminosity distance redshift: The Luminosity distance of light source is given

$$H_0 d_L = \frac{-3k_3\,(2n+1)}{(2n-2)}\left[1 - (1+z)^{\frac{2n-2}{2n+1}}\right](1+z)\,. \tag{56}$$

Figure 6 shows that the Luminosity distance increase faster with the redshift.

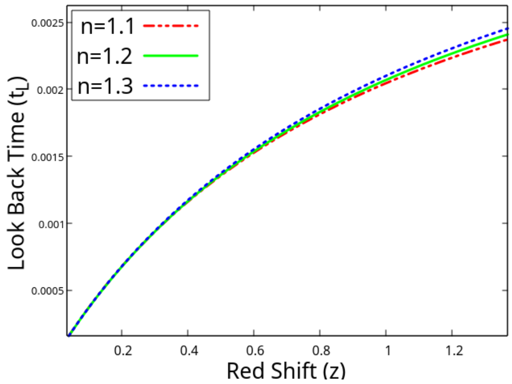


Figure 5. Plot of look-back time vs. redshift for $k_3 = 0.1$, $H_0 = 75$.

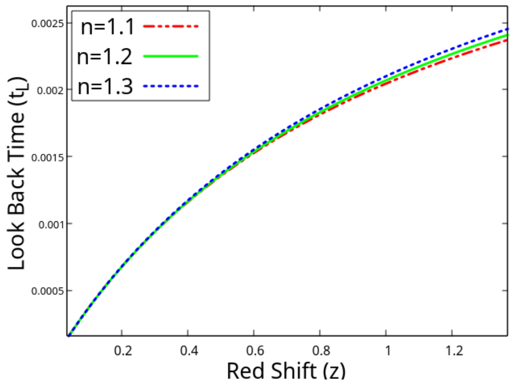


Figure 6. Plot of luminosity distance time vs. redshift for $k_3 = 0.1$, $H_0 = 75$.

Stability Analysis of Models except Cosmic String

The stability of the model are discuss by observing the ratio of sound speed given by $dp/d\rho$.

When the ratio $dp/d\rho$ is positive, we have stable model whereas when the ratio $dp/d\rho$ is negative, we have an unstable model.

(i) If we use the equation of state to obtain the values of p and ρ, then the value of the ratio $dp/d\rho$ is observed to be positive hence the model must be stable.

(ii) If we use the values of p and ρ obtained from equations (4) and (7), then the ratio $dp/d\rho$ is observed to be negative hence the model is unstable.

11 Conclusion

Energy momentum tensor and space-time associated with them have cosmological interest due to their important applications in structure formation of the universe. Also, it is well known that scalar fields have considerable effects in the early stages of revolutionary universe. Here we have presented Bianchi type VI_0 cosmological models in self creation theory of gravitation proposed by Barber [1] with cosmic string, perfect fluid, thick domain walls, wet dark fluid and macroscopic body. The models in four energy momentum tensor are similar and behave alike except macroscopic body. Also discussed some physical and kinematical properties and graphical representation of cosmic string, perfect fluid, thick domain walls and wet dark fluid. All these models studied here will be useful for a better understanding of self-creation cosmology and structure formation of the universe. Also, when p and ρ are obtained from equation of state, the value of the ratio $dp/d\rho > 0$ is observed to be positive hence the given cosmological model must be stable. Otherwise the value of the ratio $dp/d\rho$ is negative hence the model is unstable.

It may be observed that at initial moment, when $t = 0$, the spatial volume will be zero while energy density and pressure diverge. When t tends to zero, then the expansion scalar, shear scalar and Hubble's parameters tends to infinity. For large value of t, we observe that expansion scalar, shear scalar and Hubble parameters become zero.

Also, $\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$.

The model does not approach isotropy for large value of t.

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